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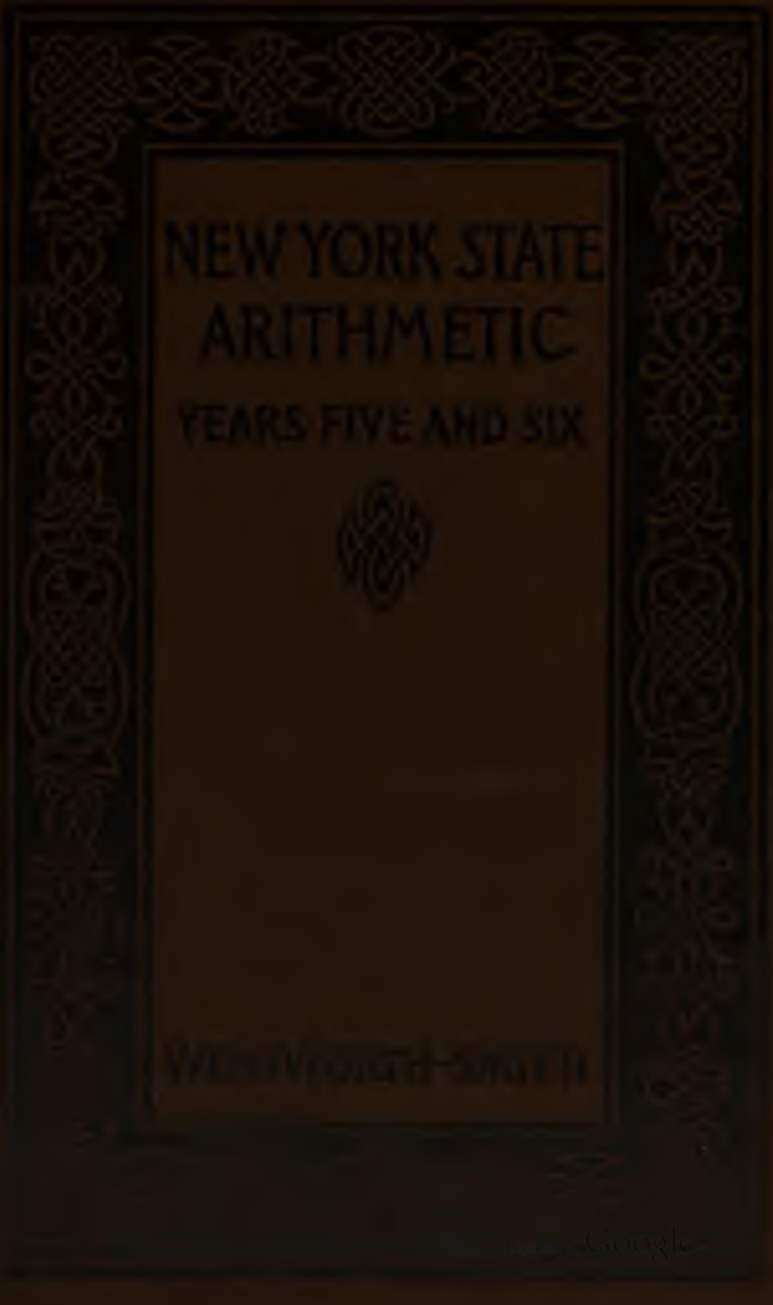

NEW YORK STATE ARITHMETIC

YEARS FIVE AND SIX

WENTWORTH-SMITH

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WENTWORTH-SMITH MATHEMATICAL SERIES

NEW YORK STATE ARITHMETIC

YEARS FIVE AND SIX

BY

GEORGE WENTWORTH

AND

DAVID EUGENE SMITH

GINN AND COMPANY
BOSTON · NEW YORK · CHICAGO · LONDON

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PREFACE

In deciding upon the plan for a textbook to meet the needs of the schools of New York state, the authors have been guided by the Syllabus in Arithmetic issued by the State Education Department. With the Syllabus constantly before them they have prepared what they believe to be a thoroughly usable book. They believe that this book, with its clear-cut topical arrangement, will appeal to teachers and pupils alike, and that it will meet the needs of the schools in every respect.

The rule that has guided the authors in their other works is followed in this one: Present the reason briefly but clearly, and then furnish such an amount of practice that the pupil cannot forget the principle. Theory is reduced to a minimum, and practice is abundant, with both abstract and concrete numbers, since the experience of a century proves beyond question that each of these two lines must be treated fully to prepare the pupils for their future work. This is the spirit of the Syllabus, and it is a thoroughly sound one, representing as it does the sanest modern educational thought.

The authors have always insisted that all applied problems should appeal as far as possible to the pupil's interests. In conformity with the spirit of this principle the concrete problems are modern in the best sense. Those that seem to be real are real; modern business customs are followed, and the needs of the future citizen are always kept in mind. But in this book is found no trace of the unfortunate tendency of some of our recent writers to arrange their work without system, to withhold from the pupil the feeling of mastery which is his due, and to consider a problem concrete when it treats of topics or contains technicalities that no pupil understands and few teachers are expected to know.

The book covers exactly the topics, grade by grade, laid down in the Syllabus, and also provides for a review of the fundamental operations with integers (pages 1–56), to be used or not as occasion demands. Under each topic will be found an unusually large number of well-graded examples, and at frequent intervals collections of problems without numbers have been inserted as exercises in terse mathematical statement.

Any corrections or suggestions relating to the work will be thankfully received.

The authors hope that teachers who recognize the strong features of the topical arrangement, who wish for an abundance of well-graded problems requiring thought in their solution, who are sympathetic with the movement to replace what is obsolete by the genuine applications of arithmetic to the American life of to-day, who are opposed to certain of the extreme attempts of the present time that are sure to result disastrously to scholarship — that such teachers will find in this work a sane, modern, and helpful treatment of the subject.

GEORGE WENTWORTH
DAVID EUGENE SMITH

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THE NEW YORK STATE SYLLABUS

The following synopsis shows how the work is arranged to meet the requirements of the Syllabus. Schools not feeling the necessity for reviewing the work in the fundamental operations with integers should begin with page 57.

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NEW YORK STATE ARITHMETIC

YEARS FIVE AND SIX

CHAPTER I

WRITING AND READING NUMBERS

1. Units. The standards by which we count or measure are called *units.*

Thus in speaking of $4 the unit is $1; in speaking of 5 eggs the unit is 1 egg, but in speaking of 10 dozen eggs the unit is 1 dozen eggs; in speaking of 40 thousand bricks the unit is a thousand bricks; in counting 1, 2, 3, 4, and so on, the unit is 1; in measuring long distances the unit may be 1 mile.

2. Numbers. That which shows how many times a unit is taken, or what part of a unit is taken, is called a *number.*

Thus 3 indicates that 1 has been taken 3 times; $6 indicates that $1 has been taken 6 times; $\frac{4}{5}$ indicates that four fifths of 1 is taken; hence 3, $6, and $\frac{4}{5}$ are all numbers.

3. Abstract and Concrete Numbers. A number used without reference to any particular unit is called an *abstract number;* if the unit is named, the whole expression is called a *concrete number.*

For example, 5 is an abstract number, but $5 and 5 bushels are concrete numbers. It must be understood, however, that the true number part is always abstract. That is, in $5 the true number part is 5. It is convenient, however, in explaining the solution of problems to make the distinction between abstract numbers and concrete numbers.

4. Integers. Numbers applied to whole units are called *whole numbers*, *integral numbers*, or *integers*.

Fractional numbers, already familiar to the class from work in the earlier grades, and mentioned in § 2, are treated later.

5. Notation and Numeration. The writing of numbers is called *notation;* the reading of numbers is called *numeration.*

6. Kinds of Notation. The class is already familiar with the common or Arabic notation (so called because it came to Europe through the Arabs), and the Roman notation.

The Arabic characters, sometimes called *figures* or *ciphers*, originated in India. They were employed without the zero more than 2000 years ago, and the zero or naught has been in use over 1200 years. These figures have been known in Europe more than 900 years, but did not become generally used until about 400 years ago, when they finally displaced the clumsy notation of the ancient Romans.

7. Numerals. The characters used in any system of notation are called *numerals.*

Thus we speak of Arabic numerals and of Roman numerals. The Arabic numerals are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The Roman numerals are I, V, X, L, C, D, M. The Arabic characters, excluding 0, are sometimes called *digits.*

8. Place Value. The value of a figure in a number written in Arabic numerals depends upon its position in the number and is called the *place value* of the figure.

Thus, in the number 53, the 3 occupies units' place and means 3 units; the 5 occupies tens' place and means 5 tens. In the Roman system VIII, although made up of V (five) and III (three), like 53, means only eight, because the numerals have no such place value.

9. Orders of Units. Each successive place in a number that may be occupied by a figure is called an *order of units.*

These orders increase from right to left in a tenfold ratio. Hence the Arabic system is called a *decimal system* (from the Latin *decem*, ten). The ones (or units) are called *units of the first order;* the tens, *units of the second order;* and so on.

10. Names of the Orders. As already learned in school, the number names of the first few orders are as follows:

Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units
1	4	3	2	6	5	7	1

11. Periods. When the figures of a number are five or more, we separate them into groups of three figures each by commas or by spaces, beginning at the right. These groups are called *periods*.

Thus we write thirty-two thousand, seven hundred fifty-six, 32,756; it is often printed with a space instead of the comma, thus, 32 756.

12. Separatrix. The comma separating two periods of a number is called the *separatrix*.

13. Names of the Periods. Beginning to name the periods at the right, the first period is called the *units' period;* the second, the *thousands' period;* the third, the *millions' period;* and the fourth, the *billions' period.*

We rarely use number names above millions. The names of the periods above billions are trillions, quadrillions, quintillions, sextillions, septillions, and so on; but these need not be learned.

14. Reading Numbers. In reading numbers we read each period by itself and add the name of the period. Avoid all useless words.

Thus we read 2,341,406 "two million, three hundred forty-one thousand, four hundred six." It is customary to omit the name of the units' period.

EXERCISE 1

Read aloud the following numbers:

1. 46.
2. 85.
3. 70.
4. 24.
5. 132.
6. 201.
7. 357.
8. 777.
9. 1234.
10. 1001.
11. 2052.
12. 3030.
13. 52,176.
14. 31,427.
15. 50,050.
16. 22,222.
17. 215,215.
18. 600,600.
19. 303,303.
20. 123,456.
21. 1,250,000.
22. 2,500,000.
23. 3,333,333.
24. 10,010,010.
25. 12,000,112.
26. 25,150,150.
27. $25,000,000.
28. $32,125,000.
29. $57,000,250.
30. $92,350,206.

Read aloud the following sentences:

31. The area of Alabama is 52,250 square miles.
32. The area of Arkansas is 53,850 square miles.
33. The area of Florida is 58,680 square miles.
34. The area of Georgia is 59,475 square miles.
35. The area of Indiana is 36,350 square miles.
36. The area of Iowa is 56,025 square miles.
37. The area of Kansas is 82,080 square miles.
38. The area of Kentucky is 40,400 square miles.
39. The area of Minnesota is 83,365 square miles.
40. The area of Nevada is 110,700 square miles.
41. The area of New York State is 49,170 square miles.
42. The area of Texas is 265,780 square miles.
43. The area of the United States is 3,616,484 square miles.

15. Writing Numbers in Arabic Figures. The class has already learned to write numbers. Beginning at the left, we write each period by itself, followed by a comma, putting a zero in every vacant place.

Thus, to write thirty million, forty thousand, seven hundred six, we first write the order of millions, 30, followed by a comma; then 40 thousand, placing a zero in the place of hundred-thousands; then, after the comma, 706. The number then stands as follows:

30,040,706.

EXERCISE 2

Write in figures, arranged in periods, the following:

1. Five thousand, eight.
2. Nine thousand, fifty-seven.
3. Seven thousand, two hundred.
4. Eight thousand, one hundred one.
5. Twelve thousand, four hundred forty.
6. Fifteen thousand, six hundred fifty-six.
7. Eighteen thousand, seven hundred forty-nine.
8. Twenty-five thousand, eight hundred eleven.
9. One hundred fifty-two thousand, eighty-eight.
10. Two hundred six thousand, four hundred one.
11. Four hundred forty thousand, one hundred six.
12. Eight hundred seventy-five thousand, twenty.
13. Nine hundred ninety-nine thousand, eighty-four.
14. One million, one thousand, one hundred one.
15. Three million, three hundred fifty-five thousand, six.
16. Twenty-three million, two hundred forty-eight thousand, six hundred forty-four.

17. Eighty-four million, seven hundred six.

18. Forty-two million, one thousand, one hundred.

19. Seventy-eight million, two hundred forty thousand.

20. Sixty-eight million, eight hundred seventy-three thousand.

21. Ninety-one million, two hundred thousand, one hundred fifty-one.

22. One hundred forty million, seven hundred forty-six.

23. Two hundred thirty-three million, one hundred seventeen thousand, eight.

24. Two hundred twenty-two million, two thousand, two hundred two.

25. Three hundred twenty-four million, five hundred sixty-seven thousand, eight hundred ninety.

26. Three hundred ninety-two million, six hundred forty thousand, five hundred seventy-nine.

27. Four hundred forty-four million, two hundred sixty-seven thousand, three hundred eighty-seven.

28. Five hundred twenty-eight million, six hundred twenty-six thousand, eight hundred forty-seven.

29. Nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one.

30. Seven hundred seventy-two million, four hundred seventy-three thousand, five hundred eighty-nine.

31. Eight hundred eighty-eight million, eight hundred eighty-eight thousand, eight hundred eighty-eight.

32. One hundred ninety-eight billion, seven hundred sixty-five million, four hundred thirty-two thousand, one hundred fifty-seven.

16. Writing United States Money. In writing United States money the *dollar sign* ($) is written before the number. A period, called a *decimal point*, is placed between the dollars and the dimes.

For example, 125 dollars is written $125; 10 dollars and 48 cents (that is, 10 dollars, 4 dimes, 8 cents) is written $10.48; 5 dollars and 8 cents is written $5.08.

EXERCISE 3

Read aloud the following:

1. $15.
2. $24.
3. $40.
4. $90.
5. $2.70.
6. $1.30.
7. $4.60.
8. $5.20.
9. $2.25.
10. $3.75.
11. $6.42.
12. $7.28.
13. $5.05.
14. $7.09.
15. $3.04.
16. $9.08.
17. $21.00.
18. $35.25.
19. $42.75.
20. $33.03.
21. $125.50.
22. $325.75.
23. $2450.25.
24. $3575.50.
25. $4825.25.
26. $7275.05.
27. $427.82.
28. $5207.60.
29. $4270.07.
30. $500.05.
31. $5000.05.
32. $9999.99.

33. In 1800 the public debt of our country was $15.63 for each inhabitant; in 1900 it was $14.52.

34. The average amount for each depositor in the savings banks of the United States recently was $429.64.

35. Recently the public debt of the United States was $2,492,231,518.54.

36. The average pay of locomotive engineers on the railroads in our country is about $4.35 a day; conductors average about $3.75 a day.

37. In a certain year the United States produced iron and steel to the value of $804,034,918.75, the value of that produced by the Middle Atlantic States being $478,687,519.80.

17. The Roman Numerals. The Roman notation is used chiefly for numbering chapters and for dates. It employs seven capital letters, as follows:

Letters,	I	V	X	L	C	D	M
Values,	1	5	10	50	100	500	1000

The first nine numbers are written thus:

I, II, III, IV or IIII, V, VI, VII, VIII, IX.

The tens are written thus:

X, XX, XXX, XL, L, LX, LXX, LXXX, XC.

The hundreds are written thus:

C, CC, CCC, CD, D, DC, DCC, DCCC, DCCCC or CM.

The numbers from eleven to nineteen are written thus:

XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, XIX.

The following are examples of other numbers:

XXII = 22
XXXIV = 34
XLIX = 49
XCVIII = 98
LXXV = 75
LXXXI = 81
CLXV = 165
CCLIX = 259
MDCCXC = 1790
MDCCCCXI or MCMXI = 1911

EXERCISE 4

Write in Arabic figures:

1. XIV.
2. XXIX.
3. XXXI.
4. LIX.
5. LXVI.
6. LXXV.
7. LXXX.
8. XCVI.
9. DCCXI.
10. MDCC.
11. MDCXX.
12. MDCCCCX.

Write in Roman numerals:

13. 26.
14. 32.
15. 50.
16. 69.
17. 81.
18. 99.
19. 135.
20. 241.
21. 363.
22. 525.
23. 1892.
24. 1928.

CHAPTER II

ADDITION OF INTEGERS

18. Addition. You have already learned how to add numbers and are now ready to read the definition of addition. The operation of finding a number equal to two or more numbers taken together is called *addition.*

Thus, to add 7 and 8 is to find a number, 15, that is equal to these numbers taken together.

$$\begin{array}{r}7\\8\\\hline 15\end{array}$$

19. Sum. The result obtained by adding is called the *sum.*

Thus 15 is the sum of 7 and 8.

20. Symbol. The symbol of addition is this form of a cross, +. It is read *plus,* a word meaning *more.*

$$\begin{array}{r}7\\+8\\\hline 15\end{array}$$

The expression $7 + 8 = 15$, as you have long ago learned, is read "7 plus 8 equals 15." Sometimes in school the plus sign is used to indicate addition in a column, as here shown.

21. Like Numbers. Numbers applied to the same unit are called *like numbers.*

Thus $3 and $5 have the same unit, $1; 3 feet and 2 feet have the same unit, 1 foot; 7 and 15 have the same unit, 1.

22. Addends. The numbers added are called *addends. Only like numbers can be added and the sum is like the addends.*

That is, we cannot add $2 and 3 yards, or 7 pounds and 4 books. If we add $12 and $13, the sum must be like the addends; that is, it must be $25, not 25 feet. It is true that we can put 4 books with 7 crayons, but we cannot add them so as to get 11 books or 11 crayons; we have to think of them only as *things*, and say that we have 11 things.

23. Addition of Digits. You have already learned how to add digits, but the following directions will help you:

(1) *First add from the bottom upwards and write the sum.*

(2) *Then add from the top downwards, to see that you have made no mistake.*

This is called *checking* the result, and is very important in all calculations.

(3) *Try to group digits in adding, thus allowing you to work more rapidly.*

For example, in this case we may begin with 6 and add each digit in order, thinking thus: "6, 15, 18, 25, 29." It is better, however, to look at the 6 and 9 and think "15"; then look at the 7 and 3 and think of 10 as added to the 15, saying to ourselves "25"; then to look at 4 and think "29."

$$\begin{array}{r}4\\7\\3\\9\\6\\\hline 29\end{array}$$

EXERCISE 5

Add the following, checking your work and timing yourself:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
2	3	4	3	8	9	9	8	4	2	4	2
3	4	2	9	2	8	2	8	6	1	2	8
5	7	8	7	8	7	3	8	5	4	6	7
7	6	9	7	3	6	1	2	5	7	3	6
9	2	3	4	9	5	4	2	3	3	4	3

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
3	7	6	4	4	7	5	6	7	3	8	4
4	2	8	1	6	7	2	8	8	4	2	7
2	4	3	2	4	8	8	3	2	2	9	8
6	1	4	8	5	8	7	9	8	8	1	3
4	1	9	7	3	2	2	4	9	7	7	9
3	2	2	3	2	3	4	5	3	9	3	2

Copy the numbers in lines 25 to 35 inclusive, and add; then add the columns numbered 36 to 43 inclusive:

	36.	37.	38.	39.	40.	41.	42.	43.
25.	2	3	2	2	3	9	8	1
26.	1	4	1	9	8	1	4	3
27.	3	7	3	3	4	2	9	5
28.	4	7	4	9	4	8	2	6
29.	2	8	3	4	1	3	7	6
30.	8	2	2	1	9	4	3	6
31.	7	3	4	2	8	1	6	9
32.	6	4	8	4	3	9	5	2
33.	4	6	6	6	2	7	8	3
34.	1	2	1	3	7	6	5	4
35.	2	3	4	2	6	2	7	5

Proceed in the same way for lines numbered 44 to 57 inclusive, and for columns numbered 58 to 65 inclusive:

	58.	59.	60.	61.	62.	63.	64.	65.
44.	1	8	6	2	4	7	3	5
45.	2	5	9	3	8	4	7	6
46.	3	4	1	5	8	6	9	7
47.	2	7	4	1	7	6	3	5
48.	7	1	5	2	8	3	9	4
49.	1	3	5	6	2	8	4	7
50.	2	4	6	3	9	5	8	7
51.	4	2	3	8	5	7	9	6
52.	3	1	2	8	6	4	7	5
53.	2	6	6	4	7	8	3	9
54.	7	9	1	6	2	4	3	5
55.	8	2	3	6	9	5	7	4
56.	1	4	5	2	8	7	3	6
57.	9	1	4	6	2	5	7	3

24. Addition of Two-Figure Numbers. You have also learned how to add two-figure numbers, but the following directions may help you to understand the work:

(1) *Write the addends so that units of the same order are in the same column.*

(2) *Add each column by itself, beginning at the right, and write the sum, if it is less than 10, under the column added.*

(3) *If the sum of any column is 10 or more, write the unit figure under the column added, and add the tens with the next column.*

67	
86	
34	
17	sum of units
17	sum of tens
187	sum

This is done in full in the example shown above. In practice, however, we do not write the sum of each column by itself, but proceed as here shown. We think of the work as follows: "4, 10, 17," writing the 7 and adding the 1 to the tens' column; then think: "1, 4, 12, 18," and write the 8 under the tens' column and the 1 in the hundreds' place.

$67
86
34
$187

The dollar sign before the top number in a column indicates that all the numbers in the column represent dollars.

EXERCISE 6

Add the following and check your work:

1.	2.	3.	4.	5.	6.
21	30	6	34	72	$32
32	47	57	9	89	21
46	22	61	68	67	49

7.	8.	9.	10.	11.	12.
34	62	75	62	47	$25
62	48	25	48	63	30
81	31	30	8	92	35
92	43	7	75	81	40

Copy the numbers in lines 13 to 22 inclusive and add; then add the columns numbered 23 to 30 inclusive:

	23.	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
13.	10	11	14	34	15	20	31	12
14.	22	12	33	21	22	34	6	9
15.	13	21	20	12	31	26	15	34
16.	31	23	6	51	28	35	42	55
17.	42	31	14	62	61	77	82	69
18.	14	43	35	71	38	56	77	86
19.	31	37	7	33	46	59	68	77
20.	22	42	62	14	84	73	82	91
21.	30	61	70	68	59	38	47	65
22.	50	12	15	73	77	82	39	46

Proceed in the same way for lines numbered 31 to 45 inclusive, and for columns numbered 46 to 53 inclusive:

	46.	**47.**	**48.**	**49.**	**50.**	**51.**	**52.**	**53.**
31.	12	20	23	14	86	77	80	78
32.	6	10	15	17	83	85	90	94
33.	7	9	12	23	87	88	91	93
34.	13	15	14	36	42	23	31	46
35.	21	31	21	41	28	17	16	20
36.	42	24	33	23	14	23	41	26
37.	9	16	40	31	15	17	48	62
38.	8	33	62	14	26	33	27	15
39.	10	27	19	17	14	23	73	85
40.	23	18	28	62	17	15	52	38
41.	37	26	30	31	13	89	59	74
42.	52	58	42	48	58	77	84	80
43.	46	39	71	73	72	83	69	54
44.	51	62	16	92	86	77	9	7
45.	73	47	86	87	85	83	10	16

25. Addition of Larger Numbers. You have learned how to add numbers of more than two figures, and the directions given on page 12 for adding two-figure numbers apply to all cases.

Thus, to add 2936 and 8798 we might proceed as shown in the left-hand solution, but the process shown in the right-hand solution is better because it is shorter and allows the work to be performed more rapidly.

2936	
8798	
14	sum of units
12	sum of tens
16	sum of hundreds
10	sum of thousands
11734	sum

2936
8798
11734

EXERCISE 7

Add the following, checking your work and timing yourself:

1.	2.	3.	4.	5.
307	607	426	462	336
268	73	427	617	774
111	803	700	296	340
702	727	99	287	760

6.	7.	8.	9.	10.
4732	2732	596	4884	8492
6239	698	69	6939	3687
5268	1596	3492	4008	4009
3761	998	9873	7227	698

11.	12.	13.	14.	15.
64,283	41,294	28,009	43,218	52,988
2,796	58,706	77,986	37,298	13,678
12,426	14,261	20,811	42,361	60,211
20,431	2,143	30,021	20,000	72,010

Add the following:

16.	**17.**	**18.**	**19.**	**20.**
73,296	28,428	44,307	30,072	78,327
41,427	73,211	62,320	14,826	4,162
33,291	70,092	40,072	4,999	899
41,627	61,778	63,142	12,000	10,090
20,623	31,732	40,012	30,200	41,009

21. Recently we had 9196 steamers and 12,831 other American vessels engaged in trade along our coast. What was the total number?

22. In our country there were reported as built recently in one year 147 sailing vessels, 674 steam vessels, 62 canal boats, and 274 barges, not counting many smaller boats. What was the total of those reported?

23. In a recent year our country sold abroad $32,305,412 worth of manufactured cotton, and $2,287,936 worth of manufactured wool. What was the total value?

The following table shows certain statistics relating to our country in 1800, and the increase in 100 years. Add the increase to the column for 1800 and find the statistics for 1900.

	1800	Increase
24. Number of square miles	827,844	2,198,945
25. Number of bales of cotton grown	155,556	9,280,860
26. Receipts of post offices	$280,804	$102,073,775
27. Gold and silver money	$16,000,000	$736,856,806
28. Total circulation of money	$26,500,000	$2,028,650,998
29. Population	5,308,483	70,994,904

26. Addition of United States Money. In adding United States money simply proceed as with ordinary integers, keeping units of the same order in the same column as here shown.

$12.50
0.75
23.
1.05
$37.30

We can easily keep the units of the same order in the same column by placing the decimal points under one another.

EXERCISE 8

Add the following:

1.	2.	3.	4.	5.
$2.34	$1.45	$7.40	$5.06	$6.92
1.35	0.75	3.80	1.04	7.21
4.26	3.00	6.20	3.07	3.87
5.72	1.26	5.90	4.83	4.

6.	7.	8.	9.	10.
$2.93	$6.42	$5.42	$2.50	$23.50
3.81	7.08	0.58	3.62	7.50
4.26	3.91	0.39	0.48	18.75
5.35	0.75	3.27	4.71	32.10
0.75	0.25	0.58	3.	4.05
4.	6.	1.25	1.96	5.10

11.	12.	13.	14.	15.
$6.42	$4.08	$5.25	$2.93	$14.08
7.81	2.75	1.27	4.62	0.92
3.10	3.02	3.06	8.21	26.29
2.07	6.72	4.81	4.26	53.07
0.56	8.05	2.72	8.02	62.75
0.82	9.	3.25	7.33	41.05
1.49	0.33	4.05	0.85	35.
7.23	6.44	6.00	3.82	14.75

CHAPTER III

SUBTRACTION OF INTEGERS

27. Subtraction. The process of finding what number must be added to one of two numbers to make the other, or of taking one number from another, is called *subtraction.*

28. Symbol. The symbol of subtraction is a short horizontal line, —. It is read *minus,* a word meaning *less.*

Thus 17 — 9 = 8 is read "17 minus 9 equals 8."

29. Terms. The smaller of the two given numbers in subtraction is called the *subtrahend;* the larger one is called the *minuend;* the result of the subtraction is called the *remainder* or *difference.* To *check* the result, add the subtrahend and remainder; the result should be the minuend.

30. The Addition Method of Subtraction. This method, also called the Austrian or making-change method, is recommended in the New York State Syllabus. It consists simply in finding, in the case of 468 — 143, what must be added to 143 to make 468. We proceed as follows: 3 + 5 = 8; 4 + 2 = 6; 1 + 3 = 4.

468	minuend
143	subtrahend
325	remainder

31. The Taking-away Method. This is, at present, a more familiar method of subtraction. Here we say: 3 from 8, 5; 4 from 6, 2; 1 from 4, 3.

Of the two methods the first is always used in making change, it is the more rapid when thoroughly learned, and it is growing in favor. When a pupil has already acquired one method, however, and works rapidly and accurately, it is not good policy to attempt to change it.

EXERCISE 9

Subtract, checking your work and timing yourself:

1.	**2.**	**3.**	**4.**	**5.**
673	742	396	485	981
261	420	104	172	651
6.	**7.**	**8.**	**9.**	**10.**
908	659	485	398	667
702	128	152	287	421
11.	**12.**	**13.**	**14.**	**15.**
$7.25	$5.36	$9.28	$6.75	$8.82
1.02	2.15	1.05	1.25	2.31
16.	**17.**	**18.**	**19.**	**20.**
$15.75	$23.50	$37.55	$52.85	$75.88
2.20	2.40	15.20	21.70	13.52
21.	**22.**	**23.**	**24.**	**25.**
$82.96	$95.63	$86.75	$73.92	$48.52
71.42	15.02	14.25	13.92	24.10
26.	**27.**	**28.**	**29.**	**30.**
$175.40	$142.65	$235.75	$468.92	$536.48
21.30	21.02	101.20	107.02	121.25

31. 62,815 – 11,702.
32. 81,482 – 70,141.
33. 96,348 – 62,123.
34. 27,984 – 14,521.
35. 31,628 – 20,407.
36. 82,489 – 70,126.
37. 68,925 – 61,420.
38. 34,876 – 10,724.
39. 42,929 – 30,009.
40. 99,999 – 47,628.

41. A grocer had 1360 pounds of sugar and sold 1040 pounds. How many pounds had he left?

42. A farmer had 2445 bushels of corn and sold 2120 bushels. How many bushels had he left?

43. A man's income for a year was $1975 and his expenses were $1630. How much had he left?

44. A dealer had 2796 tons of coal and sold 1385 tons. How many tons had he left?

45. A dry goods merchant had 1440 spools of thread and sold 1200 spools. How many spools had he left?

46. A city school had 1296 pupils of whom 184 were in the first-year class. How many were in all the other classes together?

47. A real estate dealer had 1278 acres of land for sale, and sold 1163 acres. How many acres had he left unsold?

48. The distance from New York to Albany is 143 miles, and from New York to Chicago is 975 miles. How far is it from Albany to Chicago by this route?

49. The area of Indiana is 36,350 square miles, and that of Rhode Island 1250 square miles. What is the difference in area?

50. The area of Alabama is 52,250 square miles and that of California is 158,360 square miles. What is the difference in area?

51. The area of Nebraska is 77,510 square miles, and that of South Dakota is 77,650 square miles. What is the difference in area?

52. In a recent year Pennsylvania produced 31,816,496 bushels of oats, and Massachusetts 214,472 bushels. What was the difference in the amounts?

32. General Case of Subtraction. Suppose we wish to find the difference between 652 and 476. Here we may use either of the methods stated in § 30 and § 31.

(1) The addition (Austrian) method uses this principle: *If the same number is added to both minuend and subtrahend, the remainder is not changed.* Thus 8 – 2 gives the same result as 18 – 12 or 108 – 102.

Taking the case of 652 – 476, we now consider what number added to 6 makes 2. Since it is impossible to find such a number, what number added to 6 makes 12? Evidently 6, which we write below. But we increased the minuend from 2 to 12 by adding 10, so we add 10 to the subtrahend, making the 7 into an 8. Then, since no number added to 8 makes 5, what number added to 8 makes 15? Evidently 7, which we write below. But we increased the minuend from 5 tens to 15 tens, by adding 100 (or 10 tens), so we add 100 to the subtrahend, making the 4 a 5. Then 5 + 1 = 6, and we write the 1 below.

$$\begin{array}{rl} 652 & \text{minuend} \\ \underline{476} & \text{subtrahend} \\ 176 & \text{remainder} \end{array}$$

(2) The taking-away method. Considering the same problem, we proceed as follows: Since we cannot take 6 from 2 we change one of the 5 tens to units, and this with the 2 units makes 12 units; then 12 – 6 = 6. Since we have used one of the tens in the 5 tens we have 4 tens left, from which we cannot take 7 tens. We change one of the 6 hundreds to tens, and this with the 4 tens makes 14 tens; then 14 tens – 7 tens = 7 tens. We now have 5 hundreds left of the 6 hundreds, and 5 hundreds – 4 hundreds = 1 hundred. The remainder is therefore 176. It will make the work clearer to see the numbers separated as here shown, but this is not done in practice.

$$\begin{array}{rcrcr} 500 & + & 140 & + & 12 \\ 400 & + & 70 & + & 6 \\ \hline 100 & + & 70 & + & 6 \end{array}$$

33. Special Cases. In a case like 5000 – 2765 we may shorten the work by thinking of the 5000 as 4990 + 10, and of 2765 as 2760 + 5, beginning to subtract at the left.

This amounts to taking the 2 from 4 and every other figure from 9 except the units, which we take from 10. The remainder is 2235.

$$\begin{array}{rrrr} 4 & 9 & 9 & 10 \\ 2 & 7 & 6 & 5 \\ \hline 2 & 2 & 3 & 5 \end{array}$$

In subtracting United States money place the decimal points under one another and proceed as with integral numbers.

EXERCISE 10

Subtract, checking your work and timing yourself:

1. 274 — 62.
2. 378 — 69.
3. 465 — 57.
4. 582 — 44.
5. 673 — 65.
6. 766 — 49.
7. 857 — 28.
8. 939 — 47.
9. 354 — 62.
10. 268 — 85.
11. 325 — 63.
12. 436 — 94.
13. 742 — 70.
14. 819 — 37.
15. 352 — 63.
16. 603 — 25.
17. 760 — 75.
18. 504 — 68.
19. 320 — 33.
20. 404 — 27.
21. 570 — 92.
22. 700 — 30.
23. 600 — 43.
24. 300 — 56.
25. 400 — 90.
26. 500 — 99.
27. 638 — 126.
28. 427 — 127.
29. 636 — 318.
30. 342 — 225.
31. 596 — 379.
32. 758 — 449.
33. 342 — 207.
34. 467 — 208.
35. 621 — 416.
36. 733 — 329.
37. 570 — 444.
38. 392 — 288.
39. 340 — 225.
40. 628 — 419.
41. 492 — 284.
42. 462 — 373.
43. 707 — 528.
44. 981 — 694.
45. 606 — 428.
46. 702 — 513.
47. 603 — 427.
48. 402 — 236.
49. 520 — 348.
50. 630 — 469.
51. 800 — 532.
52. 900 — 687.
53. 2782 — 120.
54. 3648 — 439.
55. 5276 — 168.
56. 8496 — 229.
57. 2349 — 256.
58. 4872 — 380.
59. 6463 — 291.
60. 4486 — 395.
61. 7286 — 193.
62. 2872 — 385.
63. 5342 — 264.
64. 3353 — 296.
65. 4374 — 287.
66. 6681 — 599.
67. 6720 — 535.
68. 7872 — 5785.
69. 3526 — 2239.
70. 7280 — 2385.
71. 6807 — 4938.
72. 38,265 — 23,406.
73. 34,635 — 25,296.
74. 48,406 — 26,573.
75. 62,068 — 48,679.
76. 85,723 — 67,836.
77. 92,631 — 75,842.
78. 80,050 — 69,967.

EXERCISE 11

1. The sum of two numbers is 2872 and the lesser is 929. What is the greater number?

2. The greater of two numbers is 17,214 and their difference is 2685. What is the lesser number?

3. If the minuend is 15,293 and the subtrahend 12,525, what is the remainder?

4. If the subtrahend is 32,658 and the minuend 61,229, what is the remainder?

5. If the minuend is 21,712 and the remainder 2564, what is the subtrahend?

6. If the subtrahend is 16,567 and the remainder 7543, what is the minuend?

7. A merchant sold some goods for $2962.50, and gained $548.75. What did they cost him?

8. What must be added to the sum of $175.50 and $62.75 to make $300?

9. What must be added to the sum of $62.30, $41.75, and $25, to make $500?

10. How much greater is the sum of $75.30 and $21.75, than the difference between these amounts?

11. What is the difference between $725.25 and the sum of $15.50, $17.92, $25.36, $48.75, $19.36, and $42.75?

12. What must be subtracted from $1000 to make the remainder equal to the sum of $175 and $42.50?

13. A man borrowed $1250 and paid back $125 at one time and $960 at another. How much did he still owe?

14. A man having $375.50 in the bank deposited at different times $175, $320, and $15.40. He then drew out $645. How much had he left in the bank?

CHAPTER IV

MULTIPLICATION OF INTEGERS

34. Multiplication. The process of taking a number as many times as there are units in another number is called *multiplication.*

This means multiplication by an integer. Multiplication by a fraction is considered later.

35. Terms. The number that is multiplied is called the *multiplicand;* the number that shows how many times the multiplicand is taken is called the *multiplier;* the result of the multiplication is called the *product.*

4 multiplicand
3 multiplier
12 product

36. Multiple. The product of two or more integers is called a *multiple* of each.

For example, 15 is a multiple of 3 and 5, and 40 is a multiple of 2, 4, 5 and also of 8, 10, 20.

37. Factors. Integral numbers whose product is a given number are called *factors* of that number; the multiplicand and the multiplier, for example, are factors of the product.

38. Symbol. The symbol of multiplication is an oblique cross, ×. It is read *times* or *multiplied by.*

Thus $3 \times 4 = 12$ is read "3 times 4 equals 12."

The multiplier may be written either before or after the multiplicand, but the tendency in this country is to write the multiplier first, as we naturally read it. For example, 3 × $4 is read "3 times $4," and $4 × 3 is read "$4 multiplied by 3."

39. The Multiplication Table. The following table, already learned, must be reviewed so that you can tell instantly any product when the teacher tells you the factors:

$2 \times 1 = 2$
$2 \times 2 = 4$
$2 \times 3 = 6$
$2 \times 4 = 8$
$2 \times 5 = 10$
$2 \times 6 = 12$
$2 \times 7 = 14$
$2 \times 8 = 16$
$2 \times 9 = 18$
$2 \times 10 = 20$

$3 \times 1 = 3$
$3 \times 2 = 6$
$3 \times 3 = 9$
$3 \times 4 = 12$
$3 \times 5 = 15$
$3 \times 6 = 18$
$3 \times 7 = 21$
$3 \times 8 = 24$
$3 \times 9 = 27$
$3 \times 10 = 30$

$4 \times 1 = 4$
$4 \times 2 = 8$
$4 \times 3 = 12$
$4 \times 4 = 16$
$4 \times 5 = 20$
$4 \times 6 = 24$
$4 \times 7 = 28$
$4 \times 8 = 32$
$4 \times 9 = 36$
$4 \times 10 = 40$

$5 \times 1 = 5$
$5 \times 2 = 10$
$5 \times 3 = 15$
$5 \times 4 = 20$
$5 \times 5 = 25$
$5 \times 6 = 30$
$5 \times 7 = 35$
$5 \times 8 = 40$
$5 \times 9 = 45$
$5 \times 10 = 50$

$6 \times 1 = 6$
$6 \times 2 = 12$
$6 \times 3 = 18$
$6 \times 4 = 24$
$6 \times 5 = 30$
$6 \times 6 = 36$
$6 \times 7 = 42$
$6 \times 8 = 48$
$6 \times 9 = 54$
$6 \times 10 = 60$

$7 \times 1 = 7$
$7 \times 2 = 14$
$7 \times 3 = 21$
$7 \times 4 = 28$
$7 \times 5 = 35$
$7 \times 6 = 42$
$7 \times 7 = 49$
$7 \times 8 = 56$
$7 \times 9 = 63$
$7 \times 10 = 70$

$8 \times 1 = 8$
$8 \times 2 = 16$
$8 \times 3 = 24$
$8 \times 4 = 32$
$8 \times 5 = 40$
$8 \times 6 = 48$
$8 \times 7 = 56$
$8 \times 8 = 64$
$8 \times 9 = 72$
$8 \times 10 = 80$

$9 \times 1 = 9$
$9 \times 2 = 18$
$9 \times 3 = 27$
$9 \times 4 = 36$
$9 \times 5 = 45$
$9 \times 6 = 54$
$9 \times 7 = 63$
$9 \times 8 = 72$
$9 \times 9 = 81$
$9 \times 10 = 90$

It is also useful to know the following:

$11 \times 1 = 11$
$11 \times 2 = 22$
$11 \times 3 = 33$

$11 \times 4 = 44$
$11 \times 5 = 55$
$11 \times 6 = 66$

$11 \times 7 = 77$
$11 \times 8 = 88$
$11 \times 9 = 99$

$11 \times 10 = 110$
$11 \times 11 = 121$
$11 \times 12 = 132$

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$

$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$

$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 9 = 108$

$12 \times 10 = 120$
$12 \times 11 = 132$
$12 \times 12 = 144$

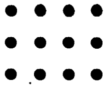
40. Multiplying by a One-Figure Multiplier. Required the product of 587 by 6.

We may multiply each part of 587 by 6 and add the results, as shown in the upper solution. This method is long, so we use it only for explanation, and write the work as in the lower solution. Beginning at the right, we have 6 × 7 = 42; we write the 2 in the units' place and reserve the 4 tens to add to the product of the tens. Then 6 × 8 tens = 48 tens, which, with the 4 tens reserved, makes 52 tens, or 5 hundreds and 2 tens. We therefore write the 2 tens in the tens' place, and reserve the 5 hundreds to add to the product of the hundreds. Then 6 × 5 hundreds = 30 hundreds, which, with the 5 hundreds reserved, makes 35 hundreds, or 3 thousands and 5 hundreds, which we write in the thousands' and hundreds' places. The product is therefore 3522.

587	multiplicand	
6	multiplier	
42	6 × 7	
48	6 × 8	(tens)
30	6 × 5	(hundreds)
3522	6 × 587	product

587
6
3522

41. Order of the Factors. The product is the same whatever the order of the factors.



That is, 3 × 4 = 4 × 3. If we read these dots by rows, we have 3 × 4 dots; if we read by columns, we have 4 × 3 dots. But the dots do not change, so 3 × 4 = 4 × 3. This is true for any number of rows and columns.

For this reason we may always use the smaller factor as the multiplier, thus making the work easier.

Thus, instead of multiplying 3 by 478, we would multiply 478 by 3. In the same way, if we wished to find the product of 478 × $3, we would take the product of 3 × $478, the answer being the same.

42. Nature of the Factors. Since the multiplier shows how many times the multiplicand is taken, we have the following:

The multiplier is always an abstract number.

The multiplicand may be either abstract or concrete, and it and the product will always be like numbers.

In 3 × $4 = $12, 3 is abstract and $4 and $12 are like numbers.

EXERCISE 12

Find the product of:

1. 2 × 207.
2. 3 × 409.
3. 4 × 306.
4. 5 × 704.
5. 6 × 508.
6. 7 × 802.
7. 8 × 605.
8. 9 × 909.
9. 2 × 437.
10. 3 × 526.
11. 4 × 823.
12. 5 × 715.
13. 6 × 412.
14. 7 × 813.
15. 8 × 612.
16. 9 × 730.
17. 2 × 749.
18. 3 × 684.
19. 8 × 777.
20. 7 × 648.
21. 2 × 6087.
22. 3 × 6074.
23. 4 × 8096.
24. 5 × 7048.
25. 6 × 3409.
26. 7 × 4706.
27. 8 × 6808.
28. 9 × 4307.
29. 2 × 2763.
30. 3 × 4496.
31. 4 × 8277.
32. 5 × 7879.
33. 6 × 9236.
34. 7 × 8943.
35. 8 × 9448.
36. 9 × 7367.
37. 6 × 9982.
38. 7 × 7347.
39. 8 × 8888.
40. 9 × 9999.
41. 2 × 41,203.
42. 3 × 26,207.
43. 4 × 35,705.
44. 5 × 42,609.
45. 6 × 30,926.
46. 7 × 40,289.
47. 8 × 70,443.
48. 9 × 80,625.
49. 2 × 77,234.
50. 3 × 82,629.
51. 4 × 47,726.
52. 5 × 83,921.
53. 6 × 72,277.
54. 7 × 63,429.
55. 8 × 48,296.
56. 9 × 36,436.
57. 6 × 78,879.
58. 7 × 89,986.
59. 8 × 56,789.
60. 9 × 99,999.

61. What will 48 sheep cost at $7 each?

62. What will 480 dozen pairs of hose cost at $5 a dozen?

63. A farmer shipped 9 cases, each containing 360 eggs. How many eggs did he ship?

64. A farmer had 7 acres of onions, and the yield was 562 bushels per acre. How many bushels did he get in all?

43. Multiplying United States Money. The process here is the same as with other numbers. Simply keep the decimal point after the dollars.

$$\begin{array}{r} \$23.05 \\ 7 \\ \hline \$161.35 \end{array}$$

Thus, to multiply $23.05 by 7, proceed as here shown. The explanation is the same as in § 40.

EXERCISE 13

Find the product of:

1. 3 × $1.25.
2. 3 × $2.70.
3. 4 × $2.65.
4. 4 × $3.25.
5. 5 × $2.63.
6. 5 × $6.31.
7. 6 × $2.48.
8. 6 × $3.72.
9. 7 × $4.28.
10. 7 × $5.75.
11. 8 × $3.62.
12. 8 × $6.41.
13. 9 × $3.25.
14. 9 × $7.28.
15. 452 × $7.
16. 326 × $6.
17. 281 × $5.
18. 429 × $8.
19. 532 × $3.
20. 799 × $9.
21. 867 × $4.
22. 3 × $15.25.
23. 3 × $21.70.
24. 4 × $12.62.
25. 4 × $32.15.
26. 5 × $17.20.
27. 5 × $23.11.
28. 6 × $18.90.
29. 6 × $37.50.
30. 7 × $41.16.
31. 7 × $37.75.
32. 8 × $21.63.
33. 8 × $82.76.
34. 9 × $35.50.
35. 9 × $47.63.
36. 2142 × $6.
37. 3142 × $5.
38. 4126 × $3.
39. 2107 × $2.
40. 4340 × $7.
41. 2989 × $9.
42. 5842 × $8.
43. 3 × $121.50.
44. 3 × $271.20.
45. 4 × $125.25.
46. 4 × $306.70.
47. 5 × $321.51.
48. 5 × $248.75.
49. 6 × $328.90.
50. 6 × $542.75.
51. 7 × $128.75.
52. 7 × $305.20.
53. 8 × $211.75.
54. 8 × $425.25.
55. 9 × $372.72.
56. 9 × $987.68.
57. 42,341 × $2.
58. 21,421 × $3.
59. 26,401 × $4.
60. 31,620 × $5.
61. 77,217 × $7.
62. 60,509 × $9.
63. 84,748 × $6.

64. At $3 each, what will 37 kegs of nails cost?

65. At $3.25 each, what will 7 door casings cost?

66. How much will 2 doors for a house cost at $6.75 each?

67. How much will 7 windows for a house cost at $3.70 each?

68. At $4.50 per thousand, what will 7 thousand shingles cost?

69. At $17.75 per thousand, what will 8 thousand feet of lumber cost?

70. At $42.75 per thousand, what will 4 thousand feet of pine siding cost?

71. If a man builds 4 cottages costing $1728.75 each, what is the total cost?

72. At $35.50 per thousand, what will 2 thousand feet of matched pine flooring cost?

73. At $2.28 per thousand square feet, what will 3 thousand square feet of building paper cost?

74. At $4.48 each for doors, and $5.62 each for windows, what will 6 doors and 9 windows cost?

75. If a dealer sells 8 building lots at an average price of $287.50 each, how much does he receive for all?

76. If a real estate dealer pays taxes of $37.75 on each of 9 lots, what is the total tax he pays?

77. How much will it cost to give some rooms in a house 3 coats of paint, if the cost of each averages $37.50?

78. If a builder has 7 men employed at $2.75 a day and 3 at $3.50 a day, how much does he pay them all a day?

79. If a contractor employs 8 men at $3.75 a day, 6 men at $2.90 a day, and 7 men at $2.65 a day, how much does he pay them all a day?

44. Powers. The result of taking a number any number of times as a factor is called a *power* of the number.

For example, 3 × 3 = 9, and 9 is called the *second power*, or *square*, of 3. So 3 × 3 × 3, or 27, is the *third power*, or *cube*, of 3. The successive powers of 10 are 10, 100, 1000, and so on.

45. Multiplying by Powers of 10. To multiply an integer by 10, annex a 0; by 100, two 0's; by 1000, three 0's.

To multiply United States money by 10, move the decimal point 1 place to the right; by 100, 2 places; by 1000, 3 places.

27	$25.16	360
40	300	400
1080	$7548	144000

That is, to multiply 27 by 40, multiply by 4 and annex a 0.

Also to multiply $25.16 by 300, multiply by 3 and move the decimal point 2 places to the right.

And, in general, consider the case of 400 × 360. Here we see the multiplicand is 36 × 10, that is, 360; the multiplier is 4 × 100, that is, 400; therefore the product is 144 × 1000, that is, 144,000.

Therefore, *when the multiplicand and multiplier are integers ending in zeros, or either ends in zeros, multiply without regard to the zeros, but annex to the product as many zeros as there are at the end of the multiplicand and of the multiplier.*

EXERCISE 14

Find the product of:

1. 30 × 59.
2. 40 × 477.
3. 60 × 238.
4. 50 × 470.
5. 60 × 930.
6. 20 × 765.
7. 50 × 642.
8. 70 × $430.
9. 80 × $3.55.
10. 90 × $4.88.
11. 70 × $2.66.
12. 80 × $4.32.
13. 70 × $7.80.
14. 90 × $5.60.
15. 300 × 427.
16. 500 × 640.
17. 700 × 4200.
18. 900 × 3500.
19. 800 × $3.75.
20. 500 × $4.80.
21. 900 × 99,000.

46. Multiplying by Larger Multipliers. Required to multiply 4674 by 649.

The multiplier is 600 + 40 + 9. The product is obtained by multiplying by 9, then by 40, and then by 600, and adding the products.

	(1)	(2)
	4674	4674
	649	649
9 times the multiplicand =	42066	42066
40 times the multiplicand =	186960	18696
600 times the multiplicand =	2804400	28044
649 times the multiplicand =	3033426	3033426

Zeros at the right of the partial products, since they do not affect the result of the addition, should be omitted as in (2).

Care must be taken, however, to put the right-hand figure of each partial product directly under the figure of the multiplier used to obtain it.

Find the product of 2007 times $45.87.

The products corresponding to the zeros in the multiplier will be zero, and therefore they need not be written. With this exception the multiplication is performed as in the preceding example. The decimal point is placed before the cents as in the multiplicand.

$45.87
2007
321.09
9174
$92061.09

EXERCISE 15

Find the product of:

1. 21 × 2648.
2. 31 × 4872.
3. 42 × 2634.
4. 62 × 3108.
5. 44 × 2177.
6. 54 × 3623.
7. 47 × 3926.
8. 56 × 7793.
9. 67 × 2987.
10. 74 × 3692.
11. 78 × 8269.
12. 89 × 4871.
13. 93 × 3642.
14. 96 × 2996.
15. 73 × 8989.
16. 48 × 7642.
17. 54 × 8387.
18. 73 × 9874.
19. 36 × 7448.
20. 95 × 6877.
21. 68 × 7482.
22. 73 × 8929.
23. 85 × 4832.
24. 63 × 6309.

25. 84 × 7470.
26. 96 × 3998.
27. 74 × 8647.
28. 98 × 8766.
29. 99 × 9999.
30. 211 × 7361.
31. 311 × 4872.
32. 401 × 3664.
33. 503 × 2763.
34. 630 × 4876.
35. 580 × 6924.
36. 770 × 3871.
37. 860 × 3926.
38. 722 × 3322.
39. 421 × 7232.
40. 512 × 3211.
41. 643 × 7806.
42. 428 × 3605.
43. 826 × 8207.
44. 407 × 8643.
45. 502 × 8926.
46. 708 × 3971.
47. 637 × 9286.
48. 843 × 8792.
49. 695 × 5764.
50. 783 × 9871.
51. 847 × 6476.
52. 972 × 3502.
53. 866 × 4009.
54. 707 × 8763.
55. 5002 × 7983.
56. 6008 × 3976.
57. 4003 × 6648.
58. 5009 × 9987.
59. 300 × $75.35.
60. 400 × $82.25.
61. 500 × $61.32.
62. 600 × $43.35.
63. 700 × $27.73.
64. 800 × $51.36.
65. 900 × $83.42.
66. 800 × $4200.
67. 700 × $7500.
68. 600 × $3200.
69. 500 × $9600.
70. 283 × $48.25.
71. 342 × $29.75.
72. 276 × $49.42.
73. 396 × $64.40.
74. 428 × $75.19.
75. 532 × $81.62.
76. 437 × $29.86.
77. 522 × $62.23.
78. 729 × $72.30.
79. 8004 × $91.76.
80. 9006 × $38.92.
81. 7052 × $47.80.

82. What will 175 chairs cost at $2.35 each?

83. What will 630 barrels of salt cost at $3.08 a barrel?

84. What will it cost to cart 168 loads of produce at $3.15 a load?

85. What will 475 bushels of clover seed cost at $3.12 a bushel?

86. What will a wholesale dealer have to pay for 240 yards of velvet at $1.85 a yard, 480 yards of silk at $1.38, 520 yards of lace at $1.62, and 250 yards of broadcloth at $1.75?

EXERCISE 16

Review Problems

1. How much will 30 cows cost at $35 a head?

2. A mile contains 5280 feet. How many feet in 8 miles?

3. How much will 440 bales of hops weigh at 200 pounds a bale?

4. If a square lot is 66 feet on a side, what is the distance around it?

5. If sound travels 1125 feet in 1 second, how far will it travel in 20 seconds?

6. A farmer bought 7 bushels of grass seed at $3.40 a bushel. How much did it all cost?

7. A city school bought 20 dictionaries at $9.75 each. How much did they all cost?

8. If a salesman receives $25 a week and works 48 weeks a year, what is his income for the year?

9. At $85 a month, how much will the rent of a city apartment amount to in a year?

10. A lady rents a piano for $4.25 a month. How much is the rent for 6 months?

11. At $250 a month, what will a man's salary amount to in a year?

12. If a man grinds 35 bushels of corn of 56 pounds each, how many pounds does he grind?

13. A farmer adds to his property a field of 65 acres, paying $75 an acre for it. How much does the field cost?

14. A dealer sells 15 automobiles at $925 each. How much does he receive for them all?

15. If 38 cows average 408 pounds of milk apiece a month, how many pounds do the cows produce in a month?

16. There being 12 inches in 1 foot, how many inches are there in 175 feet?

17. There being 5280 feet in 1 mile, how many feet are there in 24 miles?

18. There being 144 square inches in 1 square foot, how many square inches are there in 93 square feet?

19. There being 128 cubic feet in a cord, how many cubic feet are there in 152 cords of wood?

20. If a farm averages 28 bushels of buckwheat to the acre, how many bushels will 175 acres produce?

21. If a farm averages 18 bushels of winter rye to the acre, how many bushels will 136 acres produce?

22. A manufacturer sells 175 sets of harness at $24 a set. How much does he receive for them all?

23. There being 640 acres in 1 square mile, how many acres are there in 135 square miles?

24. If a farm averages 2256 pounds of hay to the acre, how much hay will 275 acres produce?

25. A cubic foot of hard coal weighing 84 pounds, how much will 246 cubic feet of hard coal weigh?

26. A cubic foot of ice weighing 58 pounds, how much will 1070 cubic feet of ice weigh?

27. Allowing 5 ounces to a cubic inch, how much will a cubic foot (1728 cubic inches) of copper weigh?

28. If a man averages 2265 steps in walking a mile, how many steps will he take in walking 23 miles?

29. A barrel of flour weighing 196 pounds, how much will 144 barrels weigh?

30. Find the cost of 42 head of cattle at $37.50 each.

31. If a farmer buys 75 acres of land at $62.25 an acre, what is the total cost?

32. How much will 65 horses cost at $82.75 each?

33. At $5.75 a ton, how much will 356 tons of coal cost?

34. A book dealer sells 206 copies of a certain book at $1.25 each. How much does he receive for them all?

35. At $2.98 a ton at the mine, how much will 478 tons of iron cost?

36. When sound travels 1125 feet in a second, how far will it travel in 2 minutes?

37. If a cubic foot of cast iron weighs 447 pounds, how much will 12 posts weigh, each containing 3 cubic feet?

38. If a cubic foot of wrought iron weighs 486 pounds, how much will 17 beams of 2 cubic feet each weigh?

39. If a cubic foot of ice weighs 58 pounds, what will 32 cakes weigh, each containing 3 cubic feet?

40. The earth moves about the sun 101,090 feet in a second. How many feet does it move in a minute?

41. If a steam engine uses 2 tons of coal a day, how many tons will it consume in January if it runs every day?

42. If a man earns $3.50 a day, and works 6 days in the week for 48 weeks, how much does he earn in a year?

43. A merchant bought 7 pieces of cloth of 33 yards each and paid $1.60 a yard for it. What was the total cost?

44. A grocer sold 4 boxes of oranges, each box containing 196 oranges, at 4 cents apiece. How much did he receive for all?

45. Which will cost the more, 48 cows at $37.50 each, or 22 horses at $88 each? How much more?

46. If electricity travels on a wire at the rate of 288,000 miles per second, how long would a wire have to be in order that an electric current would travel from one end to the other in one minute?

CHAPTER V

DIVISION OF INTEGERS

47. Nature of Division. In multiplication two factors are given to find the product. In division the product and one of two factors are given to find the other factor.

That is, in the case of $2 \times \$4 = \$8$, we have given the factors 2 and \$4, from which to find the product \$8.

In division we have given the product \$8 and the factor 2, to find the other factor \$4; or the factor \$4, to find the other factor 2.

48. Division. If the product and one of two factors are given, the process of finding the other factor is called *division.*

49. Symbol. The symbol for division is ÷. It is read *divided by.*

For example, $15 \div 5 = 3$. This is also indicated by another symbol, thus: $\frac{15}{5} = 3$. That is, $15 \div 5$ and $\frac{15}{5}$ have the same meaning.

50. Dividend. The given product, that is, the number to be divided, is called the *dividend.*

51. Divisor. The given factor, that is, the number by which we divide, is called the *divisor.*

52. Quotient. The required factor, that is, the number found by division, is called the *quotient.*

53. Remainder. Often a divisor does not exactly divide the dividend. Then the part of the dividend that is left is called the *remainder.*

Thus $14 \div 3 = 4$ with remainder 2. The quotient is usually written $4\frac{2}{3}$, meaning 4 with 2 still to be divided into 3 equal parts.

54. Two Kinds of Division. Since 2 × \$4 = \$8, therefore \$8 ÷ \$4 = 2, and \$8 ÷ 2 = \$4. That is,

If the dividend and the divisor are concrete, they must be like numbers, and the quotient is abstract.

For example, \$8 ÷ \$4 = 2. This is sometimes called *measuring* because the \$8 is measured by the \$4.

If the dividend is concrete and the divisor abstract, the quotient is like the dividend.

For example, \$8 ÷ 2 = \$4. This is sometimes called *partition*, because the \$8 is separated into 2 equal parts.

55. Check. If there is no remainder, the dividend is the product of the quotient and the divisor. To check the work in division, therefore, we multiply the quotient by the divisor; this product added to the remainder, if any, should equal the dividend.

For example, 45 ÷ 5 = 9; then 5 × 9 = 45. Again 48 ÷ 5 = 9 with remainder 3; then 5 × 9 = 45, and 45 + 3 = 48.

56. Short Division. When the divisor is so small that the work may be done mentally the process is called *short division*.

(1) Required to divide 342 by 3.

$$\begin{array}{r@{\,}l} 3 & \underline{)300 + 30 + 12} \\ & \;\;100 + 10 + \;\;4 = 114 \end{array}$$

If 342 is separated as shown, we can divide each part by 3. This is what we do mentally below. We write the numbers as shown; then 3 is contained in 3 once (that is, 100 times in 300); 3 is contained in 4 once with remainder 1 (that is, 10 times in 40 with remainder 10); this 1 is 10 units, and with the 2 makes 12. Then 3 is contained in 12 four times. Therefore the quotient is 114. We think: "3 in 3, 1; in 4, 1; in 12, 4."

$$\begin{array}{r@{\,}l} 3 & \underline{)342} \\ & \;\;114 \end{array}$$

(2) Required to divide 42,364 by 7.

$$\begin{array}{r@{\,}l} 7 & \underline{)42364} \\ & \;\;\;\;6052 \end{array}$$

Here 7 is not contained in 4, but 7 is contained in 42 six times. Then 7 is not contained in 3, so we write a 0 beneath the 3. Then 7 is contained in 36 five times with remainder 1, and in 14 twice. Therefore the quotient is 6052.

(3) Required to divide 35,282 by 9.

$$\begin{array}{r} 9\,\underline{)35282} \\ 3920 \end{array}$$

Here 9 is contained in 35 three times with remainder 8; in 82 nine times with remainder 1; in 18 twice; in 2 no times with remainder 2. Therefore the quotient is 3920 with remainder 2, and this is usually written 3920$\frac{2}{9}$.

(4) Required to divide $27.53 by 4.

$$\begin{array}{r} 4\,\underline{)\$27.53} \\ \$6.88 \end{array}$$

Dividing as before, we have \$27 ÷ 4 = \$6, with \$3 remaining; \$3.50 ÷ 4 = \$0.8 with \$0.30 remaining; \$0.33 ÷ 4 = \$0.08, with \$0.01 remaining. Therefore the quotient is \$6.88$\frac{1}{4}$.

EXERCISE 17

Find the quotient of:

1. 232 ÷ 2.
2. 345 ÷ 3.
3. 416 ÷ 4.
4. 325 ÷ 5.
5. 738 ÷ 6.
6. 924 ÷ 7.
7. 592 ÷ 8.
8. 837 ÷ 9.
9. 3242 ÷ 2.
10. 4122 ÷ 3.
11. 6004 ÷ 4.
12. 3015 ÷ 5.
13. 4032 ÷ 6.
14. 3073 ÷ 7.
15. 7368 ÷ 8.
16. 8019 ÷ 9.
17. 5555 ÷ 2.
18. 6014 ÷ 3.
19. 4213 ÷ 4.
20. 2893 ÷ 5.
21. 6271 ÷ 6.
22. 7356 ÷ 7.
23. 9123 ÷ 8.
24. 8077 ÷ 9.
25. $47.24 ÷ 2.
26. $33.15 ÷ 3.
27. $28.72 ÷ 4.
28. $32.75 ÷ 5.
29. $21.30 ÷ 6.
30. $29.47 ÷ 7.
31. $65.68 ÷ 8.
32. $83.61 ÷ 9.
33. 81,004 ÷ 4.
34. 29,005 ÷ 5.
35. 31,266 ÷ 6.
36. 40,040 ÷ 7.
37. 70,328 ÷ 8.
38. 32,013 ÷ 9.
39. 31,201 ÷ 2.
40. 34,078 ÷ 3.
41. 26,231 ÷ 4.
42. 70,023 ÷ 5.
43. 80,335 ÷ 6.
44. 29,321 ÷ 7.
45. 42,345 ÷ 8.
46. 76,209 ÷ 9.
47. $35.21 ÷ 2.
48. $72.43 ÷ 3.
49. $61.23 ÷ 4.
50. $72.36 ÷ 5.
51. $82.01 ÷ 6.
52. $42.03 ÷ 7.
53. $65.00 ÷ 8.
54. $93.65 ÷ 9.

EXERCISE 18

1. If 3 head of cattle cost $117, how much will 1 head cost?

2. It is 108 feet around a square. What is the length of each side?

3. There being 7 days in a week, how many weeks are there in 364 days?

4. There being 3 feet in a yard, how many yards are there in 432 feet?

5. There being 4 quarts in a gallon, how many gallons are there in 236 quarts?

6. It is 132 feet around a triangle whose sides are all equal. What is the length of each side?

7. At $9 a dozen, how many dozen cups and saucers can be bought for $1116?

8. At $4 a dozen, how many dozen handkerchiefs can be bought for $1152?

9. A dealer pays $1224 for 9 wagons. What is the average price?

10. A wholesale grocer pays $1728 for a shipment of chocolate at $3 per dozen cans. How many dozen cans does he buy?

11. A wholesale grocer pays $4320 for some cocoa at $6 per dozen cans. How many dozen cans does he buy?

12. A hotel pays $13.20 for 8 dozen packages of breakfast food. How much does it pay per dozen?

13. If a grocer pays $12.15 for 9 dozen packages of breakfast cereal, how much does he pay per dozen?

14. If a grocer pays $19.84 for 8 dozen tins of biscuit, how much does he pay per dozen?

57. Dividing by 10 and its Powers. To divide by 10 is merely to find how many tens there are in the number, and this we can tell at a glance.

Thus in 1230 there are 123 tens and no units. In 2373 there are 237 tens and 3 units remainder; that is, $2373 \div 10 = 237\frac{3}{10}$.

Therefore, *to divide an integer by 10 cut off the right-hand figure of the dividend. The result is the quotient, and the right-hand figure expresses the remainder.*

To divide an integer by any power of 10 cut off as many figures from the right of the dividend as there are zeros at the right of the divisor. The result is the quotient, and the right-hand figures express the remainder.

Thus $12{,}400 \div 100 = 124$; $43{,}203 \div 100 = 432\frac{3}{100}$; $624{,}000 \div 1000 = 624$; $736{,}117 \div 1000 = 736\frac{117}{1000}$.

58. Dividing by Multiples of Powers of 10. In this case we divide by the power of 10, as in § 57, and then by the number indicating the multiple.

$$\begin{array}{r} 2\cancel{0}\cancel{0})\underline{428\cancel{0}\cancel{0}} \\ 214 \end{array}$$

$$\begin{array}{r} 3\cancel{0}\cancel{0})\underline{274\cancel{0}\cancel{7}} \\ 91\frac{107}{300} \end{array}$$

For example, to divide 42,800 by 200 we cut off two zeros from divisor and dividend and then divide by 2.

So to divide 27,407 by 300 we cut off two zeros from the divisor and two figures from the right of the dividend. We then divide by 3, and the quotient is 91, with remainder 1 in the hundreds' place, and to this we join the 07 cut off, obtaining the remainder 107.

EXERCISE 19

Find the quotient and the remainder (if any):

1. $340 \div 10$.
2. $720 \div 20$.
3. $540 \div 30$.
4. $640 \div 40$.
5. $750 \div 50$.
6. $840 \div 60$.
7. $840 \div 70$.
8. $960 \div 80$.
9. $360 \div 90$.
10. $720 \div 90$.
11. $273 \div 10$.
12. $423 \div 20$.

Find the quotient and the remainder (if any):

13. 325 ÷ 30.
14. 275 ÷ 40.
15. 236 ÷ 50.
16. 327 ÷ 70.
17. 542 ÷ 70.
18. 391 ÷ 80.
19. 278 ÷ 90.
20. 777 ÷ 90.
21. 20,300 ÷ 100.
22. 27,400 ÷ 200.
23. 64,200 ÷ 300.
24. 72,400 ÷ 400.
25. 31,501 ÷ 500.
26. 27,403 ÷ 600.
27. 59,721 ÷ 700.
28. 42,348 ÷ 800.
29. 98,207 ÷ 900.
30. 126,000 ÷ 3000.
31. 328,000 ÷ 4000.
32. 465,000 ÷ 5000.
33. 564,700 ÷ 6000.
34. 429,820 ÷ 7000.
35. 742,623 ÷ 8000.
36. 857,542 ÷ 9000.

37. At $90 each, how many horses can I buy for $2160?

38. There being 60 minutes in an hour, how many hours are there in 1020 minutes?

39. There being 60 seconds in a minute, how many minutes are there in 1620 seconds?

40. At $30 each, how many harnesses must a manufacturer sell to receive $1110?

41. A farmer pays $6600 for some land at $40 an acre. How many acres does he buy?

42. A dealer sells some village lots for $10,200, at $600 a lot. How many lots does he sell?

43. An investor has $87,750 with which to buy some bonds at $500 each. What is the greatest number he can buy, and how much will he have left?

44. An automobile dealer has $35,100 to invest in automobiles at $2000 each. What is the greatest number he can buy for this sum, and how much will he have left?

45. A ton is 2000 pounds. If a coal pocket contains 583,260 pounds of coal, how many tons does it contain, and how many pounds are left over?

EXERCISE 20

PROBLEMS OF THE FARM

1. A farmer sowed 60 acres of land, using 780 quarts of timothy seed. How much seed did he use per acre?

2. He also sowed 70 acres, using 1050 quarts of timothy and clover mixed. How much seed did he use per acre?

✓ 3. Another farmer sowed 90 acres, using 2520 quarts of timothy and redtop mixed equally. How many quarts of each kind did he use per acre?

4. A Maryland farmer experimented with Italian rye grass, using 3250 pounds to sow 50 acres. How many pounds did he use per acre?

5. A farmer found that 30 sheep ate 11,550 pounds of dry fodder in a year. What was the average amount per sheep?

6. He found that 20 horses consumed 162,400 pounds of hay and grain in a year. What was the average amount per horse?

7. A dealer found that 60 bushels of white clover seed weighed 3780 pounds. How much did it weigh per bushel?

✓ 8. He told a customer 1080 pounds would seed 90 acres. How much is that per acre? If he mixed other seed with the clover so as to use this amount to cover 4 acres, how much clover seed would he use per acre?

9. A farmer found that it took 980 pounds of red clover for 70 acres, and 1200 pounds of crimson clover for 80 acres. What was the amount of each used per acre?

✓ 10. How many pounds of Paris green will be needed to spray 300 trees, allowing 5 gallons to a tree, if every 150 gallons contains 1 pound of Paris green?

59. Long Division. When the divisor has two or more figures, except in such cases as are given in § 57 and § 58, it is customary to use a process known as *long division.*

We shall first consider the simplest case, that in which the units' figure is 1.

Required to divide 7182 by 21.

If all the work should be written, it would appear as in the first of these forms, but for practical purposes the second is used.

(1)

$$
\begin{array}{rrl}
 & 342 & \\
21) & \overline{7182} & \\
 & \underline{6300} & = 300 \text{ times the divisor} \\
 & 882 & \\
 & \underline{840} & = 40 \text{ times the divisor} \\
 & 42 & \\
 & \underline{42} & = 2 \text{ times the divisor}
\end{array}
$$

(2)

$$
\begin{array}{rr}
 & 342 \\
21) & \overline{7182} \\
 & \underline{63} \\
 & 88 \\
 & \underline{84} \\
 & 42 \\
 & \underline{42}
\end{array}
$$

First we will consider (1), the more complete form. We see that 21 is evidently not contained in 7, so there are no thousands in the quotient. The divisor is contained 3 times in 71, and since the 71 is hundreds, the 3 is written above, over the hundreds' place. Then $300 \times 21 = 6300$, and there remains 882 still to be divided; 21 is contained in 88 four times, and since the 88 is tens, the 4 of the quotient is written over the tens' place. Then $40 \times 21 = 840$, and there remains 42 to be divided; 21 is contained in 42 twice, and 2 is written over the units' place. The quotient is therefore 342. It will be seen that 6300, 840, and 42 make up 342×21, as should be the case.

Consider now the second form, which is always used in practice. Since at any time in this example we need only two figures of the dividend or two figures of such a partial dividend as 882, we may omit the zeros in 6300 and 840, and use only the 88 in the remainder 882, thus making the work shorter.

We may check our result by multiplying 342 by 21. Then $21 \times 342 = 7182$, the dividend.

The pupil should check his work in enough examples to become thoroughly familiar with the process.

Required to divide 6447 by 21.

$$\begin{array}{r} 307 \\ 21\overline{)6447} \\ 63 \\ \hline 147 \\ 147 \\ \hline \end{array}$$

Here we have 64 ÷ 21 = 3, and since the 64 is hundreds, the 3 is written over the hundreds' place. Then 3 × 21 = 63, and when 63 is subtracted, there is a remainder 1. Bringing down one figure as before, we have 14 tens to be divided by 21. Since 21 is not contained in 14, we write 0 in the tens' place to show that there are no tens, and bring down the next figure 7. Then 147 ÷ 21 = 7, and 7 × 21 = 147 exactly, so there is no remainder.

EXERCISE 21

Find the quotient of:

1. 352 ÷ 11.
2. 451 ÷ 11.
3. 682 ÷ 11.
4. 968 ÷ 11.
5. 858 ÷ 11.
6. 957 ÷ 11.
7. 737 ÷ 11.
8. 913 ÷ 11.
9. 252 ÷ 21.
10. 294 ÷ 21.
11. 441 ÷ 21.
12. 672 ÷ 21.
13. 777 ÷ 21.
14. 819 ÷ 21.
15. 987 ÷ 21.
16. 868 ÷ 31.
17. 779 ÷ 41.
18. 765 ÷ 51.
19. 1037 ÷ 61.
20. 1312 ÷ 41.
21. 1562 ÷ 71.
22. 2673 ÷ 81.
23. 3367 ÷ 91.
24. 2331 ÷ 21.
25. 2583 ÷ 21.
26. 3472 ÷ 31.
27. 2121 ÷ 21.
28. 6789 ÷ 31.
29. 8547 ÷ 21.
30. 3317 ÷ 31.
31. 10,731 ÷ 21.
32. 14,322 ÷ 31.
33. 16,461 ÷ 31.
34. 25,297 ÷ 41.
35. 18,879 ÷ 31.
36. 28,741 ÷ 41.
37. 40,953 ÷ 51.
38. 49,349 ÷ 61.
39. 18,696 ÷ 41.
40. 52,582 ÷ 61.
41. 62,267 ÷ 71.
42. 63,048 ÷ 71.
43. 71,280 ÷ 81.
44. 90,090 ÷ 91.
45. 90,909 ÷ 91.
46. 50,337 ÷ 51.
47. 50,898 ÷ 51.
48. 62,937 ÷ 81.
49. 71,928 ÷ 81.
50. 60,939 ÷ 61.
51. 34,587 ÷ 61.
52. 73,629 ÷ 81.
53. 65,448 ÷ 81.
54. 73,619 ÷ 91.

60. Long Division Continued. We have considered the simple case in which the units' figure is 1, and we shall now consider the more general case of a two-figure divisor.

Required to divide 19,224 by 54.

$$\begin{array}{r}
356\\
54\overline{)19224}\\
\underline{162}\\
302\\
\underline{270}\\
324\\
\underline{324}
\end{array}$$

The divisor is not contained in 1 or in 19, so there are no ten-thousands and no thousands in the quotient. But since 3 × 54 = 162, the divisor is contained 3 times in 192 with remainder 30. Since this is hundreds, the quotient figure 3 is written over the hundreds' place. Bringing down the next figure of the dividend, 2, we have 302 tens to be divided. Since 5 × 54 = 270, the divisor is contained 5 times in 302, with remainder 32. Since this is tens, the quotient figure 5 is written over the tens' place. Bringing down the next figure of the dividend, 4, we have 324 still to be divided. Since 324 is exactly 6 × 54, the units' figure is 6 and there is no remainder. Hence the quotient is 356.

61. Product Tables. It is sometimes helpful for beginners to prepare a table of products of the divisor, as follows:

2 × 54 = 108 4 × 54 = 216 6 × 54 = 324 8 × 54 = 432
3 × 54 = 162 5 × 54 = 270 7 × 54 = 378 9 × 54 = 486

With such a table made out in advance, we can tell at once, without making several trials, what the next figure of the quotient will be.

EXERCISE 22

Find the quotient of:

1. 3796 ÷ 52.
2. 8464 ÷ 92.
3. 6510 ÷ 62.
4. 4599 ÷ 73.
5. 8772 ÷ 43.
6. 2275 ÷ 35.
7. 2736 ÷ 57.
8. 5929 ÷ 77.
9. 2451 ÷ 19.
10. 8514 ÷ 66.
11. 22,311 ÷ 67.
12. 31,950 ÷ 75.
13. 63,204 ÷ 69.
14. 74,994 ÷ 87.
15. 98,901 ÷ 99.
16. 41,831 ÷ 59.
17. 55,821 ÷ 69.
18. 54,439 ÷ 77.
19. 118,216 ÷ 56.
20. 301,184 ÷ 64.
21. 891,891 ÷ 99.

22. If a steamship sails 6336 miles in 18 days, what is the rate per day?

23. How many 25-pound packages can be made from 18,000 pounds of paper?

24. How many 49-pound sacks can be filled from 150,675 pounds of flour?

25. The product of two numbers is 98,441, and one of them is 49. What is the other?

26. A man paid $28,800 for a tract of land at $45 an acre. How many acres did he buy?

27. A man paid $19,125 for a farm, the price being $85 an acre. How many acres did he buy?

28. A certain book averages 29 lines to a page and contains 10,034 lines in all. How many pages are there?

29. If a desk company sells $20,184 worth of desks in a year at $29 each, how many desks does it sell?

30. If a factory sells $18,648 worth of harnesses in a season at $37 a set, how many sets does it sell?

31. A canning factory ships 30,000 cans of fruit in boxes containing 4 dozen cans each. How many boxes does it require?

32. There being 27 cubic feet in a cubic yard, how many loads (cubic yards) of earth must be removed to excavate 108,675 cubic feet?

33. One of the routes around the world is 18,424 miles long. If a man makes the trip in 56 days, what is the average distance per day?

34. If the circumference of a wagon wheel is 15 feet, and there are 5280 feet in a mile, how many times will the wheel turn in going 9 miles? How many times in going 25 miles?

62. Long Division Continued. We may now consider divisors of three or more figures, and cases of division in which there are remainders.

Required to divide 341,597 by 248.

```
      1377
248)341597
    248
     935
     744
     1919
     1736
      1837
      1736
       101
```

The divisor is not contained in 3 or in 34, so there are no hundred-thousands or ten-thousands in the quotient. But it is contained in 341 once with remainder 93. Since this is thousands, the quotient figure, 1, is written over the thousands' place. Bringing down the next figure of the dividend, 5, we have 935 hundreds to be divided. It might seem as if the next figure would be 4, and here a product table (§ 61) would be helpful; but 4 × 248 = 992, and this is larger than 935. Trying 3 for the next figure, we have 3 × 248 = 744, and there is a remainder 191. Since we have divided hundreds by 248, the 3 is hundreds and is written over that place. Bringing down the next figure, we find that 1919 ÷ 248 = 7, with 183 remaining. Since the 7 is tens, we write it over the tens' place. Then, bringing down the last figure, we have 1837 ÷ 248 = 7, with 101 remaining. Therefore the quotient is 1377, and the remainder is 101. The complete quotient may be written $1377\frac{101}{248}$.

We may check the work by adding 101 to the product of 248 × 1377, the result being 341,597, the dividend.

63. General Directions. It may assist us if we now state the general directions we have found for long division:

Write the divisor to the left of the dividend.

Take for the first partial dividend the least number of left-hand figures that contains the divisor, and write the quotient over the right-hand figure of this partial dividend.

Multiply the divisor by this quotient, and place the product under the partial dividend used.

Subtract this product, and to the remainder annex the next figure of the dividend for a new partial dividend.

Divide as before, and continue this process until all the figures of the dividend have been used.

EXERCISE 23

Find the quotient of:

1. 25,284 ÷ 301.
2. 51,742 ÷ 631.
3. 47,348 ÷ 623.
4. 153,186 ÷ 211.
5. 475,950 ÷ 501.
6. 972,360 ÷ 111.
7. 161,463 ÷ 321.
8. 324,360 ÷ 306.
9. 140,850 ÷ 225.
10. 183,750 ÷ 175.
11. 148,555 ÷ 407.
12. 234,901 ÷ 503.
13. 208,575 ÷ 309.
14. 870,672 ÷ 204.
15. 367,848 ÷ 786.
16. 102,465 ÷ 495.
17. 472,932 ÷ 604.
18. 349,206 ÷ 407.
19. 117,440 ÷ 320.
20. 419,640 ÷ 538.
21. 287,712 ÷ 216.
22. 615,000 ÷ 328.
23. 685,542 ÷ 286.
24. 997,108 ÷ 478.
25. 965,496 ÷ 9852.
26. 692,356 ÷ 8764.
27. 7684 ÷ 69.
28. 8522 ÷ 76.
29. 8562 ÷ 23.
30. 20,452 ÷ 56.
31. 18,113 ÷ 64.
32. 14,797 ÷ 57.
33. 19,140 ÷ 46.
34. 26,523 ÷ 47.
35. 27,559 ÷ 59.
36. 47,078 ÷ 69.
37. 50,602 ÷ 82.
38. 101,160 ÷ 232.
39. 110,370 ÷ 124.
40. 252,150 ÷ 412.
41. 120,830 ÷ 653.
42. 260,380 ÷ 422.
43. 192,892 ÷ 232.
44. 211,428 ÷ 5873.
45. 287,712 ÷ 5328.
46. 313,205 ÷ 8465.
47. 411,075 ÷ 5481.
48. 616,693 ÷ 8009.
49. 965,496 ÷ 9852.
50. 1,599,560 ÷ 2836.
51. 2,402,728 ÷ 6257.
52. 5,191,776 ÷ 5947.

53. If a man receives a salary of $1728 a year, how much does he receive a month?

54. If a man receives a salary of $1092 for the 52 weeks of a year, how much does he receive a week?

55. If a man pays $8569 for 209 acres of land, how much does it cost per acre?

56. If a book jobber pays $3402 for 42 sets of an encyclopedia, how much does he pay per set?

57. If a stock dealer buys some cattle for $71,961, paying $51 a head, how many head does he buy?

58. If a jobber sells during the year $82,350 worth of shoes at $61 a case, how many cases does he sell?

59. If a dealer sells $142,560 worth of linseed oil during a year at $22 a barrel, how many barrels does he sell?

60. If a factory sells 3684 ready-made suits of a certain grade for $92,100, what is the price per suit?

61. If the product of two numbers is 2,137,512, and one of them is 372, what is the other number?

62. If a state pays $2,325,798 for constructing 471 miles of model road, what is the average cost per mile?

63. If it costs $5,236,602 to construct 287 miles of railroad, what is the average cost per mile?

64. What is the smallest number that must be subtracted from 615,476 so that the result divided by 1875 gives a quotient with no remainder?

65. If a company employs 8009 men on the same schedule of wages, and the pay roll for these men is $6,247,020 a year, what is the annual wages of each?

66. If a company employs 250 men on the same schedule of wages, and the pay roll for these men is $270,000 a year, what is the monthly wages of each?

64. Long Division of United States Money. Division of United States money does not involve any new principle. We divide as with other numbers, placing the decimal point in the quotient above that in the dividend.

Required to divide $21,375.12 by 372.

$$
\begin{array}{r}
\$57.46 \\
372\overline{)\$21375.12} \\
\underline{1860} \\
2775 \\
\underline{2604} \\
1711 \\
\underline{1488} \\
2232 \\
\underline{\underline{2232}}
\end{array}
$$

Since $2, $21, and $213 cannot be divided by 372 with an integer for a quotient, we divide $2137 by 372, and the quotient is $5, with remainder $277. Since the $2137 is in the tens' place of dollars, we write the $5 over that place of the dividend.

The rest of the division is as in cases already studied. The decimal point in the quotient is placed over that in the dividend, and the quotient is $57.46.

EXERCISE 24

Find the quotient of:

1. $972.36 ÷ 111.
2. $374.12 ÷ 199.
3. $1467.50 ÷ 50.
4. $6882.92 ÷ 11.
5. $12,018.30 ÷ 21.
6. $96,899.23 ÷ 23.
7. $36,147.06 ÷ 53.
8. $33,328.80 ÷ 54.
9. $33,747.20 ÷ 64.
10. $22,310.64 ÷ 72.
11. $30,796.80 ÷ 64.
12. $49,060.08 ÷ 72.
13. $34,689.75 ÷ 69.
14. $39,832.56 ÷ 81.
15. $15,391.25 ÷ 125.
16. $14,225.40 ÷ 420.
17. $734,986.12 ÷ 836.
18. $501,738.80 ÷ 538.
19. $102,465.00 ÷ 2070.
20. $692,356.00 ÷ 8764.
21. $616,693.00 ÷ 8009.
22. $388,591.00 ÷ 9037.
23. $421,203.20 ÷ 5983.
24. $5,191,731.00 ÷ 5947.
25. $5,236,602.00 ÷ 9123.
26. $3,851,640.00 ÷ 9876.

27. A man distributed the sum of $17,975.10 equally among fifteen relatives. How much did each receive?

28. A man paid $52.08 for 24 dozen cans of peas. How much did they cost him a dozen?

29. A man paid $150.72 for 96 dozen cans of tomatoes. How much did they cost him a dozen?

30. A grocer bought 94 dozen quart jars of preserved figs for $888.30. How much did they cost him a dozen?

31. A dealer paid $517.50 for 250 dozen packages of shredded cocoanut. How much did he pay a dozen?

32. If $2083.20 is divided equally among 672 men, how much does each receive?

33. If $2652.30 is divided equally among 842 men, how much does each receive?

34. A merchant pays $553.80 for 195 yards of silk. How much does he pay a yard?

35. A merchant pays $865.08 for 324 yards of velvet. How much does he pay a yard?

36. A wholesale grocer sells 175 dozen quart bottles of vinegar for $862.75. How much does he receive a dozen?

37. If $1991 a day is paid to 724 men who have the same wages in a certain shop, how much does each receive?

38. If 624 men, working for the same wages in a certain shop, receive $1996.80 a day, how much does each receive?

39. If 485 men, working for the same wages in a certain shop, receive $1503.50 a day, how much does each receive?

40. A wholesale dealer purchased 144 dozen pineapple cheeses for $1451.52. How much did he pay a dozen?

41. A shop has 664 men on its pay roll, receiving the same wages. For 6 days this pay roll amounts to $9960. How much does each man receive a day?

65. Principles of Division. It will be seen by the following examples that some interesting relations exist among the dividend, divisor, and quotient. If we consider the case of $48 \div 6 = 8$, we see that

(1) *Multiplying the dividend multiplies the quotient by the same number.*

That is, $96 \div 6 = 16$. The dividend, 48, has been multiplied by 2, and the quotient, 8, has also been multiplied by 2.

(2) *Dividing the dividend divides the quotient by the same number.*

That is, $24 \div 6 = 4$. The dividend, 48, has been divided by 2, and the quotient, 8, has also been divided by 2.

(3) *Multiplying the divisor divides the quotient by the same number.*

That is, $48 \div 12 = 4$. The divisor, 6, has been multiplied by 2, and the quotient, 8, has been divided by 2.

(4) *Dividing the divisor multiplies the quotient by the same number.*

That is, $48 \div 3 = 16$. The divisor, 6, has been divided by 2, and the quotient, 8, has been multiplied by 2.

(5) *Multiplying or dividing both dividend and divisor by the same number does not change the quotient.*

That is, if instead of $48 \div 6$ we have $96 \div 12$ or $24 \div 3$, the quotient will still be 8.

The principle involved in (5) is a very convenient principle and is used much in arithmetic.

For example, $\frac{50}{100} = 50 \div 100 = \frac{5}{10}$. Furthermore, instead of dividing 725 by 5 we may divide 1450 by 10 because it is much easier to multiply by 2 than to divide by 5. Likewise, if we wish to divide $11.25 by $3.25 we may multiply each number by 100 and then divide $1125 by $325, thus not being troubled with decimal points; again, if we wish to divide 27,235 by 25 we may multiply each number by 4 and divide 108,940 by 100, which can be done at a glance.

EXERCISE 25

REVIEW PROBLEMS

1. This girl bought for her mother 7 yards of silk at $1.35 a yard. How much did it cost?

2. Her mother sent her to buy 5 yards of serge at $1.75 a yard. How much did it cost? How much change should she receive from a $10 bill?

3. If the girl's purchases at another time consist of toweling at 80 cents, muslin at $1, needles at 45 cents, thread at 16 cents, beads at 25 cents, and braid at 24 cents, how much change should she receive from $5?

4. She saw a lady buy 13 yards of broadcloth and pay $45.50 for it. How much did it cost a yard?

5. If the merchant charges $34.65 for 21 yards of velvet, how much does he charge a yard?

6. If the merchant sells tweeds at $3.25 a yard, how much will he receive for 18 yards? How much for 37 yards?

7. If a lady buys 27 yards of silk at $1.45, 3 yards of velvet at $1.75, and 22 yards of serge at $1.60, how much does she pay in all?

8. A man pays $2.52 for 12 pounds of meat. How much does it cost a pound?

9. How many inches are there in 1 foot? How many feet are there in 432 inches?

10. If the average length of a man's step is 27 inches, how many steps will he take in going 837 inches?

11. If the average length of a man's step is 27 inches, how many inches will he go in taking 275 steps?

12. If a man earns $2.65 in each of the 27 working days of a certain month, how much does he earn in all?

13. A dealer sold 75 barrels of sweet potatoes at $2.25 a barrel. How much did he receive?

14. A dealer sold 64 barrels of sweet potatoes at $1.80 a barrel, and 36 barrels at $2.40 a barrel. How much did he receive for them all?

***15.** If 3 farmers, having equal amounts of oats to thresh, pay $38.10 for the use of a threshing machine, how much should each pay?

16. If the altitude of Mt. Everest is 29,002 feet, and that of Mt. McKinley 20,464 feet, how much higher is Mt. Everest than Mt. McKinley?

17. If the total railway mileage in the United States was 193,346 miles at one time, and 229,951 miles a few years later, how much was the increase in mileage?

18. If a school buys 3 dozen blackboard pointers at $1.15 a dozen, a dictionary holder at $6.45, a dictionary at $6.75, and a teacher's desk at $13.50, what is the total cost?

19. If the greatest known depth of the Atlantic Ocean is 27,366 feet, and Mt. Washington is 6279 feet high, how high does Mt. Washington stand above the bottom of the Atlantic at its greatest known depth?

20. A grocer sold 7 barrels of flour for $38.43. How much did he receive per barrel?

21. In making the voyage from Southampton to New York, an ocean steamer consumes 2375 tons of coal in 5 days. What is the average amount consumed per day?

22. A special train made the distance from Chicago to New York in 16 hours. The distance by the route taken was 960 miles. What was the average speed per hour?

23. Two trains leave Chicago at the same time. One travels east at the rate of 48 miles an hour and the other travels west at the rate of 39 miles an hour. How far apart are they in 4 hours?

24. Two trains start at the same time, one from New York and the other from Chicago. They travel towards each other on the same line, the former at the rate of 53 miles an hour and the latter at the rate of 48 miles an hour. The distance between New York and Chicago being 980 miles by this route, how far apart are the trains in 4 hours?

25. A farmer sowed 68 acres to clover, using 13 pounds of seed to the acre. How many pounds did he use in all? If the crop averaged 4 tons to the acre, how many tons did the farmer cut in all?

26. A coal dealer buys coal at $2.87 a ton. He pays $0.67 a ton for railway transportation and $0.78 a ton for delivery, and sells the coal for $5.25 a ton. How much profit does he make per ton?

27. A watering trough holds 118 gallons. It is cracked so that water leaks out at the rate of 24 gallons an hour, but water flows in through a feed pipe at the rate of 83 gallons an hour. If the trough is empty when the water is turned on, how long will it take to fill the trough?

EXERCISE 26

WRITTEN REVIEW QUESTIONS

1. Define units and give three illustrations.

2. Define numbers and give examples of two kinds.

3. What do you mean by integers, and what other names can you give to them?

4. What are abstract numbers? concrete numbers?

5. Distinguish between notation and numeration.

6. What are the two kinds of notation known to you? Why are they so called?

7. What do you mean by numerals? by digits?

8. What is meant by the place value of a figure?

9. What is the difference between orders and periods in writing numbers? Illustrate each.

10. How are periods indicated in the writing of numbers? Illustrate in two ways.

11. In writing United States money, how are the dollars separated from the cents? Illustrate.

12. What are the characters used in writing Arabic numbers, and those used in writing Roman numbers? Write four hundred forty-nine in each system.

13. What name is given to numbers that are to be added? What is the name of the result?

14. What is meant by like numbers? Illustrate by three pairs of like numbers.

15. How do you check the work in addition?

16. What names are given to the three numbers used in subtraction? Illustrate.

17. How do you check the work in subtraction?

18. What do you mean by multiplication? Show that it is a short process of addition.

19. What names are given to the three numbers used in multiplication? Illustrate.

20. What are factors? Illustrate.

21. May the multiplicand be abstract? May it be concrete? Illustrate both cases.

22. What is the effect of changing the order of the addends? also of the factors of a number? Illustrate by the cases of $4 + 7 + 9$, and $4 \times 7 \times 9$.

23. What is meant by a power of a number? by the square of a number? by the cube of a number? Illustrate.

24. What is the short way of multiplying an integer by 100? of multiplying United States money by 100?

25. What do you mean by division? In what way does division differ from multiplication?

26. Name the terms used in division and give an example showing the use of each.

27. If the dividend and divisor are concrete, what else may be said of their nature? What is the nature of the quotient? Illustrate.

28. May the dividend be concrete and the divisor abstract? Illustrate.

29. What is the short process of dividing an integer by 10? Illustrate by dividing 4750 by 10.

30. What is the effect on the quotient of multiplying the dividend or dividing the divisor? Illustrate each case.

31. What is the effect on the quotient of dividing the dividend or multiplying the divisor? Illustrate each case.

CHAPTER VI

DECIMAL FRACTIONS

66. Meaning of Decimals. We have already seen that \$2.25 means \$2 + 25 cents, which is the same as $\$2\frac{25}{100}$ or $\$2\frac{1}{4}$.

In the same way, 2.25 miles means $2\frac{25}{100}$ miles, or $2\frac{1}{4}$ miles; 1.50 means $1\frac{50}{100}$, or $1\frac{1}{2}$; 1.75 means $1\frac{75}{100}$, or $1\frac{3}{4}$; 1.5 means $1\frac{5}{10}$, or $1\frac{1}{2}$; 2.125 means $2\frac{125}{1000}$, or $2\frac{1}{8}$; and 0.125 means $\frac{125}{1000}$, or $\frac{1}{8}$. (See § 65.)

That is, 0.5 or .5 is a fraction whose denominator is not written, it being understood to be 10 from the fact that 5 occupies the first place to the right of the decimal point.

We therefore have the following:

0.5 means $\frac{5}{10}$, for the 5 extends to the 10th's place;
0.25 means $\frac{25}{100}$, for the 25 extends to the 100th's place;
0.125 means $\frac{125}{1000}$, for the 125 extends to the 1000th's place.

The names of the places are, in part, as follows:

Thousands	Hundreds	Tens	Units	(Decimal point)	Tenths	Hundredths	Thousandths	Ten-thousandths
1	3	4	5	.	2	7	6	5

This number is read "one thousand three hundred forty-five and two thousand seven hundred sixty-five ten-thousandths." The orders beyond ten-thousandths are hundred-thousandths, millionths, ten-millionths, and so on.

67. Decimal Fractions. A fraction whose denominator is not written, but is some power of 10, is called a *decimal fraction,* or more often simply a *decimal.*

Thus 0.25, which has the same value as $\frac{25}{100}$, is a decimal fraction.

An integer and a decimal together form a *mixed decimal;* for example, 2.25.

68. Decimal Point. The period written at the left of the tenths is called the *decimal point.*

For example, $0.7 = \frac{7}{10}$; $0.08 = \frac{8}{100}$, or $\frac{2}{25}$; $0.005 = \frac{5}{1000}$, or $\frac{1}{200}$.

It was not necessary to write a zero at the left of the decimal point in the above cases, for 0.5 means the same as .5. The zero is often written there to call attention more quickly to the decimal point.

69. Reading Decimals. We read a decimal precisely as if it were a whole number, and then give it the name of the lowest decimal place.

It is best to pronounce the word *and* at the decimal point only. Thus 100.023 is read "one hundred and twenty-three thousandths," while 0.123 is read "one hundred twenty-three thousandths."

EXERCISE 27

Read aloud the following:

1. 0.2.
2. 0.7.
3. 0.9.
4. 0.12.
5. 0.65.
6. 0.70.
7. 0.85.
8. 0.09.
9. 1.6.
10. 2.9.
11. 3.5.
12. 4.75.
13. 5.35.
14. 6.40.
15. 7.08.
16. 7.10.
17. 6.125.
18. 7.335.
19. 0.475.
20. 5.500.
21. 5.005.
22. 6.075.
23. 6.750.
24. 0.875.
25. 0.1225.
26. 0.3025.
27. 2.0025.
28. 0.0007.
29. 9.0009.
30. 200.025.
31. 0.225.
32. 220.005.

Read aloud as dollars and decimals of a dollar:

33. \$1.25. **34.** \$25.05. **35.** \$26.70. **36.** \$125.50.

70. Writing Decimals. Although we write a decimal fraction without its denominator, the denominator is shown by the decimal point and is indicated in reading.

Thus 0.9 has no printed denominator, but when we say "nine tenths" we indicate the fact that the denominator is ten as well as state that the numerator is nine.

Therefore, *write only the numerator of the decimal, inserting such zeros as are necessary to indicate the denominator, and place the decimal point before the tenths.*

Thus, to write in figures two hundred one hundred-thousandths, we write 201, and prefix two zeros, preceded by a decimal point, so that the last figure is in hundred-thousandths' place. We then have 0.00201.

EXERCISE 28

Write in figures:

1. Seven tenths; one tenth; five tenths; nine tenths.

2. Twelve hundredths; fifty-five hundredths; nine hundredths; one hundredth; twenty hundredths.

3. One hundred twenty-five thousandths; forty-seven thousandths; ten thousandths; five thousandths.

4. One thousand two hundred fifty ten-thousandths; seven hundred sixty-five ten-thousandths.

5. 5 and 63 thousandths; 7 and 70 thousandths.

6. 1352 and 135 thousandths; 60 and 60 thousandths.

7. 1000 and 1 thousandth; 1000 and 10 thousandths.

Express in figures as dollars and cents:

8. Two and seventy-five hundredths dollars.

9. Seventy-one and two hundredths dollars.

10. One hundred and eight hundredths dollars.

11. One thousand three and three hundredths dollars.

12. Two hundred four and two hundredths dollars.

71. Reduction of Decimals. Since decimals are only a special form of fractions, we may reduce them as we reduce common fractions.

For example, since $\frac{5}{10} = \frac{50}{100}$ (§ 65), therefore 0.5 = 0.50.

72. Annexing Zeros. Because, as just shown, 0.5 = 0.50, therefore

Annexing zeros to a decimal does not change its value.

73. Similar Decimals. Decimals that have the same number of decimal places are called *similar decimals.*

Thus 0.75 and 0.25 are similar decimals, and so are 0.150 and 0.275; but 0.15 and 0.275 are dissimilar decimals.

74. Reduction of Dissimilar Decimals to Similar Decimals. Since we may give any decimals the same denominators (understood, not written) by annexing or cutting off zeros,

Therefore, *to reduce dissimilar decimals to similar decimals, give them the same number of decimal places by annexing or cutting off zeros.*

Thus 0.4, 0.72, and 0.125 may all be reduced to thousandths as follows: 0.400, 0.720, and 0.125. Similarly, 0.40 and 0.600 are equal to 0.4 and 0.6.

EXERCISE 29

Reduce to similar decimals:

1. 0.2, 0.17.
2. 0.8, 0.08.
3. 0.7, 0.002.
4. 0.9, 0.130.
5. 0.6, 0.123.
6. 0.5, 0.700.
7. 0.4, 0.050.
8. 0.1, 0.357.
9. 0.12, 0.103.
10. 0.72, 0.008.
11. 0.69, 0.070.
12. 0.30, 0.700.
13. 0.99, 0.999.
14. 0.75, 0.630.
15. 0.50, 0.400.
16. 0.85, 0.392.
17. 0.001, 0.1100.
18. 0.207, 0.2376.
19. 0.412, 0.9320.
20. 0.681, 0.6881.
21. 0.900, 0.7000.
22. 0.620, 0.5700.
23. 0.090, 0.6550.
24. 0.584, 0.5834.

75. Reduction of Decimals to Common Fractions. We have $0.75 = \frac{75}{100}$, which may be reduced to $\frac{3}{4}$ (§ 65). In a similar way any decimal may be expressed as a common fraction.

Therefore, *to reduce a decimal to a common fraction, omit the decimal point, write the denominator of the decimal, and then reduce the common fraction to lowest terms.*

In a case like that of $0.33\frac{1}{3}$ we proceed as follows:

$$0.33\tfrac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{1}{3}.$$

EXERCISE 30

Reduce to common fractions in lowest terms or to mixed numbers:

1. 0.2.
2. 0.4.
3. 0.6.
4. 0.8.
5. 0.10.
6. 0.15.
7. 0.25.
8. $0.12\frac{1}{2}$.
9. $0.16\frac{2}{3}$.
10. $0.66\frac{2}{3}$.
11. 0.125.
12. 0.625.
13. 0.250.
14. 0.500.
15. 0.375.
16. 0.735.
17. 0.875.
18. $0.3\frac{1}{3}$.
19. $1.16\frac{2}{3}$.
20. $1.66\frac{2}{3}$.
21. 1.125.
22. 1.025.
23. 1.375.
24. 1.050.
25. 1.625.
26. 1.750.
27. 1.180.
28. $1.33\frac{1}{3}$.
29. $2.66\frac{2}{3}$.
30. $2.16\frac{2}{3}$.
31. 2.25.
32. 2.50.
33. 3.75.
34. 6.25.
35. 1.875.
36. 18.75.
37. 187.5.
38. $3.37\frac{1}{2}$.
39. 3.375.
40. $3.3\frac{3}{4}$.

Reduce to sixteenths:

41. $\frac{3}{4}$.
42. 0.75.
43. 0.25.
44. 0.500.

Express as mixed numbers:

45. 1.50.
46. 2.25.
47. 3.20.
48. 5.75.
49. 2.40.
50. 6.80.
51. 4.65.
52. 5.35.

76. Reduction of Common Fractions to Decimals. Since a common fraction indicates division, we may divide the numerator by the denominator and thus reduce the fraction to the decimal form. For example, consider the fraction $\frac{3}{4}$.

We have found that we may annex zeros after the decimal point without changing the value; that is, $3 = 3.00$. We may now divide exactly as in United States money, and the result is 0.75. That this is allowable may be seen from the following: 3.0 means 30 tenths, and 30 tenths $\div 4 = 7$ tenths with remainder 2 tenths. But 2 tenths $= 20$ hundredths $(\frac{2}{10} = \frac{20}{100})$, and 20 hundredths $\div 4 = 5$ hundredths. Therefore $\frac{3}{4} = 0.75$.

$$\begin{array}{r} 4\overline{)3.00} \\ 0.75 \end{array}$$

In the same way, $0.1\frac{3}{4} = 0.175$.

Therefore, *to reduce a common fraction to a decimal, place a decimal point after the numerator, and divide by the denominator as in United States money.*

A common fraction cannot always be reduced to an exact decimal. Thus $\frac{1}{3} = 0.3\frac{1}{3} = 0.33\frac{1}{3} = 0.333\frac{1}{3}$. It is customary to carry the division as far as necessary and then write a common fraction or the plus sign to indicate that the work is not complete; thus $\frac{1}{3} = 0.333\frac{1}{3}$, or $0.333 +$. Often this is written $0.33\dot{3}$, using as many places as necessary.

No common fraction equals an exact decimal if the denominator, when the fraction is reduced to lowest terms, contains other prime factors than 2 and 5.

EXERCISE 31

Reduce to decimals, carrying the reduction only to thousandths in case of inexact decimals:

1. $\frac{1}{2}$.
2. $\frac{1}{4}$.
3. $\frac{1}{8}$.
4. $\frac{3}{4}$.
5. $\frac{3}{8}$.
6. $\frac{5}{8}$.
7. $\frac{7}{8}$.
8. $\frac{1}{5}$.
9. $\frac{2}{5}$.
10. $\frac{3}{5}$.
11. $\frac{4}{5}$.
12. $\frac{1}{16}$.
13. $\frac{3}{16}$.
14. $\frac{5}{16}$.
15. $\frac{7}{16}$.
16. $\frac{15}{16}$.
17. $\frac{1}{32}$.
18. $\frac{3}{32}$.
19. $\frac{1}{3}$.
20. $\frac{2}{3}$.
21. $\frac{1}{6}$.
22. $\frac{5}{6}$.
23. $\frac{3}{25}$.
24. $\frac{7}{25}$.
25. $1\frac{9}{25}$.
26. $3\frac{4}{75}$.
27. $0.1\frac{3}{8}$.
28. $6.2\frac{7}{8}$.
29. $1.3\frac{4}{5}$.
30. $6.1\frac{1}{4}$.

77. United States Money. The system of United States money is arranged on the decimal plan. Thus,

10 mills (m.) = 1 cent (ct. or ¢)
10 cents = 1 dime (d.)
10 dimes or 100 cents = 1 dollar ($)

This table, like the common operations with United States money, is already familiar to the pupil, and is inserted only for reference.

The unit of United States money is the dollar, and the dimes and cents are written as decimal parts of the dollar.

Thus $2.48 means 2 dollars and 48 hundredths of a dollar, or 2 dollars and 48 cents, the cent being one hundredth of a dollar. Similarly, $2.08 means 2 dollars and 8 hundredths of a dollar, or 2 dollars and 8 cents.

78. Coins. The coins of the United States, as issued at present, are as follows:

Gold		*Silver*	
Double eagle,	$20.00	Dollar,	$1.00
Eagle,	10.00	Half dollar,	.50
Half eagle,	5.00	Quarter dollar,	.25
Quarter eagle,	2.50	Dime,	.10

Nickel	*Bronze*
5-cent piece (the "nickel")	1-cent piece

79. Paper Money. Bank bills, United States treasury notes, and gold and silver certificates are largely used in place of coins. These are issued in denominations of $1, $2, $5, $10, $20, $50, $100, $500, $1000, $5000, and $10,000.

Formerly paper money of smaller denominations was issued, as well as the gold dollar, the silver 20-cent, 5-cent, and 3-cent pieces, the nickel 3-cent piece, the bronze 2-cent piece, and the copper half cent. These are no longer issued. The mill was never coined.

80. Reduction of Dollars to Cents. Required to reduce \$3.75 to cents.

Since \$1 = 100 cents, therefore \$3 = 3 × 100 cents, or 300 cents; then 300 cents + 75 cents = 375 cents.

Therefore, *to reduce dollars to cents, move the decimal point two places to the right.*

81. Reduction of Cents to Dollars. Required to reduce 1425 cents to dollars, or dollars and cents.

Since 1 cent = $\$\frac{1}{100}$, therefore 1425 cents = $\$\frac{1425}{100}$ = \$14.25.

Therefore, *to reduce cents to dollars, or to dollars and cents, move the decimal point two places to the left.*

A whole number may always have a decimal point placed after it without changing its value, so if there is no decimal point given it may be placed at the right and then moved two places to the left.

EXERCISE 32

Reduce to cents:

1. \$7.
2. \$29.
3. \$35.
4. \$175.
5. \$2.40.
6. \$3.60.
7. \$9.70.
8. \$12.80.
9. \$3.45.
10. \$9.65.
11. \$8.05.
12. \$29.63.
13. \$141.75.
14. \$243.85.
15. \$692.05.
16. \$900.09.

Reduce to dollars, or to dollars and cents:

17. 300¢.
18. 450¢.
19. 575¢.
20. 606¢.
21. 732¢.
22. 522½¢.
23. 633⅓¢.
24. 166⅔¢.
25. 2642¢.
26. 4900¢.
27. 7275¢.
28. 9654¢.
29. 2¢.
30. 5¢.
31. 55¢.
32. 75¢.
33. Reduce \$27.63 to mills.
34. Reduce 5240 mills to dollars and cents.
35. Reduce 2760 dimes to dollars; to cents.

82. Addition of Decimals. We add numbers containing decimals just as we add United States money.

For example, required the sum of 14.7, 3.62, 5.25, and 26.

Arranging the addends so that units of the same order are in the same column, which is easily done by placing the decimal points under one another, we add exactly as with integers, placing the decimal point in the sum directly under the decimal points in the addends.

```
14.7
 3.62
 5.25
26.
-----
49.57
```

Therefore, *to add numbers containing decimals, write like orders under one another and add as with integers.*

If there are common fractions as well as decimals, these either may be reduced to decimals and added, or else may be added separately as is the more convenient.

EXERCISE 33

Add the following:

1.	2.	3.	4.	5.
22.05	19.26	3.	5.27⅓	4.33⅓
3.75	3.54	34.17	6.42¼	1.66⅔
16.23	7.08	16.89	7.13⅛	3.12½
4.78	16.	17.32	5.16¾	5.37½
9.6	32.89	4.08	4.29¼	4.25¾
3.09	1.96	10.	2.53⅜	9.36¼

6.	7.	8.	9.	10.
23.41	14.02	38.92	42.17⅓	33.33⅓
36.98	148.36	64.87	68.13½	16.66⅔
124.75	49.79	49.81	7.14¼	66.66⅔
4.92	31.64	324.06	18.21¼	9.95⅓
6.07	5.50	4.27	16.82¾	3.37½
13.53	20.00	82.46	7.16⅓	16.52⅓
4.81	1.97	73.20	8.23	5.23⅛

Add the following :

11.	**12.**	**13.**	**14.**	**15.**	**16.**
2.75	1.92	5.77	2.07	2.70	6.342
3.40	3.48	2.83	4.81	3.00	4.921
4.62	8.08	4.69	6.72	5.02	8.713
5.96	6.52	8.36	3.96	0.79	6.984
7.25	2.09	1.64	5.83	0.62	9.862
6.60	8.76	5.31	4.89	7.88	8.691
5.38	7.91	7.17	5.72	5.55	4.447
4.04	1.24	4.23	6.08	9.99	1.206

17.	**18.**	**19.**	**20.**	**21.**	**22.**
0.75	4.44	2.27	4.44	1.29	2.723
0.62	0.26	7.73	5.62	3.42	4.681
3.08	8.75	4.81	3.81	8.71	2.807
0.04	5.92	5.19	7.29	6.58	9.087
2.72	3.27	6.42	8.84	4.44	4.620
3.86	8.02	3.58	2.73	3.33	3.557
4.41	5.72	9.27	1.46	5.56	7.277
2.89	6.00	0.93	2.98	6.67	5.319
8.64	9.87	5.81	3.27	1.11	7.193
5.23	3.81	1.11	4.52	2.22	0.913
8.37	4.04	3.08	2.00	6.67	1.823

23. 1.2 + 3.9 + 6.8 + 5.7 + 0.7 + 6.75 + 9.81.

24. 4.82 + 5.18 + 6.97 + 3.03 + 4.44 + 5.56.

25. 1.04 + 3.08 + 9.09 + 6.67 + 5.70 + 9.87.

26. 4.035 + 2.817 + 3.628 + 4.926 + 5.2.

27. 1.009 + 0.007 + 0.809 + 0.209 + 0.007.

28. 12.006 + 3.07 + 9.001 + 4.7 + 9.00 + 40 + 3.25 + 2.06.

29. 54.476 + 28.927 + 39.824 + 91.643 + 27.123 + 92.877.

30. 127.635 + 298.436 + 481.723 + 4.590 + 3.2706 + 2.8.

31. $96.45 + 88.34\frac{3}{4} + 2.91\frac{1}{8} + 6.33\frac{1}{8}$.

32. $72.63 + 19.81\frac{1}{3} + 4.66\frac{2}{3} + 2.87\frac{3}{5}$.

33. $24.75 + 16.37\frac{1}{2} + 6.33\frac{1}{3} + 5.44\frac{1}{6}$.

34. $75\frac{1}{6} + 25.16\frac{2}{3} + 17\frac{2}{3} + 250 + 0.33\frac{1}{3}$.

35. $42\frac{1}{5} + 67.80 + 19.75 + 135\frac{1}{4} + 0.08 + 100$.

36. $33\frac{1}{3} + 25\frac{1}{2} + 17.66\frac{2}{3} + 128.50 + 0.16\frac{2}{3} + 0.83\frac{1}{3}$.

37. $82.75 + 14\frac{1}{4} + 19\frac{1}{2} + 16.50 + 67 + 0.66\frac{2}{3} + 0.33\frac{1}{3}$.

38. The distance from Canajoharie to Canastota is 67.3 miles, and from Canastota to Syracuse 20.8 miles. How far is it from Canajoharie to Syracuse by way of Canastota?

39. The distance from Syracuse to Lyons is 45.8 miles, and from Lyons to Buffalo 102.3 miles. How far is it from Syracuse to Buffalo by way of Lyons?

40. Taking the distances as given in Exs. 38 and 39, find how far it is from Canajoharie to Buffalo by this route.

41. The distance from Buffalo to Albany is 296.64 miles, and from Albany to Boston 202 miles. How far is it from Buffalo to Boston by way of Albany?

42. The distance from Weehawken to Newburgh is 56.7 miles, from Newburgh to Kingston 31.4 miles, and from Kingston to Albany 53.2 miles. How far is it from Weehawken to Albany by this route? from Newburgh to Albany? from Weehawken to Kingston?

43. If a man has 168.41 acres of land and buys $22\frac{1}{2}$ acres more of one man, and $16\frac{2}{3}$ acres of another, how many acres does he then have?

44. A man has a deed to a farm that states the area to be $198\frac{3}{4}$ acres. He has the five fields in the farm measured and finds that their areas are 21.25 acres, 33.4 acres, 82.6 acres, 58 acres, and $3\frac{1}{2}$ acres. Is the area stated in the deed correct?

83. Subtraction of Decimals. We subtract numbers containing decimals just as we subtract United States money.

Required the difference between 26.32 and 17.754.

```
26.320
17.754
 8.566
```

We first reduce the numbers to similar decimals by annexing zeros as may be necessary. Then arranging the numbers so that units of the same order are in the same column, which is easily done by placing the decimal points under one another, we subtract exactly as with integers.

Therefore, *to subtract one number from another when either contains or both contain decimals, reduce to similar decimals, write like orders under one another, and subtract as with integers.*

Any common fractions may be subtracted separately if they cannot easily be reduced to decimals. Thus $4.6\frac{2}{3} - 1.3\frac{1}{3} = 3.3\frac{1}{3}$.

EXERCISE 34

Subtract:

1.	**5.**	**9.**	**13.**	**17.**
26.75	71.81	27.6	36.91	29.433
3.96	4.68	13.01	17.935	17.5
2.	**6.**	**10.**	**14.**	**18.**
43.15	24.8	61.3	82.	48.224
7.23	4.32	27.25	16.273	9.0005
3.	**7.**	**11.**	**15.**	**19.**
62.81	34.9	34.75	48.007	72.2
9.75	6.43	16.08	23.25	65.025
4.	**8.**	**12.**	**16.**	**20.**
63.42	48.7	42.27	83.422	0.72834
4.73	4.08	17.752	16.307	0.1976

21. 7 − 0.2698.

22. 3 − 0.2709.

23. 6 − 1.0273.

24. $6.2 - 1.33\frac{1}{3}$.

25. $8.4 - 1.66\frac{2}{3}$.

26. $9.5 - 4.12\frac{1}{2}$.

27. A man having a farm of 175.25 acres sells a lot of 12½ acres. How much has he left?

28. The English sovereign is worth $4.8665. How much does this lack of $5?

29. If 629.33⅓ barrels of water are in a tank that holds 948.25 barrels, how much does it lack of being full?

30. The United States $5 gold piece contains 116.1 grains of pure gold, and the English sovereign 113.0003 grains. Find the difference in the weight of the pure gold.

31. A man has three adjacent lots with a frontage of 148.5 feet. One of the lots has a frontage of 52.75 feet and another of 49.5 feet. What is the frontage of the third lot?

32. A man who had a farm of 215.8 acres bought an adjacent farm of 192¾ acres. He sold a field of 72⅓ acres and another of 68.25 acres. How much land had he left?

33. The perimeter of a triangle is 27.25 feet, and two of the sides are 8.33⅓ feet and 9.75⅔ feet. What is the length of the third side?

34. The distance from Niagara Falls to Buffalo is 22 miles, and from Niagara Falls to North Tonawanda 10.9 miles. How far is it from Buffalo to North Tonawanda? In this and the following examples the cities are on the same line.

35. The distance from Chicago to Lincoln is 561.4 miles, and from Chicago to Colorado Springs 1072.2 miles. How far is it from Lincoln to Colorado Springs?

36. The distance from Salt Lake City to Riverside, California, is 723.7 miles, and to Pasadena by way of Riverside 791.2 miles. How far is it from Riverside to Pasadena?

37. The distance from Louisville to Greencastle is 146.5 miles, and from Louisville to Chicago 324.3 miles. How far is it from Greencastle to Chicago?

84. Multiplication of a Decimal by an Integer. In multiplying a decimal by an integer we proceed exactly as in multiplying United States money.

For example, required to multiply 2.375 by 7.

$$\begin{array}{r} 2.375 \\ 7 \\ \hline 16.625 \end{array}$$

Since 5 thousandths multiplied by 7 is 35 thousandths, we know that the 5 will go in the thousandths' place. Proceeding with the multiplication in the ordinary way, we finally place the decimal point before the 625 so as to have the 5 come in the thousandths' place as above stated.

85. Multiplication of a Decimal by a Mixed Number. Multiplication of a decimal by a mixed number may be illustrated by the case of $3\frac{1}{2} \times 2.44$.

$$\begin{array}{r} 2.44 \\ 3\frac{1}{2} \\ \hline 1.22 \\ 7.32 \\ \hline 8.54 \end{array}$$

First taking $\frac{1}{2}$ of 2.44, we know that $\frac{1}{2}$ of $\frac{244}{100} = \frac{122}{100}$, or 1.22. Then 3×4 hundredths $= 12$ hundredths, so we write the 2 in the hundredths' place. Proceeding in the ordinary way, the sum of the partial products is 8.54.

EXERCISE 35

Multiply:

1. 2×6.7.
2. 4×9.8.
3. 7×0.9.
4. 8×0.6.
5. 5×3.25.
6. 7×2.75.
7. $6 \times 3.3\frac{1}{3}$.
8. $9 \times 6.6\frac{2}{3}$.
9. 8×2.125.
10. 4×6.375.
11. 7×5.025.
12. 8×7.009.
13. 5×6.285.
14. $3 \times 3.33\frac{1}{3}$.
15. $6 \times 1.66\frac{2}{3}$.
16. $6 \times 6.66\frac{2}{3}$.
17. 8×4.125.
18. 9×3.627.
19. 8×5.328.
20. 7×6.132.
21. 8×2.312.
22. $9 \times 0.033\frac{1}{3}$.
23. $7 \times 0.277\frac{1}{7}$.
24. $5 \times 0.044\frac{2}{5}$.
25. $9 \times 6.666\frac{2}{3}$.
26. $7 \times 1.425\frac{5}{7}$.
27. 25×0.125.
28. $24 \times 0.166\frac{2}{3}$.
29. $63 \times 3.333\frac{1}{3}$.
30. 70×1.4314.
31. 143×7.007.
32. 125×1.022.
33. 225×3.123.
34. $16\frac{1}{2} \times 3.025$.
35. $35\frac{1}{4} \times 2.376$.
36. $64\frac{1}{3} \times 2.032$.
37. $48\frac{3}{4} \times 2.372$.
38. $75\frac{1}{2} \times 4.2162$.
39. $12\frac{1}{2} \times 8.8888$.

40. What is the perimeter of a triangle if each side is 19.25 inches? if each side is 16.33⅓ inches?

41. What is the perimeter of a square if each side is 6.12½ inches? if each side is 5.31⅛ inches?

42. At $0.33⅓ a yard, how much will 15 yards of cloth cost? How much will 27 yards cost?

43. If a French meter is 39.37 inches, how many inches in 12 meters? in 25 meters? in 37 meters?

44. If a French kilogram is 2.204 lb., how many pounds in 75 kilograms? in 127 kilograms?

45. If a French franc is worth $0.193, what are 125 francs worth in our money?

46. If a square foot is 0.0929 of a square meter, how many square meters in 35 square feet?

47. If an English sovereign is equivalent to $4.8665, what is the value in our money of 27 sovereigns?

48. At the rate of 49.74 miles an hour, how far will an express train travel in 6⅔ hours?

49. At the rate of 52.66⅔ miles an hour, how far will an express train travel in 3 hours?

50. If the thickness of a book is 1.15 inches, how many inches will 2 dozen such books occupy on a dealer's shelf?

51. If two sides of a rectangular field are respectively 96.8 feet and 137.6 feet, what is the perimeter of the field?

52. If a man cuts a piece of land into 17 building lots, each having an area of 0.72 of an acre, what is the area of the whole property?

53. It was estimated recently that the average cost of the actual necessities of life for each person in this country was $108.955 a year. At this rate what would be the cost for 75 persons?

86. Multiplication of a Decimal by Multiples of 10. In multiplying 3.1416 by 300 we may proceed as in § 84, but there is a shorter process that may be used when the multiplier is any multiple of 10.

Since moving the decimal point one place to the right changes tenths into units, hundredths into tenths, and so on, it multiplies the number by 10. Similarly, moving the decimal point two places to the right multiplies by 100, and so on.

$$\begin{array}{r} 3.1416 \\ 300 \\ \hline 942.48 \end{array}$$

Hence, to multiply by 300 we need only to multiply by 3 and move the decimal point two places to the right.

When necessary, zeros must be annexed to the product, as in the case of 300 × 3141.6.

EXERCISE 36

Multiply:

1. 200 × 3.42.
2. 300 × 6.28.
3. 500 × 9.36.
4. 200 × 2.48.
5. 200 × 24.8.
6. 700 × 4.062.
7. 800 × 0.317.
8. 600 × 5.214.
9. 600 × 52.14.
10. 600 × 521.4.
11. 400 × 0.1250.
12. 600 × 0.166⅔.
13. 800 × 0.012½.
14. 700 × 7.9775.
15. 8000 × 674.3.

16. A meter being 39.37 inches, how many inches are there in 4000 meters?

17. At $37.75 a front foot, what will a piece of land of 300 feet front cost?

18. If a cubic foot of water weighs 62.5 pounds, what will 2500 cubic feet weigh?

19. If the thickness of each board is 0.875 of an inch, what is the height of a pile of 80 boards?

20. The circumference of a circle being 3.1416 times the diameter, what is the circumference of a circular water tank 40 feet in diameter?

87. Multiplication by a Decimal. Let us first consider the case of 0.5 of 0.7, or 0.5 × 0.7.

$$\begin{array}{r} 0.7 \\ \underline{0.5} \\ 0.35 \end{array}$$

We learned in primary arithmetic that $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$, the numerators being multiplied together for a new numerator, and the denominators for a new denominator.

In the same way, writing the given decimals as common fractions, we have

$$\tfrac{5}{10} \times \tfrac{7}{10} = \tfrac{35}{100}.$$

Therefore 0.5 × 0.7 = 0.35.

This we see without using common fractions, for 5 × 7 = 35, and a tenth of a tenth is a hundredth, so that we have 35 hundredths.

$$\begin{array}{r} 3.24 \\ \underline{0.21} \\ 324 \\ \underline{648} \\ 0.6804 \end{array}$$

Next consider the case of 0.21 × 3.24.

We multiply in the ordinary way, and notice that 1 (hundredth) × 4 (hundredths) = 4 (ten-thousandths), and that the 4 of the product must therefore go in the ten-thousandths' place.

Therefore, *to multiply by a decimal, multiply as if by an integer, and point off as many decimal places in the product as there are decimal places in both factors together.*

Zeros should be prefixed as necessary to give the required number of places; that is, 0.07 × 0.069 = 0.00483.

EXERCISE 37

Multiply:

1. 0.7 × 0.7.
2. 0.9 × 0.9.
3. 0.8 × 1.7.
4. 0.6 × 3.9.
5. 0.5 × 7.5.
6. 0.8 × 6.7.
7. 2.3 × 4.1.
8. 3.2 × 5.7.
9. 8.7 × 9.6.
10. 0.6 × 0.31.
11. 0.8 × 0.72.
12. 0.3 × 0.88.
13. 0.6 × 1.29.
14. 0.4 × 3.21.
15. 3.17 × 4.22.
16. 7.39 × 5.81.
17. 9.75 × 6.92.
18. 3.82 × 9.89.
19. 0.7 × 0.127.
20. 0.8 × 0.125.
21. 0.3 × $0.33\frac{1}{3}$.
22. 0.6 × $0.66\frac{2}{3}$.
23. 0.9 × 2.735.
24. 4.35 × 5.006.
25. 8.25 × 9.901.
26. 7.37 × 0.073.
27. 8.24 × 0.0009.

28. A cubic foot of water weighs 62.5 pounds. Ice is 0.92 as heavy as water. What does a cubic foot of ice weigh?

29. Taking the weight of water as in Ex. 28, and knowing that cork is 0.24 as heavy as water, find the weight of a cubic foot of cork.

30. Taking the weight of water as in Ex. 28, and knowing that a certain kind of granite is 2.78 times as heavy as water, find the weight of a cubic foot of the granite.

31. Taking the weight of water as in Ex. 28, and knowing that copper is 8.95 times as heavy as water, what is the weight of a cubic foot of copper?

32. The circumference of a circle is 3.1416 times the diameter. What is the circumference of a wheel whose diameter is 42 inches?

33. From the statement made in Ex. 32, find the circumference of the earth, assuming its diameter to be 8000 miles.

34. From the statement made in Ex. 32, find the circumference of the sun, assuming its diameter to be 866,500 miles.

35. In a year when the average yield of oats in the United States was 31.2 bushels per acre, and the average price on the farm was 31.7 cents per bushel, what was the average value on the farm of the oat crop per acre?

36. In a year when the average yield of corn in the United States was 30.3 bushels per acre, and the average price on the farm was 39.9 cents per bushel, what was the average value on the farm of the corn crop per acre?

37. In a year when the average yield of wheat in the United States was 15.5 bushels per acre, and the average price on the farm was 66.7 cents per bushel, what was the average value on the farm of the wheat crop per acre?

EXERCISE 38

DRILL WORK IN MULTIPLICATION

1. 2×0.73.
2. 4×1.25.
3. 5×7.20.
4. 7×3.42.
5. $9 \times 3.3\frac{1}{3}$.
6. 12×8.75.
7. 16×9.25.
8. 25×8.80.
9. $50 \times 7.12\frac{1}{2}$.
10. 75×6.38.
11. 86×0.07.
12. 88×12.5.
13. $96 \times 33.3\frac{1}{3}$.
14. 0.3×48.75.
15. 0.7×6.281.
16. 0.9×351.1.
17. 1.2×4.211.
18. 2.1×3.922.
19. 75×44.88.
20. 7.5×448.8.
21. 32×2.111.
22. 3.2×21.11.
23. 3.2×2.111.
24. 4.5×8.275.
25. 0.45×2.45.
26. 0.07×0.095.
27. 20.6×1.27.
28. 0.75×0.84.
29. 7.5×0.84.
30. 7.5×8.40.
31. 1.25×240.
32. 12.5×2.4.
33. 0.125×24.
34. 123×23.1.
35. 12.3×2.31.
36. 4.06×4.06.
37. 40.6×40.6.
38. 2.73×4.80.
39. 27.3×0.48.
40. 4.51×2.36.
41. 57.2×0.96.
42. 43.4×0.27.
43. 2.73×0.26.
44. 0.98×0.98.
45. 5.27×63.5.
46. 4.88×0.75.
47. 2.36×0.83.
48. 0.98×9.80.
49. 17.9×62.8.
50. 2.43×2.43.
51. 6.39×1.62.
52. 89.1×6.21.
53. 5.31×3.15.
54. 2.97×3.51.
55. 5×1.278.
56. 0.5×1.278.
57. 0.005×0.007.
58. 0.003×0.123.
59. 4.72×1.0001.
60. 5.27×2.1111.
61. 65.5×48.88.
62. 6.55×4.888.
63. 275×5600.
64. 27.5×5600.
65. 2.75×5.600.
66. 5.43×1.0271.
67. 6.28×2.0002.
68. 1.01×6.0005.
69. 3.04×40.002.
70. 5.11×2.0101.
71. 7.75×6.4480.
72. 5.5×3.1416.
73. 0.55×4.7238.
74. 2.75×4.8888.
75. $5.33\frac{1}{3} \times 6.666$.
76. $2.66\frac{2}{3} \times 3.333$.
77. $8.12\frac{1}{2} \times 4.888$.
78. $1.16\frac{2}{3} \times 36.36$.

88. Division of a Decimal by an Integer. In dividing a decimal by an integer we proceed exactly as in dividing United States money.

For example, required to divide 1.561 by 7.

Here 1 ÷ 7 gives no units in the quotient, so we write 0 in the units' place; 15 tenths ÷ 7 = 2 tenths, and we write 2 in the tenths' place, considering the remainder (1 tenth) as 10 hundredths. Then 16 hundredths ÷ 7 = 2 hundredths, and we write this 2 in the hundredths' place, considering the remainder as 20 thousandths. Then 21 thousandths ÷ 7 = 3 thousandths. Hence the quotient is 0.223.

$$\begin{array}{r} 7\overline{)1.561} \\ 0.223 \end{array}$$

Zeros may be annexed as needed, to the right of the decimal point, to enable the division to proceed. Thus 6.1 ÷ 5 = 6.10 ÷ 5 = 1.22.

Therefore, *to divide a decimal by an integer divide as with integers, writing the decimal point in the quotient as soon as the decimal point in the dividend is reached.*

EXERCISE 39

Divide:

1. 4.52 ÷ 2.
2. 5.58 ÷ 9.
3. 3.15 ÷ 7.
4. 31.5 ÷ 7.
5. 0.315 ÷ 7.
6. 7.3425 ÷ 25.
7. 81.675 ÷ 75.
8. 92.100 ÷ 25.
9. 211.428 ÷ 36.
10. 215.016 ÷ 34.
11. 32.4714 ÷ 46.
12. 8838.80 ÷ 19.
13. 2.94996 ÷ 39.
14. 9.65496 ÷ 98.
15. 0.40715 ÷ 85.

16. If 12 dozen eggs cost $5.16, what does 1 dozen cost?

17. A train goes from New York to Philadelphia, 90.3 miles, in 2 hours. What is the average rate per hour?

18. A train leaves Poughkeepsie at 9 A.M. and reaches Hopewell Junction, 13.2 miles distant, at 9:29 A.M. What is the average rate per hour?

19. There being 231 cubic inches in a gallon, how many gallons in a tank containing 14,437.5 cubic inches?

89. Division by a Decimal. Since we may multiply both divisor and dividend by the same number without affecting the quotient (§ 65), we may, if the divisor contains decimal places, multiply both divisor and dividend by such a power of 10 as shall make the divisor an integer.

For example, the quotient of 1.25 ÷ 0.5 is the same as that of 12.5 by 5, and this we may find as in § 88.

Therefore, *to divide by a divisor containing decimal places, multiply both divisor and dividend by such a power of 10 as shall make the divisor an integer. Then divide as with integers, writing the decimal point in the quotient as soon as the decimal point in the dividend is reached.*

```
    14.4
12)172.8
   12
   --
    52
    48
    --
     48
     48
     --
```

Thus, in dividing 17.28 by 1.2, we multiply both divisor and dividend by 10, and then have 172.8 ÷ 12. The divisor is now an integer and we proceed as shown, finding the quotient to be 14.4. It should be noted that the decimal point in the quotient is placed exactly above that in the dividend. This arrangement of the work is sometimes called "the Austrian method" of division.

90. Division with a Remainder. If the divisor is not contained in the dividend without a remainder, zeros may be annexed to the dividend and the division continued.

Required to divide 5.273 by 0.24 to four decimal places.

```
    21.9708
24)527.3000
   48
   --
    47
    24
    ---
    233
    216
    ---
     170
     168
     ---
       200
       192
       ---
         8
```

Multiplying both divisor and dividend by 100, we have 527.3 ÷ 24. Dividing, the quotient is 21.9708 with remainder 8, and $\frac{8}{24} = \frac{1}{3}$. Therefore the quotient is $21.9708\frac{1}{3}$, or simply 21.9708 to four decimal places.

If the next figure of the quotient had been 5 or more, we should call the quotient 21.9709 to four decimal places, this being nearer the correct result than 21.9708. But since the fraction is only $\frac{1}{3}$, and we are required to go only to four decimal places, we drop the fraction and take 21.9708 as the result.

91. Division by Multiples of 10. If the divisor is an integer and ends in one zero or more than one zero, we may shorten the work by dividing both divisor and dividend by such a power of 10 as shall cut off the zeros.

For example, 42.08 ÷ 8000 gives the same quotient as 0.04208 ÷ 8. That is, we have divided both numbers by 1000 by cutting off three zeros from the divisor and moving the decimal point three places to the left in the dividend.

EXERCISE 40

Divide:

1. 1.12 ÷ 0.4.
2. 6.25 ÷ 0.5.
3. 5.04 ÷ 0.7.
4. 5.84 ÷ 0.8.
5. 7.02 ÷ 0.9.
6. 7.26 ÷ 0.6.
7. 8.44 ÷ 200.
8. 96.4 ÷ 400.
9. 70.2 ÷ 300.
10. 0.85 ÷ 500.
11. 2.72 ÷ 0.02.
12. 5.34 ÷ 0.03.
13. 3.64 ÷ 0.04.
14. 7.15 ÷ 0.05.
15. 27.6 ÷ 0.06.
16. 36.4 ÷ 0.07.
17. 0.425 ÷ 2.5.
18. 0.288 ÷ 1.2.
19. 0.057 ÷ 1.9.
20. 8.12 ÷ 0.029.
21. 36.08 ÷ 3.28.
22. 1.036 ÷ 0.56.
23. 62.32 ÷ 0.82.
24. 115.6 ÷ 0.34.
25. 3.509 ÷ 12.1.
26. 1.9044 ÷ 9.2.
27. 1.0989 ÷ 3.3.
28. 0.8118 ÷ 1.23.
29. 19.536 ÷ 44.4.
30. 17.139 ÷ 59.1.

31. A man pays $113 for coal at $5.65 a ton. How many tons does he buy?

32. The product of two numbers is 156.25 and one of the numbers is 5000. What is the other number?

33. The product of two numbers is 6.51021 and one of the numbers is 3.207. What is the other number?

34. If wind blows in a hurricane at the rate of 8796.6 feet per minute, what is the rate per second?

35. A knot, used at sea, is 1.152 miles. When a steamship is traveling 23.04 miles an hour, what is its rate in knots?

36. A contractor bought a shipment of bricks for $176.40, paying $7.20 a thousand. How many bricks did he buy?

37. A dealer paid $54.40 freight on some oil at $0.68 per thousand pounds. How many pounds were there?

38. A contractor bought a lot of shingles for $717.75, paying $4.12½ a thousand. How many thousand did he buy?

39. Wheat that cost 98 cents a bushel is sold for $1.04 a bushel. How much must be sold to realize a profit of $132?

40. How many times will a steam roller 4.25 feet in circumference turn in rolling the length of a lawn 136 feet long?

41. How many lengths of fence, each 12.75 feet long, will be required to inclose a square field that is 522.75 feet on a side?

42. A dealer receives a shipment of stoves, the total weight being 9.75 thousand pounds. He pays $70.20 freight. What is the freight per thousand pounds?

43. A wheel is 28 inches in diameter, and the circumference is 3.1416 times as much. Find the number of times the wheel turns in going a mile, 63,360 inches.

44. A cubic foot of a certain kind of granite weighs 173.75 pounds, and a cubic foot of water weighs 62.5 pounds. The granite is how many times as heavy as water?

45. Cedar is 0.61 as heavy as water, and oak is 1.17 times as heavy as water. Oak is how many times as heavy as cedar? Answer to the nearest hundredth.

46. Find to the nearest hundredth the weight of a cubic foot of air, water being 830 times as heavy as air, and a cubic foot of water weighing 1000 ounces.

47. A gallon contains 231 cubic inches. Alcohol is 0.84 as heavy as water. Find the number of cubic inches of alcohol that will weigh as much as a gallon of water.

EXERCISE 41

Drill Work in Division

1. 62.5 ÷ 25.
2. 62.5 ÷ 2.5.
3. 6.25 ÷ 2.5.
4. 62.9 ÷ 37.
5. 6.29 ÷ 37.
6. 62.9 ÷ 3.7.
7. 6.29 ÷ 3.7.
8. 80.5 ÷ 35.
9. 80.5 ÷ 3.5.
10. 8.05 ÷ 3.5.
11. 46.2 ÷ 22.
12. 46.2 ÷ 2.2.
13. 4.62 ÷ 2.2.
14. 600 ÷ 0.6.
15. 600 ÷ 0.06.
16. 6 ÷ 0.0006.
17. 627 ÷ 0.3.
18. 6.27 ÷ 0.3.
19. 62.7 ÷ 0.03.
20. 305 ÷ 6.1.
21. 3.05 ÷ 6.1.
22. 62.32 ÷ 8.2.
23. 623.2 ÷ 8.2.
24. 6.232 ÷ 8.2.
25. 1156 ÷ 3.4.
26. 11.56 ÷ 3.4.
27. 1.156 ÷ 3.4.
28. 7.14 ÷ 4.2.
29. 71.4 ÷ 4.2.
30. 7.14 ÷ 0.42.
31. 62.32 ÷ 7.6.
32. 88.44 ÷ 7.5.
33. 8190 ÷ 0.9.
34. 61.20 ÷ 1.2.
35. 7.200 ÷ 1.2.
36. 78.4 ÷ 0.08.
37. 3.12 ÷ 0.04.
38. 7.81 ÷ 1.1.
39. 8.91 ÷ 0.11.
40. 4.55 ÷ 0.07.
41. 2.73 ÷ 0.03.
42. 8.96 ÷ 12.8.
43. 4.41 ÷ 0.21.
44. 17.28 ÷ 14.4.
45. 6.25 ÷ 0.25.
46. 6.25 ÷ 2.50.
47. 2.002 ÷ 0.22.
48. 3.003 ÷ 7.7.
49. 2.65 ÷ 0.53.
50. 60.06 ÷ 0.33.
51. 72.5 ÷ 0.25.
52. 71.25 ÷ 2.5.
53. 7.375 ÷ 2.5.
54. 0.279 ÷ 0.09.
55. 12,325 ÷ 25.
56. 1232.5 ÷ 2.5.
57. 1.2325 ÷ 2.5.
58. 62.75 ÷ 125.
59. 6.275 ÷ 1.25.
60. 36.08 ÷ 32.8.
61. 3.608 ÷ 0.328.
62. 1.036 ÷ 0.560.
63. 882.32 ÷ 0.82.
64. 4.4116 ÷ 0.41.
65. 11,300 ÷ 5.65.
66. 6.51021 ÷ 3.207.
67. 651.021 ÷ 320.7.
68. 176.4 ÷ 0.0072.
69. 5.44 ÷ 0.0068.
70. 717.75 ÷ 4.125.
71. 4.41 ÷ 0.002.
72. 4.752 ÷ 2.2.
73. 4.752 ÷ 0.022.
74. 32.4714 ÷ 4.6.
75. 883.88 ÷ 0.19.
76. 32.4714 ÷ 2.3.
77. 324.714 ÷ 9.2.
78. 27.75 ÷ 0.25.

EXERCISE 42

MISCELLANEOUS PROBLEMS

1. If a family uses 3 barrels of flour a year, how much more will it spend by purchasing by the bag (of $\frac{1}{8}$ of a barrel) at 65¢, than by the barrel at $5?

2. When butter sells at 35¢ a pound, and at $2.80 per box of 10 pounds, how much will be saved on 10 pounds by purchasing by the box?

3. If a family uses 4 pounds of butter a week costing 35¢ per pound, how much would it save in 50 weeks by purchasing butter by the 10-pound box at $2.80 a box?

4. If a sponge cake is made in March, when eggs cost 30¢ a dozen, what will be the cost of the 4 eggs needed for the cake?

5. If the same kind of cake is made in December, when eggs are 66¢ a dozen, what will the eggs cost?

6. If flour costs $6.70 a barrel (196 pounds), and in smaller quantities 4¢ a pound, how much is saved in purchasing by the barrel, by a family that buys 196 pounds?

7. If a family's ice bill is $1.50 a month, and ice costs 30¢ per 100 pounds, how many pounds does the family use? If by having a better ice box it uses only 0.9 as much ice, how many pounds does it use?

8. In July a family used 1500 pounds of ice, paying 30¢ per 100 pounds, and would have used only 0.8 as much by taking care of the ice box. How much could it have saved?

9. A family with an income of $2000 spent $460 on food. What decimal of the income was spent on food?

10. A family with an income of $800 spent 0.45 of it on food. How much did it spend on food?

11. With an income of \$1100 a family spent \$275 for food. This was what decimal fraction of the income?

12. With an income of \$900 a family spent \$252 for food. This was what decimal fraction of the income?

13. A man whose income is \$2800 a year spends 0.26 of it for rent. How much does he spend for rent?

14. A man whose income is \$2800 a year spends \$728 for rent. This is what decimal fraction of the income?

15. A man spends 0.23 of his income for food, 0.26 of it for rent, and 0.1 of it for clothing. If his income is \$2400 a year, how much does he spend for each? for all three?

16. A man earns \$200 a month and spends 0.22 of it for rent. How much does he pay for rent in a year?

17. A grocer bought 6 barrels of sugar of respectively 313 pounds, 311 pounds, 307 pounds, 316 pounds, and 317 pounds, at $5\frac{5}{16}$¢ a pound. How much did the sugar cost?

18. The sides of a field are respectively 141.25 feet, 136.75 feet, 158.6 feet, and 149.8 feet. How many feet are there in the perimeter of the field?

19. A man owned 0.60 of a certain manufacturing plant, and sold $0.33\frac{1}{3}$ of his share for \$3000. At this rate, what was the whole plant worth?

20. Cedar is 0.61 as heavy as water, and marble is 2.7 times as heavy as water. Marble is how many times as heavy as cedar? Answer to the nearest hundredth.

21. White oak is 0.83 as heavy as water, and granite is 2.72 times as heavy as water. Granite is how many times as heavy as oak? Answer to the nearest hundredth.

22. A gallon contains 231 cubic inches. Gasoline is 0.64 as heavy as water. Find the number of cubic inches of gasoline that will weigh as much as a gallon of water.

23. A man bought 6250 (6.25 thousand) tiles at $7.20 a thousand. What was the total cost?

24. A man bought 6500 tiles at $7.50 a thousand. What was the total cost?

25. A man bought 13,750 shingles for $49.50. What did they cost per thousand?

26. A man bought 20,500 shingles for $73.80. What did they cost per thousand?

27. A dealer paid $0.74 per hundred pounds for a shipment of 4800 pounds of salt. How much did he pay in all?

28. A dealer paid $0.79 per hundred pounds on a shipment of 5900 pounds of salt. How much did he pay in all?

29. What will 320 barrels of apples cost at $\$2.12\frac{1}{2}$ a barrel? at $\$2.33\frac{1}{3}$ a barrel?

30. What will 640 pounds of sugar cost at $\$0.04\frac{1}{16}$ a pound? at $\$0.04\frac{5}{8}$ a pound?

31. A merchant bought 320 yards of silk for $790.40, and sold it at $2.85 a yard. How much did he gain?

32. The sides of a field are respectively 374.75 feet, 369.5 feet, 348.8 feet, and 353.25 feet. What is the perimeter of the field?

33. If the sides of a triangle are 148.3 feet, 134.75 feet, and 179.6 feet respectively, how much less is the sum of the first two sides than the sum of the second two?

34. If the sides of a field are 271.3 feet, 268.5 feet, and 283.7 feet respectively, how much less is the sum of the first two of these sides than the sum of the second two?

35. A manufacturer pays 35 men at the rate of $1.50 a day, 48 at the rate of $1.75, 30 at the rate of $\$2.66\frac{2}{3}$, and 10 at the rate of $3.30. How much is the pay roll for a week of 6 days?

36. Divide the sum of 4.78 and 5.22 by 3⅓.

37. Divide the product of 2.25 and 48 by the sum of 2.25 and 1.75.

38. Divide the product of 800.7 and 0.5 by the difference between 10.91 and 3.06.

39. Divide the product of 22.5 and 37.5 by the product of 36.25 and 21.25.

40. Divide the product of 1.33⅓ and 87.5 by the product of ⅓ and 3¾.

41. Divide the product of 36.25 and 21.25 by the product of 22.5 and 37.5.

42. If we know, in an example in division, that the divisor is 2.3 and the quotient is 5.7, what is the dividend?

43. If we know, in an example in division, that both the divisor and the quotient are 25.25, what is the dividend?

44. If the area of a triangle is represented by half the product of the base and altitude, and the base is represented by 17.25 and the altitude by 8.4, what number represents the area?

45. Mr. A owned 0.75 of a manufacturing plant and sold 0.33⅓ of his share to Mr. B, who in turn sold 0.2 of what he bought to Mr. C. What part of the plant did Mr. C buy?

46. A man bought three barrels of granulated sugar containing respectively 312.75 pounds, 314.75 pounds, and 316.5 pounds. He paid 5.375¢ a pound for the sugar. How much did the sugar cost him?

47. A man has to make a railway journey of 254.7 miles. After he has gone 0.33⅓ of the distance he receives a telegram calling him back to a place half the distance he has already gone. After he gets back there, how many miles is it to the end of his journey?

EXERCISE 43

Questions about Decimals

1. What is the unit of United States money, and what sign is used to indicate it?

2. How many places of decimals do cents (including dimes) occupy? How many places do dimes occupy?

3. Write two decimals showing the part of a dollar a cent is worth, and the part of a dollar a mill is worth.

4. Write three decimals showing the part of a dollar a dime is worth, the part of a dime a cent is worth, and the part of a cent a mill is worth.

5. How are dollars reduced to cents? How are dollars reduced to mills?

6. How are cents reduced to dollars? How are mills reduced to dollars?

7. How are dollars and cents arranged for addition or subtraction? Illustrate each operation.

8. In multiplying dollars and cents by an integer, how do you know where to place the decimal point in the product?

9. In dividing dollars and cents by an integer, how do you know where to place the decimal point in the quotient?

10. A unit consists of how many tenths? of how many hundredths? of how many thousandths?

11. A tenth consists of how many hundredths? of how many thousandths? of how many ten-thousandths?

12. How is the value of a figure in a number changed if it is moved one place to the left? one place to the right?

13. What decimal place is occupied by tenths? by hundredths? by thousandths? by ten-thousandths? by hundred-thousandths? by millionths?

14. How should you read a number that has an integral part and a decimal part? Illustrate by reading 200.007.

15. What effect does it have on a decimal to annex zeros to it? Explain this in the case of 0.5 and 0.50.

16. In adding and subtracting decimals, how do you arrange the decimal points? Why is this?

17. In subtraction of decimals, if the minuend has a smaller number of decimal places than the subtrahend how do you proceed? What right have you to do this?

18. What effect does it have on the value of a number if the decimal point is moved one place to the right? one place to the left?

19. What effect does it have on the value of a number if the decimal point is moved two places to the right? two places to the left?

20. In the multiplication of decimals, how many decimal places must the product have? Illustrate by the case of 0.7 and 0.25.

21. Explain why the product of 0.6 and 0.4 should have two decimal places, by reducing both to common fractions and then multiplying.

22. In division, if the divisor is not an integer, how may you make it such without altering the quotient?

23. If the divisor is an integer, how can you tell where to put the decimal point in the quotient? Illustrate by dividing 41.44 by 4.

24. What is the reason for writing the quotient over the dividend? Give an example to assist in your explanation.

25. Why do you not have to write the denominators in decimal fractions, while you do have to write them in common fractions?

92. Aliquot Parts. An exact divisor of a number is called an *aliquot part* of that number.

Thus 5, 10, 25, $12\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{1}{3}$, are aliquot parts of 100.

We may also have aliquot parts of numbers besides 100. For example, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are important aliquot parts of 10; 125, $333\frac{1}{3}$, and 250 are important aliquot parts of 1000, and so on. The aliquot parts of 100 and \$10 are, however, the ones most frequently used.

93. Important Aliquot Parts. The following fractional parts of \$1 should be memorized:

$5\text{¢} = \$\frac{1}{20} = \$0.05$
$6\frac{1}{4}\text{¢} = \$\frac{1}{16} = \$0.06\frac{1}{4}$
$8\frac{1}{3}\text{¢} = \$\frac{1}{12} = \$0.08\frac{1}{3}$
$10\text{¢} = \$\frac{1}{10} = \$0.10$
$12\frac{1}{2}\text{¢} = \$\frac{1}{8} = \$0.125$
$16\frac{2}{3}\text{¢} = \$\frac{1}{6} = \$0.16\frac{2}{3}$
$20\text{¢} = \$\frac{1}{5} = \$0.20$
$25\text{¢} = \$\frac{1}{4} = \$0.25$
$30\text{¢} = \$\frac{3}{10} = \$0.30$
$33\frac{1}{3}\text{¢} = \$\frac{1}{3} = \$0.33\frac{1}{3}$
$37\frac{1}{2}\text{¢} = \$\frac{3}{8} = \$0.375$
$40\text{¢} = \$\frac{2}{5} = \$0.40$
$50\text{¢} = \$\frac{1}{2} = \$0.50$
$60\text{¢} = \$\frac{3}{5} = \$0.60$
$62\frac{1}{2}\text{¢} = \$\frac{5}{8} = \$0.625$
$66\frac{2}{3}\text{¢} = \$\frac{2}{3} = \$0.66\frac{2}{3}$
$70\text{¢} = \$\frac{7}{10} = \$0.70$
$75\text{¢} = \$\frac{3}{4} = \$0.75$
$80\text{¢} = \$\frac{4}{5} = \$0.80$
$87\frac{1}{2}\text{¢} = \$\frac{7}{8} = \$0.875$
$90\text{¢} = \$\frac{9}{10} = \$0.90$
$100\text{¢} = \$1 = \$1.00$

The most important aliquot parts of 100, including those that should be memorized, are as follows:

$50 = \frac{1}{2}$ of 100
$25 = \frac{1}{4}$ of 100
$12\frac{1}{2} = \frac{1}{8}$ of 100
$6\frac{1}{4} = \frac{1}{16}$ of 100
$33\frac{1}{3} = \frac{1}{3}$ of 100
$16\frac{2}{3} = \frac{1}{6}$ of 100
$8\frac{1}{3} = \frac{1}{12}$ of 100
$4\frac{1}{6} = \frac{1}{24}$ of 100
$20 = \frac{1}{5}$ of 100
$10 = \frac{1}{10}$ of 100
$5 = \frac{1}{20}$ of 100
$2 = \frac{1}{50}$ of 100

Other important fractional parts of 100, but not aliquot parts, are as follows:

$75 = \frac{3}{4}$ of 100
$37\frac{1}{2} = \frac{3}{8}$ of 100
$62\frac{1}{2} = \frac{5}{8}$ of 100
$87\frac{1}{2} = \frac{7}{8}$ of 100
$66\frac{2}{3} = \frac{2}{3}$ of 100
$83\frac{1}{3} = \frac{5}{6}$ of 100

As in the case of aliquot parts, these should be memorized.

94. Multiplication by Aliquot Parts. We may simplify multiplication by 10, 100, and 1000, and by their aliquot parts, as follows:

(1) *To multiply by 10, move the decimal point one place to the right, annexing zero if necessary.*

Thus $10 \times 46.75 = 467.5$; $10 \times 75 = 750$.

(2) *To multiply by 100, move the decimal point two places to the right, annexing zeros if necessary.*

Thus $100 \times 0.275 = 27.5$; $100 \times 6.5 = 650$; $100 \times 72 = 7200$.

(3) *To multiply by 1000, move the decimal point three places to the right, annexing zeros if necessary.*

Thus $1000 \times 0.02865 = 28.65$; $1000 \times 147 = 147{,}000$.

(4) *To multiply by 50, multiply by 100 and divide by 2; to multiply by 0.5, divide by 2.*

Thus $50 \times 84 = \frac{100}{2} \times 84 = \frac{8400}{2} = 4200$; $0.5 \times 8 = \frac{1}{2}$ of $8 = 4$.

(5) *To multiply by 25, multiply by 100 and divide by 4; to multiply by 0.25, divide by 4.*

Thus $25 \times 84 = \frac{100}{4} \times 84 = \frac{8400}{4} = 2100$; $0.25 \times 8 = \frac{1}{4}$ of $8 = 2$.

(6) *To multiply by $12\frac{1}{2}$, multiply by 100 and divide by 8; to multiply by $0.12\frac{1}{2}$, divide by 8.*

Thus $12\frac{1}{2} \times 64 = \frac{100}{8} \times 64 = 800$; $0.12\frac{1}{2} \times 24 = \frac{1}{8}$ of $24 = 3$.

(7) *To multiply by 125, multiply by 1000 and divide by 8; to multiply by 0.125, divide by 8.*

Thus $125 \times 72 = \frac{1000}{8} \times 72 = 9000$; $0.125 \times 16 = \frac{1}{8}$ of $16 = 2$.

(8) *To multiply by $33\frac{1}{3}$, multiply by 100 and divide by 3; to multiply by $0.33\frac{1}{3}$, divide by 3.*

Thus $33\frac{1}{3} \times 69 = \frac{6900}{3} = 2300$; $0.33\frac{1}{3} \times 15 = \frac{1}{3}$ of $15 = 5$.

(9) *To multiply by $16\frac{2}{3}$, multiply by 100 and divide by 6; to multiply by $0.16\frac{2}{3}$, divide by 6.*

(10) *To multiply by 5, multiply by 10 and divide by 2.*

EXERCISE 44

Multiply:

1. $10 \times \$0.25$.
2. $10 \times \$8251$.
3. $10 \times \$6.25$.
4. $10 \times \$5.50$.
5. $100 \times \$0.37$.
6. $100 \times \$2.87\frac{1}{2}$.
7. $1000 \times \$0.25$.
8. $1000 \times \$6.37\frac{1}{2}$.
9. $50 \times \$64{,}224$.
10. $50 \times \$752.20$.
11. $50 \times \$861.55$.
12. $25 \times \$144$.
13. $25 \times \$640$.
14. $25 \times \$325$.
15. $675 \times \$25$.
16. $25 \times \$25.50$.
17. $25 \times \$75.25$.
18. $12\frac{1}{2} \times \$6400$.
19. $12\frac{1}{2} \times \$4880$.
20. $125 \times \$640$.
21. $125 \times \$6400$.
22. $125 \times \$2800$.
23. $33\frac{1}{3} \times 36$.
24. $33\frac{1}{3} \times 48$.
25. $33\frac{1}{3} \times 144$.
26. $33\frac{1}{3} \times \$312.30$.
27. $16\frac{2}{3} \times 12{,}630$.
28. $16\frac{2}{3} \times \$636.30$.
29. $5 \times 47{,}664.80$.
30. $5 \times \$210{,}650$.
31. $125 \times \$84{,}480$.
32. $125 \times \$844.80$.
33. $16\frac{2}{3} \times 4{,}800{,}000$.

34. What will 25 books cost at \$1.20 each?

35. What will 25 wagons cost at \$116 each?

36. What will 50 cows cost at \$54 a head?

37. What will 100 books cost at $62\frac{1}{2}$¢ each?

38. What will 25 tons of hay cost at \$12.40 a ton?

39. What will $12\frac{1}{2}$ yards of cloth cost at 36¢ a yard?

40. What will 25 dozen pencils cost at \$0.48 a dozen?

41. What will 48 office desks cost at \$25 each?

42. At \$5 an acre, what is a tract of 14,264 acres of mountain land worth?

43. How much will a man earn in 36 weeks at \$25 a week? at $\$33\frac{1}{3}$ a week?

44. How much will a dozen tables weigh at 25 pounds each? at 50 pounds each?

45. How much will 615 men earn in a week if each man earns $\$12\frac{1}{2}$? if each man earns $\$16\frac{2}{3}$?

95. Multiplication by Fractional Parts. We may simplify multiplication by fractional parts as follows:

(1) *To multiply by 0.75, take* $\frac{3}{4}$ *of the number.*

For example, $0.75 \times \$64.40 = \frac{3}{4}$ of $\$64.40 = \$48.30$. Similarly, to multiply by 75, multiply by 100 and take $\frac{3}{4}$ of the product.

(2) *To multiply by* $0.37\frac{1}{2}$, *take* $\frac{3}{8}$ *of the number.*

For example, $0.37\frac{1}{2} \times \$72 = \frac{3}{8}$ of $\$72 = \$27$. Similarly, to multiply by $37\frac{1}{2}$, multiply by 100 and take $\frac{3}{8}$ of the product. Proceed in the same way in (3), (4), and (5), following.

(3) *To multiply by* $0.62\frac{1}{2}$, *take* $\frac{5}{8}$ *of the number.*

For example, $0.62\frac{1}{2} \times 640 = \frac{5}{8}$ of $640 = 400$.

(4) *To multiply by* $0.87\frac{1}{2}$, *take* $\frac{7}{8}$ *of the number.*

(5) *To multiply by* $0.66\frac{2}{3}$, *take* $\frac{2}{3}$ *of the number.*

EXERCISE 45

Multiply:

1. 0.75×684.
2. 0.75×816.
3. $0.75 \times \$1.48$.
4. $0.75 \times \$3.20$.
5. $248 \times \$0.75$.
6. $432 \times \$0.75$.
7. 0.75 of 6884.
8. $736 \times \$0.37\frac{1}{2}$.
9. $736 \times \$0.62\frac{1}{2}$.
10. $824 \times \$0.87\frac{1}{2}$.
11. $366 \times \$0.66\frac{2}{3}$.
12. $174 \times \$0.16\frac{2}{3}$.
13. $744 \times 0.87\frac{1}{2}$.
14. $0.66\frac{2}{3} \times 3366$.
15. $0.66\frac{2}{3} \times 288.6$.
16. $0.66\frac{2}{3} \times 28.86$.
17. $48{,}800 \times 7.5$.
18. $0.12\frac{1}{2} \times 0.16$.

19. How much will 75 cows cost at $48 each?

20. How much will 75 cows cost at $55 each?

21. How much will 72 horses cost at $75 each?

22. How much will 88 horses cost at $125 each?

23. How much will 288 whalebones cost at 25¢ each?

24. How much will 275 packing boxes cost at $62\frac{1}{2}$¢ each?

25. How much will 484 yards of silk cost at 75¢ a yard?

26. How much will 364 yards of cloth cost at 75¢ a yard?

96. Division by Fractional Parts. There are certain divisors, some of them aliquot parts of 1, 10, 100, or 1000, that are used so often that it is convenient to know short methods of dividing by them.

The principal ones are 10, 100, 1000, and other powers of 10; 5, 25, 125, $12\frac{1}{2}$, and $33\frac{1}{3}$; and 0.5, 0.25, 0.125, and $0.33\frac{1}{3}$.

Since division is the inverse of multiplication, we easily derive the following from the cases already considered:

(1) *To divide by 10, move the decimal point one place to the left; by 100, two places; by 1000, three places; and so on.*

Thus $4936 \div 10 = 493.6$; $\$4876 \div 100 = \$48.76$; $29.5 \div 1000 = 0.0295$. This has already been explained in division of decimals

(2) *To divide by 5, multiply by 2 and divide by 10; to divide by 0.5, multiply by 2.*

For to divide by 5 is to multiply by $\frac{1}{5}$ or $\frac{2}{10}$. To divide 32,305 by 5 we may multiply by 2, obtaining 64,610, and cut off the zero, obtaining 6461, a process somewhat easier than actual division.

(3) *To divide by 25, multiply by 4 and divide by 100; to divide by 0.25, multiply by 4.*

For $\frac{1}{25} = \frac{4}{100}$. It is much easier to multiply by 4 and move the decimal point two places than to divide by 25. Furthermore, $0.25 = \frac{1}{4}$, and to divide by $\frac{1}{4}$ we merely multiply by 4.

(4) *To divide by 125, multiply by 8 and divide by 1000; to divide by 0.125, multiply by 8.*

Why is this true? To what common fraction is 0.125 equal? How do we divide by this common fraction?

(5) *To divide by $33\frac{1}{3}$, multiply by 3 and divide by 100; to divide by $0.33\frac{1}{3}$, multiply by 3.*

Why is this true? To what common fraction is $0.33\frac{1}{3}$ equal?

(6) *To divide by $0.66\frac{2}{3}$, multiply by 3 and divide by 2.*

Why is this true?

(7) *To divide by 0.75, multiply by 4 and divide by 3.*

EXERCISE 46

Divide:

1. $7463 ÷ 10.
2. $2896 ÷ 100.
3. $4960 ÷ 1000.
4. $2215.50 ÷ 5.
5. $3122.35 ÷ 5.
6. $172,232 ÷ $5.
7. $9125 ÷ $25.
8. 12,325 ÷ 25.
9. 31,325 ÷ 125.
10. 62,750 ÷ 125.
11. $1525 ÷ 33⅓.
12. $3275 ÷ 33⅓.
13. 76 ÷ 0.33⅓.
14. 87 ÷ 0.33⅓.
15. 96 ÷ 0.66⅔.
16. 68 ÷ 0.66⅔.
17. 96 ÷ 0.75.
18. 69 ÷ 0.75.

19. At 25¢ a yard, how many yards of gingham can a dealer buy for $124?

20. At 25¢ a yard, how many yards of percaline can a dealer buy for $160?

21. At 25¢ a dozen, how many dozen spools of darning cotton can a dealer buy for $180?

22. At 33⅓¢ a yard, how many yards of canvas can a dealer buy for $135?

23. At 33⅓¢ a yard, how many yards of jute burlap can a dealer buy for $165?

24. At $3.33⅓ a day, how many days must a man work in order to earn $130?

25. At 3⅛¢ a yard, how many yards of cheesecloth can a dealer buy for $115?

26. At 2½¢ a skein, how many skeins of cotton must a manufacturer sell in order to receive $125.75?

27. At $1.25 a box, how many boxes of fancy candies can a dealer buy for $180?

28. At $125 each, how many horses can a dealer buy for $1750? for $1375?

29. At $5 each, how many rocking-chairs must a factory sell in order to receive $3375?

30. How much will 144 boxes cost at 37½¢ each?

31. How much will 22 typewriters cost at $62½ each?

32. How much will 16 acres of land cost at $87½ an acre?

33. How much will 375 mules cost at $125 each?

34. If 25 tons of coal cost $187.50, what will be the cost of 99 tons?

35. How much will 288 dozen sewing-machine needles cost at 37½¢ a dozen?

36. How much will 250 dozen yards of millinery braid cost at 62½¢ a dozen?

37. There are 44,000 yards in 25 miles. How many yards are there in one mile?

38. How much will 18 thousand feet of oak flooring cost at $125 a thousand feet?

39. How many bushels of oats will 81 acres produce, if an average acre produces 33⅓ bushels?

40. How many bushels of potatoes will 32 acres produce, if an average acre produces 125 bushels?

41. A tailor paid $210 for some broadcloth at $3.33⅓ a yard. How many yards did he buy?

42. A dealer sold some overcoats for $2900, at the rate of $33⅓ apiece. How many overcoats did he sell?

43. How much will 16 thousand feet of mixed red and white birch flooring cost at $50 a thousand feet?

44. A manufacturer sells $1810 worth of silk at $1.25 a yard. How many yards does he sell?

45. A manufacturer sells $2777.50 worth of velvet at $2.50 a yard. How many yards does he sell?

46. A farmer owns 250 acres of land covered with a growth of white pine which he values at $166 an acre. At what amount does he value the property?

EXERCISE 47

Drill Work in Fractional Parts

Multiply:

1. 0.25 × 488.
2. 2.5 × 488.
3. 25 × 488.
4. 0.25 × 4.88.
5. 0.75 × 644.
6. 7.5 × 644.
7. 75 × 644.
8. 0.75 × 6.44.
9. 0.75 × 2436.
10. 3574 × 0.05.
11. 0.25 × 536.
12. 0.25 × 5.36.
13. 0.25 × 0.536.
14. 704 × $0.25.
15. 172 × $0.25.
16. 776 × $0.12½.
17. 0.37½ × 424.
18. 3.7½ × 424.
19. 37½ × 424.
20. 37½ × 4.24.
21. 0.62½ × 584.
22. 6.25 × 584.
23. 6.2½ × 5.84.
24. 0.62½ × 0.584.
25. 592 × $0.87½.
26. 368 × $0.12½.
27. 336 × $0.33⅓.
28. 297 × $0.33⅓.
29. 372 × $0.66⅔.
30. 492 × $0.66⅔.
31. 558 × $0.16⅔.
32. 174 × $0.16⅔.
33. 0.87½ × 728.
34. 0.87½ × 7.28
35. 8.75 × 72.8.
36. 8.7½ × 7.28.
37. 875 × 728.
38. 0.66⅔ × 546.
39. 6.6⅔ × 546.
40. 6.66⅔ × 546.
41. 2952 × 0.05.
42. 248 × 0.05.
43. 24.8 × 0.5.
44. 2.48 × 0.5.
45. 24 × 7.5.
46. 2.4 × 7.5.
47. 2.4 × 0.75.
48. 0.12½ × 0.16.

Divide:

49. 485 ÷ 0.25.
50. 526 ÷ 0.12½.
51. 17,216 ÷ 5.
52. 436 ÷ 25.
53. 528 ÷ 2.5.
54. 232 ÷ 125.
55. 232 ÷ 12.5.
56. 328 ÷ 1.25.
57. 564 ÷ 0.125.
58. 275 ÷ 33⅓.
59. 326 ÷ 3.3⅓.
60. 290 ÷ 0.33⅓.
61. 428 ÷ 0.66⅔.
62. 524 ÷ 6.6⅔.
63. 424 ÷ 6.66⅔.
64. 520 ÷ 0.6¼.
65. 390 ÷ 0.5.
66. 4.5 ÷ 0.05.
67. 288 ÷ 0.75.
68. 2.88 ÷ 7.5.
69. 666 ÷ 0.75.

97. Cancellation. The process of shortening work in division by dividing both divisor and dividend by the same number is called *cancellation.*

For example, what is the quotient when 17.8 × 45.39 is divided by 0.267?

Multiplying both divisor and dividend by 1000, to make the divisor integral, we have for the dividend 178 × 4539 (i.e. 10 times 17.8 and 100 times 45.39). Canceling, we have 2 × 1513 = 3026.

$$\frac{\overset{2 \times \cancel{89}}{\cancel{178}} \times \overset{1513}{\cancel{4539}}}{\underset{\cancel{3} \times \cancel{89}}{\cancel{267}}}$$

EXERCISE 48

1. $\dfrac{3.9 \times 0.84}{15.6}$.

2. $\dfrac{1.4 \times 0.6}{0.63}$.

3. $\dfrac{2.2 \times 2.4}{4 \times 7.7}$.

4. $\dfrac{5.5 \times 0.42}{0.77}$.

5. $\dfrac{4.5 \times 6.3}{8.1}$.

6. $\dfrac{5.6 \times 0.3}{0.63 \times 0.8}$.

7. $\dfrac{2.1 \times 0.4}{0.015}$.

8. $\dfrac{0.21 \times 0.6}{0.126}$.

9. $\dfrac{1.8 \times 5.5}{9.9 \times 0.1}$.

10. The product of 6.3, 1.4, and 5.7 is how many times as great as the product of 17.1, 0.06, and 7?

11. The product of 0.2, 0.21, and 7.5 is how many times as great as the product of 0.14, 0.3, and 1.5?

12. How many times is the product of 1.6, 0.01, and 1.5 contained in the product of 0.032, 2.25, and 3?

13. $\dfrac{0.8 \times 1.5 \times 0.6}{3.6 \times 0.2 \times 10}$.

14. $\dfrac{1.2 \times 0.39 \times 1.4}{2.6 \times 0.42 \times 2}$.

15. $\dfrac{0.2 \times 3.6 \times 1.43}{9.9 \times 0.4 \times 2.6}$.

16. $\dfrac{6.3 \times 4.2 \times .17}{2.7 \times 3.4 \times .49}$.

17. The volume of a box whose interior dimensions are 2.4 inches, 5 inches, and 1.7 inches, is how many times as great as that of one whose interior dimensions are 3 inches, 1.6 inches, and 0.85 inches?

EXERCISE 49

DRILL WORK IN DECIMALS

1. Why will there be three decimal places in the product of 0.75 multiplied by 0.3? of 2.325 multiplied by 7?

Exs. 1–9 illustrate the oral work that should be given frequently.

Without multiplying, state the number of decimal places in each of the following products:

2. 2.8 × 7.9. **4.** 2.007 × 6.09. **6.** 1.203 × 2.67.

3. 1.7 × 1.42. **5.** 1.234 × 3.456. **7.** 3.421 × 0.713.

8. Multiply 1.21 by 2; by 3; by 4; by 10; by 100.

9. Divide 1.05 by 5; by 10; by 2; by 3; by 0.1.

Add the following:

10.	**11.**	**12.**	**13.**
4.028	0.387	1.043	0.2909
0.396	0.0426	0.427	0.0628
0.285	0.0539	6.091	0.0703
4.72	0.2064	0.5	0.5076
3.942	0.0708	0.709	0.0289
0.409	0.5029	6.273	0.0079
6.029	0.0627	0.108	0.0403

Subtract:

14.	**15.**	**16.**	**17.**
1.035	0.2809	82.481	64.035
0.876	0.1589	2.909	37.029

18.	**19.**	**20.**	**21.**
86.431	102.98	62.035	80.289
28.902	72.89	51.975	79.939

Add the following, timing yourself:

22.	**23.**	**24.**	**25.**
$1000.00	$824.63	$209.09	$802.35
289.00	981.42	32.78	608.72
40.27	127.69	45.67	402.81
6.35	842.38	682.73	96.43
42.07	327.29	102.93	87.29
189.27	420.63	41.52	289.62

Copy and subtract, timing yourself:

26. $640.00 – $298.29.
27. $902.63 – $127.49.
28. $3476.42 – $1029.29.
29. $702.53 – $129.65.
30. $1000.00 – $298.75.
31. $2025.30 – $1627.92.

Multiply the following:

32. 234.5 by 8.001.
33. $683.45 by 0.2.
34. $283.75 by 42.8.
35. 0.500 by 0.600.
36. $281.42 by 6.50.
37. $333.33⅓ by 0.3.

38. At $17.52 a front foot, what will a piece of land of 28.33⅓ feet front cost?

39. A tank when full holds 42.8 gallons of water. How much does it contain when 0.37 full?

40. A certain rectangle is 0.78 as wide as it is long. It is 4.9 inches long. What is the perimeter?

41. A certain rectangle is 3.42 times as long as it is wide. It is 4.8 feet wide. What is the perimeter?

42. A city lot is 45.8 feet by 76.3 feet. What is the area of the lot? What is it worth at $1.50 a square foot?

43. If the circumference of a circle is 3.1416 times the diameter, what is the circumference of a circle the diameter of which is 42.27 feet?

44. At $1.75 each, what will 17 books cost?

45. Find the perimeter of a square 34.7 feet on a side.

46. At $75.50 an acre, what will 25.5 acres of land cost?

47. At $0.75 a yard, what will 32.2 yards of silk cost?

48. At $125 a front foot, how much will a city lot cost having a frontage of 72.3 feet?

Multiply:

49. 1.4 × 6.64.
50. 3.6 × 19.72.
51. 43.2 × 98.6.
52. 0.823 × 2946.
53. 48.1 × 32.5.
54. 26.8 × 80.8.
55. 60.9 × 90.6.
56. 4.72 × 6.93.
57. 2.87 × 5.492.
58. 16.4 × 17.25.
59. 6.81 × 7.92.
60. 8.35 × 6.423.
61. 89.1 × 49.3.
62. 72.9 × 87.09.
63. 20.5 × 60.7.
64. 20.2 × 20.2.
65. 73.7 × 73.7.
66. 42.3 × 5.37.
67. 1.5 × 553.35.
68. 5.8 × 439.002.
69. 30.5 × 57.2.
70. 8.25 × 6.37.
71. 68.7 × 40.5.
72. 53.6 × 2.889.
73. 43.7 × 74.1.
74. 1.6 × 140.64.
75. 6.2 × 53.134.
76. 17.2 × 26.35.
77. 2.175 × 6.73.
78. 0.273 × 0.625.

Divide:

79. 17.25 ÷ 500.
80. 817.6 ÷ 700.
81. 0.7364 ÷ 7000.
82. 3.248 ÷ 200.
83. 82.48 ÷ 8000.
84. $32.48 ÷ 8.
85. $26.25 ÷ 25.
86. $21.42 ÷ 18.
87. $99.54 ÷ 14.
88. $82.20 ÷ 12.
89. 20.720 ÷ 0.28.
90. 38.8692 ÷ 5.4.
91. 825.468 ÷ 4.2.
92. 888.16 ÷ 2.8.
93. 1131.264 ÷ 4.8.

In the following divisions, carry the quotient to three decimal places:

94. 17.78 ÷ 1.5.
95. 423.6 ÷ 0.27.
96. 976.34 ÷ 0.125.
97. 167.8 ÷ 3.7.
98. 62.23 ÷ 0.23.
99. 483.62 ÷ 42.7.
100. 19.26 ÷ 3.1.
101. 8.348 ÷ 2.68.
102. 826.3 ÷ 12.5.

CHAPTER VII

DENOMINATE NUMBERS

98. Denominate Number. A concrete number that expresses measure is called a *denominate number.*

For example, $7 and 2 feet 3 inches are denominate numbers, but not 5 chairs or 8 rivers.

99. Compound Number. A denominate number involving two or more different units is called a *compound number.*

For example, 2 feet 3 inches, or 3 hours 20 minutes. We do not usually speak of $2.50 as a compound number because it is written as an integer and a decimal fraction. If we should write 2¼ feet or 2.25 feet instead of 2 feet 3 inches, we should not be using compound numbers.

Some work in compound numbers is done in the primary grades, and many of the common measures are therefore already known.

100. Measures of Length. The table of common measures of length is as follows:

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
5½ yards, or 16½ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

A *hand* (4 in.) is used in measuring the height of horses; a *fathom* (6 ft.) and *cable length* (120 fathoms) in measuring depths of water; a *knot* (nautical mile, 1.152 common or statute miles, or 6080.27 ft.) in measuring distances at sea. Carpenters and mechanics usually write 2′ 6″ for 2 ft. 6 in.

101. Reduction. The process of changing the unit of a number without changing the value is called *reduction.*

Thus 2 ft. 6 in. = 30 in. = 2.5 ft.

102. Reduction Descending. Reduction from a higher to a lower denomination is called *reduction descending*.

For example, required to reduce 2 ft. 3 in. to inches.

$$\begin{aligned} 1 \text{ ft.} &= 12 \text{ in.}, \\ 2 \text{ ft.} &= 2 \times 12 \text{ in.} \\ &= 24 \text{ in.} \end{aligned}$$

Therefore 24 in. + 3 in. = 27 in.

$$\begin{array}{rr} \text{ft.} & \text{in.} \\ 2 & 3 \\ 12 & \\ \hline 24 & \\ 3 & \\ \hline 27 & \end{array}$$

For convenience the actual multiplications and additions are carried out as shown at the right.

In practical life we seldom have reductions involving more than two different units (denominations), and in this arithmetic other cases will not in general be considered.

Therefore, *in reduction descending, multiply the number of the highest denomination given, by the number showing how many units of the next lower denomination are equal to one of the higher. To the product add the given number of this lower denomination.*

Proceed thus with each successive result until the required denomination is reached.

EXERCISE 50

Reduce:

1. 54 ft. 7 in. to inches.
2. 72 ft. 9 in. to inches.
3. 28 yd. to feet; to inches.
4. 5½ yd. to feet; to inches.
5. 26 rd. to yards; to feet.
6. 320 rd. to yards; to feet.
7. 41.8 mi. to miles and feet.
8. 25.75 mi. to miles and feet.
9. 126.7 mi. to miles and feet.
10. 160 rd. to yards.
11. 5 mi. to rods.
12. 7 mi. to yards.
13. 22½ mi. to rods.
14. 3.25 mi. to feet.
15. 0.5 mi. to feet.
16. 0.6 mi. to feet.
17. 0.65 mi. to feet.
18. 0.01 mi. to yards.

19. A knot being 1.152 miles, how many miles does a ship sail in going 480 knots?

103. Reduction Ascending. Reduction from a lower to a higher denomination is called *reduction ascending.*

For example, required to reduce 104 in. to higher units.

For practical purposes the work may be arranged as shown at the left, but the explanation is better understood from the following:

12)104 (in.)
3)8 (ft.) ···8 in.
2 (yd.) ···2 ft.

Since 1 in. $= \frac{1}{12}$ ft.,
104 in. $= 104 \times \frac{1}{12}$ ft. $= 8\frac{8}{12}$ ft. $=$ 8 ft. 8 in.
Since 1 ft. $= \frac{1}{3}$ yd.,
8 ft. $= 8 \times \frac{1}{3}$ yd. $= 2\frac{2}{3}$ yd. $=$ 2 yd. 2 ft.

Therefore the answer is 2 yd. 2 ft. 8 in.

Therefore, in reduction ascending,

Divide the given number of units by the number of units required to make one of the next higher denomination.

Divide this quotient and each successive quotient in like manner until the required denomination is reached.

The last quotient with the several remainders arranged in order is the answer sought.

EXERCISE 51

Reduce to higher units:

1. 96 in.
2. 100 in.
3. 115 in.
4. 5 ft.
5. 9 ft.
6. 25 ft.
7. 11 yd.
8. 15 yd.
9. 35 yd.
10. 700 rd.
11. 960 rd.
12. 1280 rd.

Reduce to feet, or to feet and inches:

13. 150 in.
14. 165 in.
15. 360 in.
16. 400 in.
17. 600 in.
18. 720 in.
19. 960 in.
20. 972 in.

Reduce to yards, or to yards and one lower unit:

21. 60 ft.
22. 65 ft.
23. 500 ft.
24. 750 ft.
25. 72 in.
26. 84 in.
27. 50 in.
28. 6 ft. 8 in.

EXERCISE 52

Express as inches:

1. 9 ft. 6 in.
2. 36 ft. 4 in.
3. 42 ft. 9 in.
4. 28 ft. 11 in.
5. 2 ft. 6 in.
6. 5⅓ ft.
7. 3 ft. 8 in.
8. 4⅔ ft.
9. 3 yd. 2 in.
10. 4 yd. 15 in.
11. 6 yd. 22 in.
12. 8 yd. 30 in.

Express as feet:

13. 4 yd. 2 ft.
14. 22 yd. 1 ft.
15. 33 yd. 2 ft.
16. 41 yd. 1 ft.
17. 33⅓ yd.
18. 66⅔ yd.
19. 66 yd. 2 ft.
20. 75 yd. 2 ft.
21. 5 rd.
22. 51 rd. 7 ft.
23. 3 mi. 720 ft.
24. 4 mi. 8 rd.

Express as yards:

25. 22 rd.
26. 28 rd.
27. 3 mi.
28. 1 mi. 10 rd.
29. 2 mi. 57 rd.
30. 3.25 mi.

Express as rods:

31. ½ mi.
32. 1.2 mi
33. 3.75 mi.

Express as feet, or as feet and inches:

34. 48 in.
35. 480 in.
36. 360 in.
37. 275 in.
38. 325 in.
39. 450 in.
40. 635 in.
41. 590 in.
42. 1250 in.

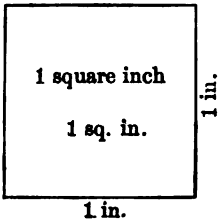
Express as yards, or as yards and one lower unit:

43. 1280 ft.
44. 7650 ft.
45. 5280 ft.
46. 1795 ft.
47. 425 in.
48. 72 ft. 27 in.

Express as rods, or as rods and feet:

49. 66 ft.
50. 330 ft.
51. 33 yd.
52. 75 ft.
53. 5 yd. 18 in.
54. 350 yd.

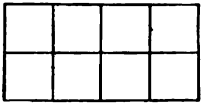
104. Square. A flat surface that has four equal sides and four equal angles is called a *square.*



If each side is 1 in. long, the square is 1 square inch; if each side is 1 ft. long, the square is 1 square foot; if each side is 1 yd. long, the square is 1 square yard; and so on.

105. Area. The number of square units in a surface is called its *area.*

For example, if this figure represents a surface 4 yd. long and 2 yd. wide, the area is 2 × 4 × 1 square yard, or 8 square yards.



106. Square Measure. The common table of square measure is as follows:

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

EXERCISE 53

Reduce to the next lower units:

1. 13 sq. ft.
2. 28 sq. ft.
3. 60 sq. ft.
4. 43 sq. yd.
5. 62 sq. yd.
6. 124 sq. rd.
7. 6 A.
8. 15 A.
9. 35 A.
10. 5 sq. mi.
11. 25 sq. mi.
12. 150 sq. mi.

13. Reduce 75 sq. ft. 36 sq. in. to square inches.

14. Reduce 2 sq. yd. 58 sq. in. to square inches.

15. Reduce 52 sq. yd. 8 sq. ft. to square feet.

16. Reduce 67,592 sq. in. to higher units.

17. Reduce 47,916 sq. ft. to higher units.

18. Reduce 22,471 sq. yd. to higher units.

EXERCISE 54

Express as square inches:

1. 1.2 sq. ft.
2. 1.7 sq. ft.
3. 1¾ sq. ft.
4. ½ sq. yd.
5. 2.5 sq. yd.
6. 0.7 sq. yd.
7. 3 sq. ft. 75 sq. in.
8. 2 sq. ft. 80 sq. in.
9. 4 sq. ft. 72 sq. in.

Express as square feet:

10. 288 sq. in.
11. 720 sq. in.
12. 864 sq. in.
13. 1008 sq. in.
14. 59 sq. yd.
15. 8.7 sq. yd.
16. 42½ sq. yd.
17. 0.3⅓ sq. yd.
18. 5 sq. rd.
19. 6½ sq. rd.
20. 7.25 sq. rd.
21. 130 sq. rd.

Express as square yards:

22. 1296 sq. in.
23. 6480 sq. in.
24. 108 sq. ft.
25. 801 sq. ft.
26. 60.5 sq. rd.
27. 121 sq. rd.
28. 605 sq. rd.
29. 242 sq. rd.
30. 1 A.
31. 0.5 A.
32. 0.75 A.
33. 0.33⅓ A.

Express as square rods:

34. 272¼ sq. ft.
35. 1089 sq. ft.
36. 5445 sq. ft.
37. 10,890 sq. ft.
38. 121 sq. yd.
39. 242 sq. yd.
40. 605 sq. yd.
41. 302½ sq. yd.
42. 7 A.
43. 1½ A.
44. 1¾ A.
45. 1⅝ A.

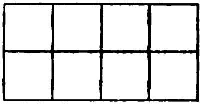
Express as acres:

46. 480 sq. rd.
47. 800 sq. rd.
48. 1120 sq. rd.
49. 4840 sq. yd.
50. 24,200 sq. yd.
51. 26,620 sq. yd.
52. 7 sq. mi.
53. 2½ sq. mi.
54. 3.75 sq. mi.

Express as square miles:

55. 1280 A.
56. 1920 A.
57. 320 A.
58. 2240 A.
59. 4480 A.
60. 5760 A.

107. Measurement of a Rectangle. If this figure represents a rectangle 4 in. long by 2 in. wide, there are evidently 4 sq. in. in the lower row of squares, and since there are 2 rows, there are 2 × 4 sq. in., or 8 sq. in., in the rectangle.



If we know the area and one dimension, we may find the other dimension; for, since 8 sq. in. = 2 × 4 sq. in., then 8 sq. in. ÷ 4 sq. in. = 2, the number of units in the other dimension.

Express the length and breadth of a rectangle in the same linear unit; the product of these two numbers will express the area in square units of the same name as the linear unit.

The number of square units in a rectangle divided by the number of linear units in one dimension gives the number of linear units in the other dimension.

This means that the numbers are to be considered as abstract. The complete solutions with concrete numbers are given above.

The expression 4″ × 7″ means and is read 4 inches *by* 7 inches.

EXERCISE 55

Find the areas of rectangles with these bases and altitudes:

1. 127 ft., 46 ft.
2. 152 in., 35 in.
3. 225 yd., 82 yd.
4. 240 in., 11 ft.
5. 6 ft. 4 in., 3 ft.
6. 7 ft. 2 in., 5 in.
7. 3 ft. 6 in., 1 ft. 4 in.
8. 9 yd. 2 ft., 3 ft. 6 in.

9. How many square rods in a rectangular field 32 rd. long and 16 rd. wide?

10. How many acres in a rectangular field 126 rd. long and 38 rd. wide?

11. A rectangular city lot is 129½ ft. long and 25 ft. 9 in. wide. Find the number of square feet in the lot.

12. A rectangular field containing 4.05 A. is 36 rd. long. How wide is it?

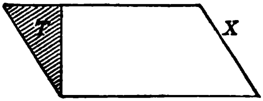
108. Triangle. A plane figure bounded by three straight lines is called a *triangle.*

The perpendicular distance from the vertex to the line of the base is called the *altitude* or *height* of the triangle.

109. Parallelogram. A plane figure bounded by two pairs of parallel lines is called a *parallelogram.*

The *base* of a parallelogram is the side on which it rests. The *altitude* is the perpendicular distance from the opposite side to the base. The opposite parallel sides of a parallelogram are equal.

If we cut off the triangle T and place it at X, the parallelogram will be changed to a rectangle of the same base and the same altitude.



Therefore, *the area of a parallelogram is equal to the product of the base and altitude.*

For example, if the base of a parallelogram is 3 ft. 8 in. and the altitude is 3 ft., what is the area?

$$3 \times 3\tfrac{2}{3} \text{ sq. ft.} = 11 \text{ sq. ft.}$$

If the area of a parallelogram is 15 sq. ft. and the base is 5 ft., required the altitude.

15 sq. ft. ÷ 5 sq. ft. = 3, the number of feet of altitude (§107).

EXERCISE 56

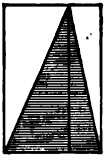
Find the areas of parallelograms with the following bases and altitudes:

1. 157 ft., 96 ft.
2. $32\frac{1}{2}$ ft., $15\frac{1}{2}$ ft.
3. 148 rd., 100 rd.
4. 27 ft. 6 in., 3 ft. 4 in.

Find the altitudes of parallelograms with the following areas and bases:

5. 972 sq. ft., 36 ft.
6. 1170 sq. ft., 45 ft.
7. 1728 sq. ft., 18 yd.
8. 40 A., 100 rd.

110. Measurement of a Triangle. We see that this triangle may be cut into two parts, each of which is half of some rectangle, and that therefore the whole triangle is half of a rectangle of the same base and altitude.



Whatever the shape of the triangle, it may, in a similar way, be shown to be half of a parallelogram of the same base and altitude, and therefore (§ 109) equal to half of a rectangle of the same base and altitude.

Therefore, *the area of a triangle equals half the product of the numbers representing the base and altitude.*

EXERCISE 57

Find the areas of triangles with the following bases and altitudes:

1. 47 ft., 28 ft.
2. 75 ft., 25 ft.
3. 82½ ft., 35 ft.
4. 160 rd., 25 rd.
5. 6 ft. 9 in., 27 in.
6. 3 yd. 18 in., 2 ft.
7. 7 ft. 2 in., 3 ft. 1 in.
8. 8 ft. 6 in., 4 ft. 8 in.
9. 12 ft. 9 in., 7 ft. 6 in.
10. 4 yd. 27 in., 2 ft. 4 in.

Find the altitudes of triangles with the following areas and bases:

11. 92 sq. ft., 23 ft.
12. 480 sq. ft., 48 ft.
13. 594 sq. ft., 54 ft.
14. 1053 sq. in., 39 in.

15. At $5.50 a square foot, what is the value of a triangular city lot with base 82 ft. and altitude 45 ft. 6 in.?

16. What is the area of a triangle with sides all equal, whose perimeter is 18 in. and altitude 5.19 in.?

17. A triangular gable to a house has a base 30 ft. and an altitude 10 ft. 6 in. Find the area of the gable.

18. What is the altitude of a triangle with area $3\frac{7}{8}$ sq. in. and base $\frac{3}{4}$ in.?

19. What is the altitude of a triangle with area $7\frac{5}{16}$ sq. in. and base $3\frac{1}{4}$ in.?

20. What is the base of a triangle with area $4\frac{7}{12}$ sq. ft. and altitude $2\frac{3}{4}$ ft.?

21. The gable of a house is a triangle with base $32\frac{1}{2}$ ft. and altitude $9\frac{1}{2}$ ft. What is the area of the gable?

22. In laying out a triangular flower bed the sides are all made equal. The perimeter is found to be 33 ft. and the altitude 9.5 ft. What is the area?

23. The four faces of a church spire are triangles, each with base 24 ft. and altitude 53 ft. How many square feet in each triangle? in the whole surface of the spire?

24. A man buys a 54-acre lot that is a parallelogram in shape. The frontage along the road is 144 rd. What is the distance to the rear line?

25. For the panels of a staircase 20 marble slabs are needed. They are parallelograms, each with base 1 ft. 6 in. and altitude 4 ft. 3 in. What is the total area of all the slabs?

26. A marble floor is made of 975 equilateral triangles, each with base 10 in. and altitude 8.6 in. What is the area of the floor space in square inches, making no allowance for cement?

27. A corner lot, where two streets meet at oblique angles, is a parallelogram in shape. Its length along one street is $63\frac{1}{2}$ ft., and the shortest distance from that front to the side parallel to it is $98\frac{1}{2}$ ft. What is the area?

28. A triangle, a square, a rectangle, and a parallelogram all have the same base and the same altitude. The side of the square is $4\frac{1}{4}$ in. What is the area of the square? of the triangle? of the rectangle? of the parallelogram?

111. Plastering, Painting, and Calcimining. The unit of plastering, painting, and calcimining is usually the square yard. A common business rule for estimating such work is as follows:

From the total area of the wall or ceiling deduct half the area of the doors, windows, and other openings, and express the result to the nearest square yard.

It would not be fair to deduct the full area of the openings because some allowance should be made for the difficulty of plastering or painting around them.

The rule varies in different places, and an exact agreement in advance is necessary when work is to be done. In some places it is the custom to deduct even the full area of the openings, while in others a certain number of square feet is allowed.

Technical rules of this nature need not be memorized. They should be referred to when necessary. Teachers will find it helpful to use local rules in matters of this kind.

In giving dimensions, architects and contractors write first the length, then the width, and then the height.

At 21¢ a square yard, find the cost of plastering a room 18′ × 16′ × 10′, if the room has a baseboard 10 in. high, two doors 7′ 4″ × 4′, and three windows 6′ 6″ × 4′.

Perimeter of room = 2 × (18′ + 16′) = 68′
Height above baseboard = 10′ − 10″ = $9\frac{1}{6}$′
Total wall area = $9\frac{1}{6}$ × 68 sq. ft. = $623\frac{1}{3}$ sq. ft.
Area of ceiling = 18 × 16 sq. ft. = 288 sq. ft.
Total area = $911\frac{1}{3}$ sq. ft.

Height of doors above baseboard = 7′ 4″ − 10″ = 6′ 6″

Area of 2 doors = 2 × 4 × $6\frac{1}{2}$ sq. ft. = 52 sq. ft.
Area of 3 windows = 3 × 4 × $6\frac{1}{2}$ sq. ft. = 78 sq. ft.
Total area of openings = 130 sq. ft.
Half area of openings = 65 sq. ft.
Area to be paid for = $846\frac{1}{3}$ sq. ft.
= $94\frac{1}{27}$ sq. yd.

At 21¢ a square yard, 94 sq. yd. will cost 94 × 21¢ = $19.74.

EXERCISE 58

Find the number of square yards to be covered in ceilings having the following dimensions:

1. 17′ × 15′.
2. 18′ × 16′.
3. 15′ × 13′.
4. 16′ × 14′.
5. 17′ × 16′.
6. 20′ × 19′.
7. 25′ × 20′.
8. 32′ × 25′.
9. 35′ × 28′.
10. 32′ × 29′.
11. 34′ × 26′.
12. 37′ × 25′.

If the cost of plastering is 22¢ per square yard, find the cost of plastering ceilings having the following dimensions:

13. 16′ × 15′.
14. 20′ × 15′.
15. 19′ × 15′.
16. 20′ × 18′.
17. 18′ × 15′.
18. 18′ × 14′.
19. 16′ × 16′.
20. 16′ × 13′.
21. 17′ × 13′.
22. 17′ × 14′.
23. 22′ × 20′.
24. 25′ × 22′.

Allowing 1½¢ per square yard for labor and ½¢ per square yard for material, find the cost of calcimining ceilings having the following dimensions:

25. 13′ × 12′.
26. 14′ × 12′.
27. 15′ × 14′.
28. 16′ × 12′.
29. 19′ × 16′.
30. 21′ × 17′.
31. 20′ × 17′.
32. 25′ × 21′.
33. 30′ × 25′.
34. 35′ × 30′.
35. 40′ × 35′.
36. 50′ × 40′.

37. How much will it cost at 33⅓¢ per square yard to plaster a room whose walls and ceiling have an area of 117 sq. yd., and have openings amounting to 9 sq. yd.?

38. A room is 18′ × 14′ × 9′, and has a baseboard 12 in. high, two doors 7′ × 3′ 6″, and two windows 6′ × 3′. How much will it cost to plaster the room at 30¢ per square yard?

39. A room is 22′ × 16′ × 10′, and has a baseboard 10 in. high, three doors 7′ 6″ × 4′, and four windows 6′ 6″ × 4′. How much will it cost to plaster the room at 25¢ per square yard?

112. Carpeting Rooms. Carpet is usually either 27 in. or 1 yd. wide, and in estimating the amount needed for a room, allowance must be made for matching the pattern.

In allowing for the waste in matching the pattern, it is not necessary to allow for loss on the first strip, since the other strips are cut to match that one. In the following examples, unless otherwise directed, the strips are to run lengthwise of the room.

For example, how much will it cost to carpet a room 13½′ × 16′, the strips to run lengthwise, and the carpet being plain color, 27 in. wide, and worth 90¢ a yard?

13½ ft. = 4½ yd., the width,
and 16 ft. = 5⅓ yd., the length.
4½ yd. ÷ ¾ yd. = 6, the number of strips.
6 × 5⅓ yd. = 32 yd.
32 × 90¢ = $28.80.

EXERCISE 59

Find the number of yards of plain carpet 27 in. wide that will be required for rooms of the following dimensions:

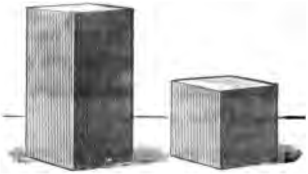
1. 24 ft., 15 ft.
2. 20 ft., 15 ft.
3. 22 ft., 18 ft.
4. 15 ft., 12 ft.
5. 17 ft., 12 ft.
6. 19 ft., 15 ft.
7. 25 ft., 18 ft.
8. 26 ft., 18 ft.
9. 32 ft., 21 ft.

10. What will it cost to carpet a room 19 ft. 8 in. long and 15 ft. wide with plain carpet 27 in. wide, at 75¢ a yard?

11. A floor 16 ft. wide is to be covered with 27-inch carpet, the strips running lengthwise. How many strips will be needed, and how much of the width of one strip will be lost by turning under?

12. What will it cost to cover a floor 18 ft. 6 in. long and 14 ft. 9 in. wide with carpet 1 yd. wide, worth $0.85 a yard, if the strips run lengthwise and ⅛ yd. is allowed on every strip except the first for matching the pattern?

113. Rectangular Solid. A solid that is bounded by six rectangles is called a *rectangular solid.*



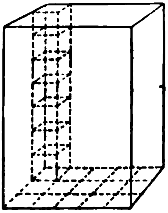
The six rectangles are called the *faces* of the solid, and their sides form the *edges* of the solid.

A rectangular solid has length, breadth, and thickness, and these are called the three *dimensions* of the solid.

114. Cube. A rectangular solid all of whose faces are squares is called a *cube.*

115. Volume. The number of cubic units a body contains is called its *volume.*

If this figure represents a rectangular solid 5 in. long, 3 in. wide, and 7 in. high, it is evident that in the column of cubes shown there are 7 cu. in. It is also evident that on the base we can place 3 × 5 such columns. Hence the volume is 3 × 5 × 7 cu. in., or 105 cu. in. Hence, if the dimensions are all expressed in the same unit,



The volume of a rectangular solid equals the product of its three dimensions.

We read 2 ft. × 3 ft. × 4 ft. "two feet by three feet by four feet," and this is a proper method of expressing dimensions, but not of expressing multiplication. The multiplier always should be an abstract number, as in 3 × 5 × 7 cu. in.

116. Cubic Measure. The common table of cubic measure is as follows:

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

A cubic yard of earth is considered a load.

EXERCISE 60

Reduce to the next lower units:

1. 2 cu. ft.
2. 25 cu. ft.
3. 75 cu. ft.
4. 125 cu. ft.
5. 33⅓ cu. ft.
6. 66⅔ cu. ft.
7. 25 cu. yd.
8. 36 cu. yd.
9. 125 cu. yd.
10. 25 cd.
11. 125 cd.
12. 500 cd.

Reduce to higher units:

13. 3456 cu. in.
14. 8640 cu. in.
15. 2457 cu. ft.
16. 4900 cu. ft.
17. 2000 cu. in.
18. 4260 cu. in.

19. Reduce 25 cu. ft. 65 cu. in. to cubic inches.

20. Reduce 15 cu. yd. 10½ cu. ft. to cubic feet.

21. Reduce 1290 cu. ft. to cords; to cubic yards.

Find the volumes of rectangular solids having the following dimensions:

22. 17″, 19″, 32″.
23. 16″, 22″, 27″.
24. 15″, 16″, 34″.
25. 17″, 19″, 29″.
26. 15′, 29′, 12′.
27. 127′, 6′, 1½′.
28. 124 ft., 6 ft., 1 ft. 9 in.
29. 135 ft., 5 ft., 2 ft. 2 in.
30. 275 ft., 4 ft., 1 ft. 10 in.
31. 2 ft. 6 in., 3 ft., 4 ft. 8 in.
32. 6 ft. 2 in., 2 ft., 3 ft. 9 in.
33. 4 ft. 3 in., 7 ft., 6 ft. 2 in.

34. In the school is an aquarium 16 in. long, 8½ in. wide, and 9½ in. deep. How many cubic inches does it contain?

35. A cellar 24 ft. by 39 ft. by 6 ft. is to be excavated. How much will it cost at 40¢ a load (cubic yard)?

36. A cellar 23½ ft. by 32½ ft. by 6½ ft. is to be excavated. How much will it cost at 37½¢ a load?

37. The front of a school building is 40 ft. long, 30½ ft. high, and 1½ ft. thick. It is built of brick. Making no allowance for openings, how many cubic feet of masonry does it contain?

38. What is the volume of a cube 16 in. on an edge?

39. How many cubic feet of air are there in a schoolroom 28 ft. long, 20 ft. wide, and 14 ft. high?

40. What is the total area of the six faces of a rectangular solid 8 in. by 7 in. by 5 in.?

41. What is the total area of the six faces of a rectangular solid 3 ft. 6 in. by 2 ft. 4 in. by 1 ft. 9 in.?

42. These children are measuring an excavation, and find it to be 6 ft. 2 in. wide, 8 ft. 4 in. long, and 4 ft. 3 in. deep. Assuming it to be practically rectangular, how many cubic feet of earth have been removed?

43. What is the weight of the water in an aquarium 1 ft. 4 in. long and 1 ft. wide, the water being 9 in. deep, if a cubic foot of water weighs 62½ lb.?

44. What is the weight of a block of marble 4 ft. 6 in. long, 2 ft. wide, and 1 ft. 4 in. thick, marble being 2.7 times as heavy as water, and a cubic foot of water weighing 62½ lb.?

45. A rectangular solid is 4 ft. 6 in. long and 2 ft. 8 in. wide, and the volume is 72 cu. ft. What is the height?

46. A rectangular solid is 6 ft. 4 in. long and 3 ft. 9 in. wide, and the volume is 285 cu. ft. What is the height?

47. A rectangular solid is 2 ft. 6 in. high and the volume is 240 cu. in. What is the area of the base?

12

117. Capacity of Bins. In finding the capacity of bins we seek either the number of cubic feet or the number of bushels. A bushel (bu.) contains 2150.42 cu. in., or approximately $\frac{5}{4}$ cu. ft., and 1 cu. ft. contains approximately $\frac{4}{5}$ of a bushel.

Therefore, *to find the approximate capacity, in bushels, of a bin, multiply* $\frac{4}{5}$ *bu. by the number of cubic feet in the bin.*

For example, how many bushels in a bin $10' \times 8' \times 5'$?

$$10 \times 8 \times 5 \times \tfrac{4}{5} \text{ bu.} = 320 \text{ bu.}$$

To find the approximate space, in cubic feet, required for a given number of bushels of grain, multiply $\frac{5}{4}$ *of a cubic foot by the number of bushels.*

For example, how many cubic feet will be required for 1000 bu.?

$$1000 \times \tfrac{5}{4} \text{ cu. ft.} = 1250 \text{ cu. ft.}$$

EXERCISE 61

Find the number of bushels in bins of the following dimensions:

1. $12' \times 6' \times 5'$.
2. $12' \times 7' \times 6'$.
3. $15' \times 9' \times 5'$.
4. $16' \times 10' \times 6'$.
5. $18' \times 10' \times 8'$.
6. $18' \times 12' \times 7'$.
7. $12\frac{1}{2}' \times 6' \times 4'$.
8. $15\frac{1}{2}' \times 8\frac{1}{2}' \times 5'$.
9. $16\frac{3}{4}' \times 9\frac{1}{4}' \times 5\frac{1}{2}'$.

Find the number of cubic feet in bins of the following capacities:

10. 600 bu.
11. 850 bu.
12. 975 bu.
13. 1500 bu.
14. 1625 bu.
15. 1860 bu.
16. 3200 bu.
17. 5600 bu.
18. 7500 bu.
19. 16,000 bu.
20. 22,320 bu.
21. 28,000 bu.
22. 37,500 bu.
23. 66,960 bu.
24. 59,200 bu.
25. 32,975 bu.

26. At 95¢ a bushel, how much is the wheat worth that fills a bin $10' \times 9' \times 4'$?

27. At $1.02 a bushel, how much is the wheat worth that fills a bin $30' \times 20' \times 10'$?

Allowing 35 cu. ft. to a ton of hard coal, find the number of tons that can be placed in bins of the following dimensions:

28. 10′ × 7′ × 6′.
29. 12′ × 10′ × 7′.
30. 13′ × 7′ × 5′.
31. 9 ft. 6 in. × 6 ft. × 5 ft.
32. 8 ft. 4 in. × 7 ft. × 6 ft.
33. 10 ft. 6 in. × 6 ft. × 6 ft.

Allowing 42 cu. ft. to a ton of soft coal, find the number of tons that can be placed in bins of the following dimensions:

34. 10′ × 9′ × 7′.
35. 12′ × 8′ × 6′.
36. 14′ × 7′ × 5′.
37. 8 ft. 4 in. × 7 ft. × 5 ft.
38. 9 ft. 2 in. × 8 ft. × 6 ft.
39. 7 ft. 3 in. × 9 ft. × 4 ft.

40. Taking a bushel as 1¼ cu. ft., and a ton of hard coal as measuring 35 cu. ft., how much should a bushel of hard coal weigh?

41. Taking a ton of soft coal as measuring 42 cu. ft., how much should a bushel of soft coal weigh?

42. How many loads in a rectangular cut for a mill flume 300 ft. long, 15 ft. wide, and 10 ft. deep?

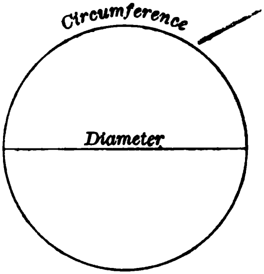
43. How many loads of sand in a gravel train of 24 cars, each car being 15′ 6″ × 5′ 8″, and the sand being 2′ deep?

44. How many loads will be removed in preparing the ground for macadamizing a street 1500′ long, 40′ wide, and 2½′ deep?

45. What is the value of the bricks in a pile 60 ft. long, 25 ft. wide, and 9 ft. high, each brick being 8 in. by 4 in. by 2 in., at $9 a thousand?

46. Allowing 231 cu. in. to the gallon, how many gallons in a watering trough that is 6 ft. long and 16 in. wide, the depth being 0.75 of the width?

118. Circumference of a Circle. If we measure around and across any circular object, we shall find that the circumference is about 3⅐ times the diameter. It is shown in geometry that 3.1416 is more nearly correct than 3⅐.



It is sufficient for most ordinary purposes to use 3⅐ instead of 3.1416.

Thus, if the diameter of a wheel is 7 ft., the circumference is 3⅐ × 7 ft., or 22 ft. If we use the multiplier 3.1416, the result is 3.1416 × 7 ft., or 21.9912 ft.

We need to consider the circle in finding the capacity of cisterns. Little or no explanation should be required of the pupil in connection with §§ 118–120.

EXERCISE 62

Using $3\frac{1}{7}$, find the circumference, given the diameter:

1. 105 ft.
2. 18.9 ft.
3. 25.9 ft.
4. 2.94 ft.
5. 0.364 in.
6. 4270 in.
7. 4970 in.
8. 51.10 in.

Using $3\frac{1}{7}$, find the circumference, given the radius:

9. 63 in.
10. 77 in.
11. 98 in.
12. 210 in.
13. 61.6 in.
14. 68.6 in.
15. 85.4 ft.
16. 8.82 ft.

Using 3.1416, find the circumference, given the diameter:

17. 27 ft.
18. 43 ft.
19. 32 yd.
20. 4.7 yd.
21. 0.62 ft.
22. 0.75 ft.
23. 2.25 ft.
24. 1.27 in.

Using 3.1416, find the circumference, given the radius:

25. 27 in.
26. 38 in.
27. 61 in.
28. 4.8 ft.
29. 1.9 yd.
30. 2.7 yd.
31. 0.48 ft.
32. 0.62 ft.

Using 3.1416, find the diameter, given the circumference:

33. 62.832 ft.
34. 345.576 in.
35. 502.656 in.

119. Area of a Circle. If we know the circumference and the radius of a circle, we can find the area.

The area of a circle equals half the product of the numbers representing the circumference and radius.

It is necessary to give this rule at this time, on account of its use in finding the capacity of cisterns. Teachers who wish to give some proof of it may refer to page 370.

For example, if the radius is 2 in. and the circumference is 12.5664 in., the area is ½ of 2 × 12.5664 sq. in., or 12.5664 sq. in.

120. Area in Terms of Radius. We can also find the area of a circle if we know merely the radius.

The area of a circle equals nearly 3.1416 times the square of the radius.

The *square* of a number is the product of the number multiplied by itself.

For example, if the radius is 7 in., the area is 3⅐ × 7 × 7 sq. in., or 154 sq. in.

Instead of 3.1416 we may use 3⅐, which gives the area somewhat less exactly.

A proof of this rule is given on page 370, where it will be better understood by the pupil.

In Exercise 63, use 3.1416 instead of 3⅐.

EXERCISE 63

1. Find the area of the circle with radius 1 in. and circumference 6.2832 in.

2. Find the area of the circle with radius 3 in. and circumference 18.8496 in.

3. Find the area of the circle with radius 2½ in. and circumference 15.708 in.

4. Find the area of the circle with diameter 8 in. and circumference 25.1328 in.

5. Find the area of the circle of which the radius is 8 in.

6. Find the area of the circle of which the radius is 6.5 in.

Find the areas of circles with the following radii and circumferences:

7. 5 in., 31.416 in.
8. 4½ in., 28.2744 in.
9. 4 in., 25.1328 in.
10. 3½ in., 21.9912 in.
11. 5½ in., 34.5576 in.
12. 10 in., 62.832 in.
13. 20 in., 125.664 in.
14. 30 in., 188.496 in.

Find the areas of circles with the following radii:

15. 1 in.
16. 4 in.
17. 3 in.
18. 5 in.
19. 10 in.
20. 20 in.
21. ½ in.
22. ⅓ in.
23. 0.1 in.
24. ¼ in.
25. ¾ in.
26. ⅔ in.
27. 2½ in.
28. 3¼ in.
29. 2.3 in.

30. What is the circumference of a water pipe that is 4 in. in diameter?

31. What is the area of a cross section of a water pipe that is 3 in. in diameter?

32. A water pipe is 4 in. in external diameter. If the iron is ¼ in. thick, what is the internal circumference?

33. A school globe is 15 in. in diameter. How long is the equator on the globe?

34. The equator on a school globe is 62.832 in. What is the radius of the globe?

35. What is the circumference of the largest circle that can be drawn in a square 10 ft. on a side?

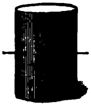
36. What is the area inclosed between the circle and the sides of the square in Ex. 35?

37. What is the diameter of a circular plot of ground of which the circumference is 691.152 ft.? What is the area?

38. A pipe is 3.75 in. in external diameter. If the iron is ¼ in. thick, what is the area of an internal cross section?

39. A pipe has an internal diameter of 3.5 in. The iron is ⅛ in. thick. What is the area of an external cross section?

121. Cylinder. A solid formed by the revolution of a rectangle about one of its sides is called a *cylinder.*



The two circles that are thus formed are called the *bases*, and the curved surface is called the *lateral surface* of the cylinder.

The distance between the two bases is called the *altitude* of the cylinder.

122. Volume of a Cylinder. We can think of 1 cu. in. on each square inch of the base. Therefore, if the base contains 4 sq. in., and if the cylinder is 1 in. high, the volume will be 4 × 1 cu. in., or 4 cu. in., and if the cylinder is 5 in. high, the volume will be 5 × 4 cu. in., or 20 cu. in.

Therefore, to find the volume of a cylinder,

Multiply the number of cubic units corresponding to the number of square units of the base, by the number of units of altitude.

That is, if the base has 9 sq. in., and the altitude is 12 in., the volume is 12 × 9 cu. in., or 108 cu. in.

If the base has a diameter of 2 in., and the altitude is 5 in., then the area of the base is 3.1416 × 1 sq. in. (§ 120), and the volume is 5 × 3.1416 × 1 cu. in., or 15.708 cu. in.

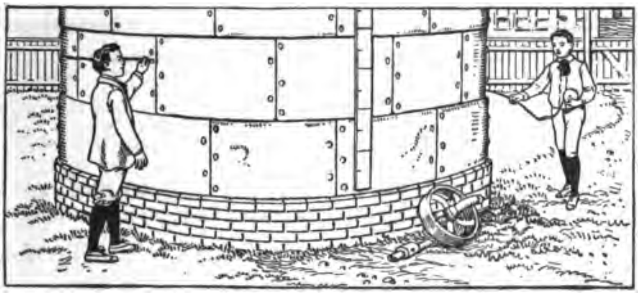
123. Lateral Area of a Cylinder. We may think of the surface of a cylinder as unrolled, like a paper reaching once around a pencil. In this case it would unfold into a rectangle, one side of which would be the circumference of the base and the other the altitude.

Therefore, to find the lateral area of a cylinder,

Multiply the number of units in the circumference by the number of units in the altitude.

For example, if the circumference is 6 in. and the altitude 4 in., the number of square units of lateral area is 4 × 6, or 24. We may express the work thus:

4 × 6 sq. in. = 24 sq. in.



EXERCISE 64

Find the volumes of cylinders whose altitudes and areas of base are as follows:

1. 13 in., 27 sq. in.
2. 17 in., 36 sq. in.
3. 38 in., 125 sq. in.
4. 3 ft. 7 in., $8\frac{1}{2}$ sq. in.
5. 4 ft. 6 in., $7\frac{1}{2}$ sq. in.
6. 8 ft. 7 in., 4 sq. ft. 24 sq. in.

Find the altitudes of cylinders whose volumes and areas of base are as follows:

7. 20 cu. ft. 324 cu. in., 8 sq. ft. 72 sq. in.
8. 190 cu. ft. 1305 cu. in., 30 sq. ft. 75 sq. in.

Find the areas of base of cylinders whose volumes and altitudes are as follows:

9. 324 cu. ft., 18 ft.
10. 323 cu. ft., 17 ft.
11. 28 cu. ft. 288 cu. in., 2 ft. 2 in.
12. 81 cu. ft. 576 cu. in., 5 ft. 4 in.

Find the volumes of cylinders whose altitudes and radii of base are as follows:

13. 27 in., 4 in.
14. 25 in., 6 in.
15. 32 in., 8 in.
16. 36 in., 9 in.
17. 2 ft. 6 in., 2 in.
18. 3 ft. 4 in., 1 ft.

124. Capacity of Cisterns. The following measurements are used in computing the capacity of cisterns:

$$\begin{aligned}1\text{ gal.} &= 231\text{ cu. in.}\\ 1\text{ cu. ft.} &= 7.48+\text{ gal.} = \text{nearly } 7\tfrac{1}{2}\text{ gal.}\\ 31\tfrac{1}{2}\text{ gal.} &= 1\text{ bbl.} = \text{nearly } 4\tfrac{1}{5}\text{ cu. ft.}\end{aligned}$$

In Exercise 65 use $7\frac{1}{2}$ gal. for 1 cu. ft., $4\frac{1}{5}$ cu. ft. for 1 bbl., and $3\frac{1}{7}$ as the multiplier in cases requiring the area of base, given the diameter.

Find the number of gallons of water in a cylindrical tank 40 ft. in diameter, the water being 35 ft. deep.

$$\begin{aligned}\text{Radius of base} &= \tfrac{1}{2}\text{ of } 40\text{ ft.} = 20\text{ ft.}\\ \text{Area of base} &= 3\tfrac{1}{7} \times 20 \times 20\text{ sq. ft. (§ 120).}\\ \text{Volume} &= 35 \times 3\tfrac{1}{7} \times 20 \times 20\text{ cu. ft. (§ 122)}\\ &= 35 \times 3\tfrac{1}{7} \times 20 \times 20 \times 7\tfrac{1}{2}\text{ gal.}\\ &= 330{,}000\text{ gal.}\end{aligned}$$

EXERCISE 65

Find the number of gallons in these rectangular tanks:

1. $4' \times 2' \times 1'$.
2. $5' \times 2' \times 1'$.
3. $6' \times 1\frac{1}{2}' \times \frac{3}{4}'$.
4. $6' \times 2\frac{1}{4}' \times 1'$.
5. $12\frac{1}{2}' \times 12\frac{1}{2}' \times 6'$.
6. $15\frac{1}{2}' \times 12\frac{1}{2}' \times 6\frac{1}{2}'$.

Find the number of barrels in the following tanks:

7. $12' \times 11' \times 9'$.
8. $15' \times 14' \times 8'$.
9. $15\frac{1}{2}' \times 12\frac{1}{2}' \times 5\frac{3}{4}'$.

Find the number of gallons in cylindrical tanks of the following diameter and depth of water respectively:

10. $6'$, $10'$.
11. $15'$, $12'$.
12. $16'$, $14'$.
13. $35'$, $40'$.
14. $48\frac{1}{2}'$, $56'$.
15. $48\frac{3}{4}'$, $45\frac{1}{2}'$.

Find the number of gallons of water in wells of the following diameter and depth of water respectively:

16. 3 ft., 8 ft.
17. 3 ft., 6 ft.
18. 4 ft., 8 ft.
19. $3\frac{1}{2}$ ft., 6 ft.
20. $3\frac{3}{4}$ ft., $5\frac{1}{4}$ ft.
21. $3\frac{3}{4}$ ft., $8\frac{3}{4}$ ft.

125. Brick Work and Stone Work. The number of bricks required for walls is estimated by the thousand, and 22 bricks of ordinary size laid in mortar are allowed to each cubic foot of wall.

A brick of ordinary size is 2″ × 4″ × 8″, and its volume is therefore 64 cu. in., or $\frac{1}{27}$ cu. ft. Allowing for mortar, however, it is reckoned as $\frac{1}{22}$ cu. ft., as stated.

Stone work is reckoned by the cubic foot or cubic yard, and sometimes by the perch.

A perch is 16½′ × 1½′ × 1′, and therefore equals 24¾ cu. ft. Practically, however, it is taken as 25 cu. ft., although this varies.

In estimating the labor for brick and stone work, masons measure the outside of the walls, thus reckoning the corners twice. This is considered fair because of the extra trouble in laying a corner. Openings are generally not considered, although sometimes an allowance for half of the openings is made, the gain to the mason being offset by the extra labor in building around them.

In estimating the material they will need, however, masons consider the corners and openings. The custom varies in these matters.

For example, find the number of bricks required for a wall 42′ × 25′ and 1 ft. thick.

$$42 \times 25 \times 1 \times 22 \text{ bricks} = 23{,}100 \text{ bricks.}$$

How many cubic feet should be allowed in estimating the labor on a cellar wall 40′ × 30′, 6′ high, and 2′ thick?

Perimeter of cellar, outside the wall, is 2 × (40 + 30) ft., or 140 ft. Therefore the number is 6 × 2 × 140 cu. ft., or 1680 cu. ft.

EXERCISE 66

Find the number of cubic feet in walls of these dimensions:

1. 62 ft., 4 ft., 1 ft.
2. 78 ft., 5 ft., 1 ft. 6 in.
3. 49 ft., 5 ft., 1 ft. 8 in.
4. 75 ft., 6 ft., 2 ft.
5. 82 ft., 6 ft., 2 ft. 2 in.
6. 96 ft., 6 ft. 6 in., 2 ft.

Find the number of cubic yards in walls having these dimensions:

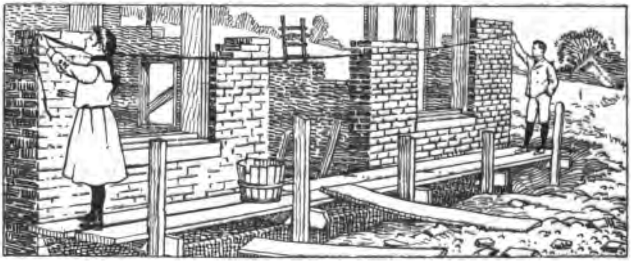
7. 4 rd., 6 ft., 15 in.

8. 120 ft., 2 yd., 1 ft.

9. 4 rd., 5 ft., 18 in.

10. 150 ft., 6 ft., 1 ft. 6 in.

11. At $8 a thousand, what will the bricks cost for a wall 120 ft. long, 5 ft. high, and 1 ft. thick?



12. These children measure this house and find it to be 30 ft. by 20 ft. They learn that the brick walls are to be 20 ft. high and 1 ft. thick, and the openings are to amount to 250 cu. ft. How many bricks will be required? How much will they cost at $8.25 a thousand?

13. How many cubic feet of stone in the cellar walls of a house 42′ × 25′, if the walls are 18″ thick and 6′ high? How many perches of 25 cu. ft. each? What will it cost at $9 a perch?

14. How many bricks will be required for the walls of a flat-roofed house 42′ × 36′ × 21′, if the walls are 1 ft. thick and if 400 cu. ft. is deducted for doors and windows? How much will they cost at $8.30 a thousand?

15. How many bricks will be required for the walls of a flat-roofed building 100′ × 60′ × 22′, if the walls are 1½ ft. thick, and if 800 cu. ft. is deducted for doors and windows?

126. Weight. The common weight used in the United States is called *avoirdupois weight.*

The table is as follows:

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

A barrel of flour weighs 196 lb. (Memorize.)

100 lb. is sometimes called a *hundredweight* (cwt.). The ton of 2000 lb. is sometimes called the *short ton*, there being an old ton of 2240 lb., known as the *long ton*, which is used in the customhouse and in some wholesale transactions in mining products. There is also the *long hundredweight* of 112 lb., but it is rarely used except in English trade. A ton of loose hay fills about 8 × 8 × 8 cu. ft., or 512 cu. ft., of space, but this varies greatly with the condition of the hay. A keg of nails, a cental of corn, or a quintal of fish weighs 100 lb. A barrel of salt weighs 280 lb., and a barrel of pork or of beef, 200 lb. A cask of lime weighs 240 lb. A cubic foot of water weighs $62\frac{1}{2}$ lb.

The laws of the several states specify the weight of a bushel of various products. In New York the following are the standards:

Barley	48 lb.	Oats	32 lb.
Buckwheat	48 "	Potatoes	60 "
Corn (shelled)	56 "	Wheat	60 "

These weights should be memorized.

A *carat* weight, used in weighing diamonds, is commonly taken in the United States as 3.2 grains. The word *carat*, meaning $\frac{1}{24}$, is used also in speaking of the purity of gold. Hence 18 carats fine means $\frac{18}{24}$ pure gold and $\frac{6}{24}$ alloy.

EXERCISE 67

1. Express 19,000 lb. as tons.
2. Express 255 T. as pounds.
3. What is the weight of 25 barrels of salt?
4. What is the weight of 250 bu. of barley?
5. What is the weight of 750 bu. of wheat?
6. What is the weight of 1250 bu. of oats?
7. What is the weight of 1675 bu. of potatoes?

8. What do 100 bu. of buckwheat weigh? What do 275 bu. weigh?

9. What do 100 bu. of barley weigh? What do 25 bu. weigh? What do 2500 bu. weigh?

10. What do 100 bu. of shelled corn weigh? What do 25 bu. weigh? What do 2500 bu. weigh?

11. What do 25 bu. of oats weigh? What do 250 bu. weigh? What do 2500 bu. weigh?

12. How many tons to 1000 bu. of wheat? of oats?

13. How many bushels of potatoes to the ton?

14. How many bushels of oats to the ton?

15. The weight of a bushel of shelled corn is what part of the weight of a bushel of wheat? of a bushel of potatoes?

16. Barley is what fraction heavier than oats? Oats are what fraction lighter than barley?

17. What fraction more does 1 bu. of shelled corn weigh than 1 bu. of barley? than 1 bu. of buckwheat?

18. Buckwheat is what fraction heavier than oats? Oats are what fraction lighter than buckwheat?

19. Potatoes are what fraction heavier than shelled corn? Barley is what fraction lighter than shelled corn?

20. Barley is what fraction lighter than wheat? Oats are what fraction lighter than wheat?

21. At $14.50 a ton how much will 25 tons of hay cost?

22. At $15.75 a ton how much will 62.5 tons of hay cost?

23. At $15.80 a ton how much will 48¾ tons of hay cost?

24. If 9½ tons of hay cost $139.65, what is the price per ton?

25. Taking the weight of 1 cu. ft. of water as 62½ lb., what is the weight of 6 cu. ft. of water? of 70 cu. ft.?

26. In this load of hay there are 2150 lb. Express this as tons and a common fraction.

27. If there were 1850 lb. in the load, what decimal of a ton would there be?

Reduce to the next lower units:

28. 16 lb. **29.** $2\frac{1}{2}$ lb. **30.** 16 T. **31.** $28\frac{1}{2}$ T.

Reduce to higher units:

32. 176 oz. **33.** 200 oz. **34.** 32,000 oz. **35.** 10,000 lb.

Reduce to ounces:

36. $29\frac{3}{4}$ lb. **37.** $56\frac{5}{8}$ lb. **38.** $54\frac{3}{16}$ lb. **39.** 27 lb. 5 oz.

Reduce to pounds:

40. 27 T. **41.** 2.7 T. **42.** 144 oz. **43.** 3 T. 700 lb.

44. What is the weight of the water in an aquarium 1 ft. 6 in. long and 1 ft. 4 in. wide, the water being $9\frac{1}{2}$ in. deep, if a cubic foot of water weighs $62\frac{1}{2}$ lb.?

45. What is the weight of a block of stone 3 ft. 4 in. long, 3 ft. 1 in. wide, and 1 ft. 2 in. thick, the stone being 2.7 times as heavy as water?

46. What is the weight of ½ cu. ft. of water? of 0.1 cu. ft.? of 0.8 cu. ft.?

47. What part of 1 cu. ft. of water weighs 12½ lb.? 25 lb.? 50 lb.? 31¼ lb.?

48. Silver is about ten times as heavy as water. About how much does 1 cu. ft. of silver weigh? 3 cu. ft.?

49. How much does 864 cu. in. of water weigh? 432 cu. in.? 216 cu. in.?

50. Water is half as heavy as sulphur. How much does 1 cu. ft. of sulphur weigh?

51. The weight of cork is $\frac{6}{25}$ of the weight of an equal volume of water. How much does 1 cu. ft. of cork weigh?

52. A certain quality of gold is 19.3 times as heavy as water. How much does 1 cu. ft. weigh?

53. If silver is 10½ times as heavy as water, how much is 1 cu. ft. of silver worth at 50¢ an ounce?

54. If good milk is 0.03 heavier than water, what is the weight of a can of such milk containing 1 cu. ft., the can itself weighing 5 lb. 10 oz.?

55. Ice is 0.92 as heavy as water. I wish 100 lb. of ice to fill my refrigerator. If the iceman sends me 2 cu. ft., he sends me how many pounds too much?

56. A man building a house wishes to put a tank for water in his attic. The tank is to be 5 ft. long, 4 ft. wide, and is to allow for 3½ ft. depth of water. Allowing 600 lb. for the weight of the tank, how much will it weigh when full?

57. A tree sparrow eats ¼ oz. of weed seed a day. Allowing 10 birds to 1 sq. mi., and supposing them to average 200 days a year in the state of New York, which has an area of 49,170 sq. mi., how many tons of weed seed are consumed a year by these birds alone?

127. Dry Measure. The common table of dry measure is as follows:

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

A bushel, as we have learned, contains 2150.42 cu. in., or about 1¼ cu. ft. A dry quart contains 67.2 cu. in., while the liquid quart contains only 57.75 cu. in. In measuring grain, seeds, and small fruits the measure must be *even* full. In measuring apples, potatoes, and other large articles the measure must be *heaping* full.

The various states prescribe the weight of certain farm products per bushel. Some of these weights have been given on page 125.

EXERCISE 68

Reduce to the next lower units:

1. 37 qt.
2. 49 qt.
3. 99 qt.
4. 256 qt.
5. 16½ qt.
6. 37½ qt.
7. 87½ qt.
8. 248½ qt.
9. 48 pk.
10. 59 pk.
11. 67¼ pk.
12. 268¾ pk.
13. 47 bu.
14. 56½ bu.
15. 92¾ bu.
16. 427¼ bu.

Reduce to higher units:

17. 72 pt.
18. 76 pt.
19. 98 pt.
20. 64 qt.
21. 60 qt.
22. 98 qt.
23. 96 pk.
24. 89 pk.
25. 75 pk.
26. 156 pt.
27. 156 qt.
28. 156 pk.

Reduce to pints:

29. 72 qt.
30. 85½ qt.
31. 68½ qt.
32. 49 pk.
33. 53½ pk.
34. 75¾ pk.
35. 2 bu.
36. 35 bu.
37. 28½ bu.
38. 2 pk. 3 qt.
39. 5 pk. 7 qt.
40. 2 bu. 1 pk.

Reduce to quarts:

41. 38 pk.
42. 48½ pk.
43. 59¾ pk.
44. 78⅛ pk.
45. 75 bu.
46. 87½ bu.
47. 2 bu. 3 pk.
48. 9 bu. 1 pk.

Reduce to pecks:

49. 72 bu.
50. 89½ bu.
51. 144 qt.
52. 576 pt.

Reduce to bushels:

53. 336 pk.
54. 516 pk.
55. 772 pk.
56. 956 pk.
57. 352 qt.
58. 1952 qt.
59. 1984 pt.
60. 6336 pt.

61. A bushel contains 2150.42 cu. in. How many bushels of wheat can be put in a bin containing 645,126 cu. in.?

62. Allowing 1¼ cu. ft. to a bushel, how many bushels of wheat can be put in a bin containing 900 cu. ft.?

63. If a bushel of wheat weighs 60 lb., what will be the weight of the wheat in Ex. 62?

64. A dry quart contains 67.2 cu. in. How many quarts in a measure containing 336 cu. in.?

65. If a bushel of oats weighs 32 lb., what will be the weight of the oats required to fill a bin that contains 875 cu. ft., allowing 1¼ cu. ft. to a bushel?

66. There are 231 cu. in. in a gallon and 537.6 cu. in. in a peck. From these measures verify the number of cubic inches in a liquid quart and in a dry quart as given in § 127 below the table.

128. Liquid Measure. The common table of liquid measure is as follows:

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

These measures also are in common use:

31½ gallons = 1 barrel (bbl.)
63 gallons = 1 hogshead (hhd.)

Casks holding from 28 gal. to 43 gal. are called barrels, and casks holding from 54 gal. to 63 gal. are called hogsheads. Whenever barrels or hogsheads are used as *measures*, a barrel means 31½ gallons, and a hogshead means 63 gallons. A barrel contains about 4⅕ cu. ft. A gallon of water weighs about 8.3 lb., and contains 231 cu. in.

EXERCISE 69

Reduce to the next lower units:

1. 27 pt.
2. 46 pt.
3. 69 qt.
4. 87 qt.
5. 89 gal.
6. 68 gal.
7. 64¼ gal.
8. 62½ qt.

Reduce to higher units:

9. 96 gi.
10. 96 pt.
11. 35 gi.
12. 85 pt.
13. 300 qt.
14. 475 pt.
15. 92½ qt.
16. 75½ pt.

Reduce to pints:

17. 396 gi.
18. 892 gi.
19. 126½ qt.
20. 79½ gal.

Reduce to quarts:

21. 792 gi.
22. 298 pt.
23. $926\frac{3}{4}$ gal.
24. 72 gal. 3 qt.

The following examples review liquid measures and other measures already studied.

Reduce to the next lower units:

25. 64 lb.
26. $5\frac{1}{2}$ cu. ft.
27. $9\frac{3}{4}$ lb.
28. $7\frac{1}{2}$ sq. ft.
29. 96 T.
30. $6\frac{1}{2}$ sq. rd.
31. $48\frac{3}{4}$ T.
32. $2\frac{1}{2}$ A.

Reduce to higher units:

33. 528 oz.
34. 624 oz.
35. 375 oz.
36. 450 oz.
37. 35,000 oz.
38. 32,176 oz.
39. 12,500 lb.
40. 75,550 lb.

Reduce to ounces:

41. $\frac{3}{4}$ lb.
42. $\frac{7}{32}$ lb.
43. $72\frac{3}{8}$ lb.
44. $29\frac{5}{32}$ lb.
45. $96\frac{7}{8}$ lb.
46. 29,75 lb.
47. 1 T. 200 lb.
48. 9 lb. 13 oz.

Reduce to pounds:

49. $42\frac{3}{4}$ T.
50. $65\frac{4}{5}$ T.
51. 3.6 T.
52. $3.3\frac{1}{3}$ T.
53. 432 oz.
54. 4832 oz.
55. 2 T. 800 lb.
56. 5 T. 650 lb.

57. How many pounds in $26\frac{1}{2}$ long tons?

58. How many pounds in 1750 hundredweight?

59. How many more pounds in 86 long hundredweight than in 86 common hundredweight?

60. Allowing $7\frac{1}{2}$ gal. to a cubic foot, and taking the weight of a cubic foot of water as $62\frac{1}{2}$ lb., what is the weight of the water needed to fill a tank containing 75,000 gal.?

61. Taking the weight of a cubic foot of water as $62\frac{1}{2}$ lb., and ice as being 0.92 as heavy as water, should we buy a piece of ice of 50 lb., or one piece of 50 lb. and another of 25 lb., in order to fill approximately an ice chamber that is 12 in. by 12 in. by 18 in.?

62. Taking the weight of 1 gal. of water as 8.3 lb., what is the weight of 25 gal. of water? of 48 gal.?

63. How much does ½ gal. of water weigh? 1 pt. of water? 3 qt. of water? 2.5 gal. of water?

64. If a 3-gallon pail weighs 3 lb., how much will it weigh when full of water? when half full? when 0.75 full?

65. How much does 125 gal. of water weigh? 355 gal.? 45.8 gal.? 1500 gal.?

66. How much does 1 bbl. (of 31½ gal.) of water weigh?

67. Milk being 0.03 heavier than water, how much does it weigh to the gallon? the quart? the pint?

68. Alcohol being 0.79 as heavy as water, how much does it weigh per gallon? per barrel of 31½ gal.?

69. Petroleum being 0.7 as heavy as water, how much does it weigh per gallon? per barrel of 31½ gal.?

70. Copper being 8.95 times as heavy as water, and a cubic foot of water weighing 62½ lb., how much does 1 cu. ft. of copper weigh? also 231 cu. in.?

71. Turpentine being 0.01 lighter than water, how much does it weigh per gallon? per barrel of 31½ gal.?

72. A tank 8 ft. long, 6 ft. wide, and 4 ft. deep contains how many cubic feet? how many cubic inches? how many gallons?

73. A tank 8 ft. long, 5.5 ft. wide, and 3 ft. deep contains how many cubic feet? how many cubic inches? how many gallons?

74. A tank 7 ft. long by 5½ ft. wide is filled with water to the depth of 3 ft. How many gallons of water in it?

75. The water of the Dead Sea is so salty that it is $1\frac{6}{25}$ times as heavy as fresh water. How much does 1 gal. of Dead Sea water weigh?

129. Equivalent Measures. In cooking, the following equivalents are often used:

4 teaspoons = 1 tablespoon, often written 4 t = 1 T;
16 tablespoons = 1 cup, often written 16 T = 1 C;
2 cups = 1 pint, often written 2 C = 1 pt.

EXERCISE 70

1. Given the number of cups of anything, how do you find the equivalent number of pints?

2. Given a certain fraction of a cup of anything, how do you find the equivalent number of tablespoons?

3. Given the number of pints of anything, how do you find the equivalent number of cups?

4. Given the number of tablespoons of anything, how do you find the equivalent number of cups?

5. Given the number of teaspoons of anything, how do you find the equivalent number of tablespoons?

6. Given a certain fraction of a tablespoon, how do you find the equivalent number of teaspoons?

7. Given a recipe, how do you make a certain fractional part of it?

8. Given a recipe, how do you double it or make any given number of times the amount?

9. How many cups will you allow to 2 pt.? to 4 pt.?

10. How many cups will you allow to 1½ pt.? to 1 qt.? to 1½ qt.? to 1¼ qt.?

11. How many tablespoons will you allow to ¾ of a cup? to 1½ cups? to 1¼ cups? to 1¾ cups?

12. How many teaspoons will you allow to ⅛ of a cup? to ¾ of a cup? to 4 tablespoons? to 8 tablespoons? to ½ of a tablespoon?

130. English Money. The following is the table of English money:

4 farthings = 1 penny (d.)
12 pence = 1 shilling (s. or /)
20 shillings = 1 pound or sovereign (£)

The *florin* is 2 shillings and the *crown* is 5 shillings. The half crown is therefore 2 shillings 6 pence, commonly called "two and six."

The value of the pound in our money is $4.866 + ; of the shilling, 24.3¢ + ; of the penny, about 2¢. We commonly think of the pound as about $5, and the shilling as about 25¢.

We write 2 pounds 4 shillings 3 pence thus: £2 4s. 3d. It is customary to write 2 shillings 6 pence thus: 2/6.

EXERCISE 71

Express as pence:

1. £1.
2. £2.
3. 5s.
4. £1 2s.
5. £1 9s.
6. £2 16s.
7. £1 2s. 6d.
8. £3 9s. 5d.
9. £4 12s. 4½d.

Express as shillings:

10. £2.
11. £5.
12. £7
13. £1 10s.
14. £3 15s.
15. £6 19s.
16. 48d.
17. 96d.
18. 240d.

Express as pounds:

19. 60s.
20. 140s.
21. 480s.
22. 240d.
23. 480d.
24. 1200d.
25. 700s.
26. 720d.
27. 1500s.

Taking $4.87 to the pound, express as dollars:

28. £48.
29. £75.
30. £475.

Taking $4.87 to the pound, express as pounds:

31. $24.35.
32. $34.09.
33. $535.70.

131. French Money. The French table of money is as follows:

100 centimes (c.) = 1 franc (fr.) = $0.193

We usually think of the franc as about 20¢, and the 5-franc piece as about $1. This system is used in several other countries besides France. In Italy, for example, 100 centesimi = 1 lira, the lira having the same value as the franc.

132. German Money. The German table of money is as follows:

100 pfennigs (pf.) = 1 mark (M.) = $0.238

We usually think of 4 pf. as 1¢, and 1 M. as about 25¢.

EXERCISE 72

Express as francs and decimals:

1. 850 c. 2. 375 c. 3. 480 c. 4. 1250 c.

Express as marks and decimals:

5. 400 pf. 6. 175 pf. 7. 1600 pf. 8. 9275 pf.

9. What part of a franc is 75 centimes?

10. What part of a mark is 75 pfennigs?

11. Taking the franc as 19.3¢, this is what part of our quarter?

12. At the same rate, the 5-franc piece is what part of our dollar?

13. Taking the mark as 23.8¢, this is what part of our quarter?

14. At the same rate, 4 marks is what part of our dollar?

15. Find $\frac{1}{4}$ of 75 francs 40 centimes. (This may be written 75.40 francs.)

16. Find 0.06 of 25 marks 50 pfennigs. (This may be written 25.50 marks.)

133. Time Measure. The table of time is as follows:

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days = 1 common year (yr.)
366 days = 1 leap year
100 years = 1 century

There are 12 months (mo.) or about 52 weeks in a year. The names of the months, called calendar months, are:

	da.		da.
1. January (Jan.)	31	7. July	31
2. February (Feb.)	28 or 29	8. August (Aug.)	31
3. March (Mar.)	31	9. September (Sept.)	30
4. April (Apr.)	30	10. October (Oct.)	31
5. May	31	11. November (Nov.)	30
6. June	30	12. December (Dec.)	31

The number of days in each month may be easily remembered by committing to memory the following lines:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
Which has but twenty-eight, in fine,
Till leap year gives it twenty-nine."

We usually write 15 minutes after 10 in the forenoon thus: 10 hr. 15 min. A.M., 10.15 A.M., or 10 15 A.M. We usually write 20 minutes before 3 in the afternoon thus: 2 hr. 40 min. P.M., 2.40 P.M., or 2 40 P.M.

A *year* is the time in which the earth performs one revolution in its orbit around the sun, and consists of 365 da. 5 hr. 48 min. 46 sec.; that is, nearly 365¼ days. As there are 365 days in a common year, a common year lacks nearly ¼ of a day of being a solar year, and this defect is made up by reckoning for some years (leap years) 366 days.

The leap years are the years whose dates are exactly divisible by 4; except in the case of complete hundreds, and these are leap years if exactly divisible by 400. Thus, 1600 and 1904 were leap years; 1800, 1900, and 1911 were not; 1928 and 1940 will be.

EXERCISE 73

Reduce to the next lower units:

1. 360 min.
2. 18 hr.
3. 36 hr.
4. 16 da.
5. 36 da.
6. $48\frac{1}{2}$ da.
7. 52 wk.
8. 26 wk.
9. $46\frac{3}{7}$ wk.
10. 8 yr.
11. 16 yr.
12. 32 yr.

Reduce to higher units:

13. 360 sec.
14. 720 min.
15. 504 hr.
16. 168 hr.
17. 132 mo.
18. $182\frac{5}{8}$ da.
19. 25 yr.
20. 365 da.

21. How many days from March 5 to June 5?

22. How many days from June 10 to November 10?

23. Allowing $365\frac{1}{4}$ da. to the year, how many days in half a century?

24. Allowing $365\frac{1}{4}$ da. to the year, how many years in 182,625 da.?

25. Name the leap years in the following: 1925, 1926, 1927, 1928, 1929, 1930, 1940, 1950, 1960, 1972, 1982, 2000.

26. January 1, 1914, being Thursday, on what day of the week does January 1, 1916, fall?

27. Reduce 15 hr. 32 min. 17 sec. to seconds.

28. Reduce 2 da. 7 hr. 29 min. to minutes.

29. Reduce 8 wk. 5 da. 2 hr. 3 min. to minutes.

30. How many days will the month of February, 1948, contain? How many hours?

31. A lunar month (the time from one new moon to the next one) contains 2,551,443 seconds. Express a lunar month in units of higher denominations.

32. A man received $2.75 a day every week day during March, in a year when that month had five Sundays. How much did he receive during the month?

33. If it takes 45 min. to bake potatoes and ⅔ as long to boil them, how many minutes does it take to boil them?

34. If 9 min. for each pound is allowed for roasting beef, how long will it take to roast a piece weighing 7¼ lb.?

35. If 12 min. for each pound is allowed for roasting mutton, how long will it take for a piece weighing 6¾ lb.?

36. If 20 min. for each pound is allowed for roasting veal, how long will it take to roast a piece weighing 8¾ lb.?

37. If bread rises from 7 P.M. to 7 A.M. at the first rising, and 1 hr. at the second rising, how long does it take for both risings?

✓ **38.** If it takes bread 55 min. to bake and it takes rolls 25 min. to bake, rolls will bake in what fraction of the time required for bread?

✓ **39.** If quick-process bread is set for its first rising at 8 A.M. and rises 3¾ hr., and if 15 min. is allowed for getting it ready for the second rising, and it then rises 1 hr. 5 min. and is baked for 50 min., at what time will it be finished?

40. When should a chicken, which needs 1 hr. 30 min. for roasting, be put into the oven to be ready to serve at 6 P.M.?

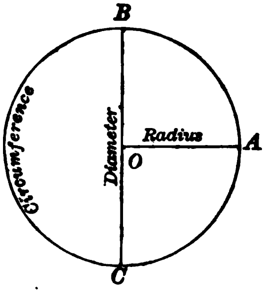
41. At what time should peas begin to boil, if they should boil 20 min. and be served at 5.50 P.M.?

42. If we allow 15 min. for each pound in roasting a chicken, how much time should we allow for a chicken weighing 5¾ lb.?

✓ **43.** If a class in cooking meets from 1.30 to 2.40 P.M., and is to make a cake that must bake 40 min., when must the cake be ready for the oven in order to be done 10 min. before the close of the class period?

44. How many minutes from 9.45 A.M. to 11.25 A.M.? to noon? to 1.15 P.M.?

134. Measuring Arcs. The circumference of every circle is divided into 360 equal parts called *degrees.*



Corresponding to every arc, like *AB* in the figure, is an angle at the center, *AOB.* Therefore to every degree on the circumference (*arc degree*) there is a degree at the center (*angle degree*), and an arc has as many degrees as its angle.

135. Right Angle and Perpendicular. The angle at the center of a circle corresponding to one fourth of a circumference is called a *right angle,* and the two lines that form a right angle are said to be *perpendicular* to each other.

Thus angle *AOB* is a right angle, and *OB* is perpendicular to *OA.* A right angle, therefore, contains ¼ of 360 degrees, or 90 degrees.

An angle less than a right angle is called an *acute angle,* and one greater than a right angle is called an *obtuse angle.*

136. Angle Measure and Arc Measure. The following are the tables of circular measure:

Angle Measure

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 4 right angles

Arc Measure

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circumference

That is, angle measures and arc measures are practically the same.

A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69.16 statute miles.

The teacher should explain the relation of this table to the study of the latitude and longitude of places represented on the map.

EXERCISE 74

Express as seconds:

1. 17′.
2. 32′.
3. 48′.
4. 1°.
5. 3°.
6. 27°.
7. 3° 5′.
8. 4° 8′.
9. 8° 5′ 8″
10. 9° 6′ 2″.
11. 15° 10′ 30″.
12. 18° 20′ 35″.
13. 90°.
14. 45°.
15. 30° 30′ 30″.

Express as minutes:

16. 480″.
17. 560″.
18. 420″.
19. 900″.
20. 360″
21. 240″.
22. 27°.
23. 35°.
24. 45°.
25. 9° 9′.
26. 10° 10′.
27. 35° 30′.
28. 27° 56′.
29. 48° 50′.
30. 59° 20′.

Express as degrees:

31. 4800″.
32. 3600″.
33. 1800″.
34. 9600″.
35. 2700′.
36. 5400′.
37. 50,400″.
38. 108,000″.

39. How many degrees in $\frac{1}{6}$ of a circumference? Draw a circumference and divide it approximately into sixths.

40. How many degrees in $\frac{1}{2}$ of a circumference? in $\frac{1}{4}$ of a circumference? Draw the figures.

41. There are 360° in the equator. How many minutes?

42. There are 90° from the north pole to the equator. How many minutes are there? How many seconds?

43. The circumference of a sailor's compass is divided into thirty-two equal parts. How many degrees are there in each part?

44. How many degrees in $\frac{1}{8}$ of the circumference of the earth? in $\frac{1}{60}$ of the circumference?

45. There are $23\frac{1}{2}$° from the north pole to the arctic circle. How many minutes?

46. There are $23\frac{1}{2}$° from the equator to the Tropic of Cancer. How many degrees from the arctic circle to the Tropic of Cancer?

137. Miscellaneous Measures. The following tables are used.

Numbers

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross
20 units = 1 score

Paper

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

The great gross and score are becoming obsolete. For convenience in counting, 500 sheets are now commonly called a ream.

EXERCISE 75

1. How many sheets in 24 quires?

2. How many pens in 18 doz.? in 9 gro.? in 35 gro.?

3. How many dozen in ½ gro.? in 2⅔ gro.? in 5¾ gro.?

4. What will a gross of padlocks cost at $5.25 a dozen?

5. How many years old is a man who is 4 score? one who is 3 score? one who is 4 score years and 10?

6. How many sheets of fine writing paper in 2 quires? in 35 quires? in 75 quires?

7. If paper costs 15¢ a quire, how much will 14 quires cost? How many sheets will there be?

8. At 20¢ a quire, how much will 7 quires of paper, with $1 worth of envelopes, cost?

9. What will 9 gro. of tablets cost at 25¢ a dozen?

10. At $7.20 a gross, what will 12 lamp chimneys cost?

11. In 7 doz. reams of 500 sheets each, how many more sheets than in 7 doz. reams of 480 sheets each?

12. At 12½¢ a quire, how much will 9½ quires of paper cost? 6½ quires? a dozen quires?

13. At 24¢ a quire for paper, and 1¢ each for envelopes, find the cost for paper, envelopes, and postage for sending 8 letters of one sheet each to some place in this country.

138. Wood Measure. Wood is sold by the cord. The standard cord is a pile 8 ft. long, 4 ft. wide, and 4 ft. high, and therefore contains 128 cu. ft.

The cord varies, often being considered to be a pile 8 ft. long and 4 ft. high, the price then varying with the length of the stick.

For example, how many cords of wood in a pile 24 ft. long, 4 ft. wide, and 6 ft. high ?

Here we have $$\frac{\overset{3}{\cancel{24}} \times \cancel{4} \times \overset{3}{\cancel{6}}}{\cancel{8} \times \cancel{4} \times \underset{2}{\cancel{4}}} \text{ cd.} = 4\tfrac{1}{2} \text{ cd.}$$

EXERCISE 76

Find the number of cords of wood in piles 4 ft. wide, whose length and height are as follows:

1. 10 ft., 4 ft.
2. 12 ft., 6 ft.
3. 15 ft., 5 ft.
4. 16 ft., 5 ft.
5. 18 ft., 4 ft.
6. 20 ft., 6 ft.
7. 24 ft., 4 ft.
8. 26 ft., 5½ ft.
9. 32 ft., 5 ft. 8 in.

10. How much are 7½ cd. of wood worth at $2.50 a cord ?

11. How much are 16¾ cd. of wood worth at $2.25 a cord ?

12. If wood is worth $2.40 a cord, how many cords can be bought for $42.60 ?

13. What is the cost of a pile of wood 22 ft. long and 5 ft. high, when wood of that length is selling for $3.50 a cord ? (Take the cord as a pile 8 ft. long and 4 ft. high.)

14. What is the cost of a pile of wood 36 ft. long and 6 ft. high, when wood of that length is selling for $3.75 a cord ?

15. A pile of wood 32 ft. long and 6 ft. high sells for $24. What is the price of wood of that length per cord ?

16. When wood of a certain length is worth $3.80 a cord, a pile 4 ft. high sells for $11.40. How long is the pile ?

17. These children are standing by a cord of wood. It is 8 ft. long and the wood is cut in 4-foot lengths. How high is the pile?

18. How many cubic feet are there in $3\frac{3}{4}$ cords?

19. A pile of 4-foot wood is 4 rd. long and 6 ft. high. How many cords does it contain?

20. A pile of 4-foot wood is 75 ft. long and 7 ft. high. How many cords does it contain?

21. How much is a pile of 4-foot wood 60 ft. long and 6 ft. high worth at $3.75 a cord?

22. How much is a pile of 4-foot wood 75 ft. long and 7 ft. high worth at $4 a cord?

23. A pile of wood 8 ft. long and 4 ft. high is usually called a cord, the price varying with the length of the wood. How many cords in a pile 28 ft. long and 6 ft. high?

24. A pile of wood is 50 ft. long and 5 ft. high. How many cords does it contain?

25. How much is a pile of wood 30 ft. long and 5 ft. high worth at $3.50 a cord?

139. Addition of Compound Numbers. The general principles for adding compound numbers are the same as those for adding integers.

The only difference arises from the fact that in integers ten units of each order always make one unit of the next higher, while in compound numbers the number of units of a lower denomination required to make one unit of the next higher varies with the denomination we are using.

Required to add 2 hr. 38 min. 42 sec., 3 hr. 25 min. 36 sec., and 5 hr. 17 min. 58 sec.

hr.	min.	sec.
2	38	42
3	25	36
5	17	58
11	22	16

The sum of the seconds is 136 sec., or 2 min. 16 sec. We write the 16 sec. under the seconds, and add the 2 min. to the minutes. The sum of the minutes is 82 min., or 1 hr. 22 min. Writing the 22 min., we add the 1 hr. to the hours. The sum of the hours is 11 hr. Hence the result is 11 hr. 22 min. 16 sec.

In practical life to-day we rarely have to add numbers of more than two denominations, and hence the examples given in this book are in general of not more than two denominations.

EXERCISE 77

1. A girl made two aprons. For one she used 1 yd. 9 in. of cloth and for the other 27 in. How much did she use in all?

Add the following:

2.

ft.	in.
13	2
24	8

3.

lb.	oz.
36	9
15	3

4.

rd.	ft.
10	4
25	11

5.

rd.	ft.
17	6½
18	10

6. If a boy weighs 73 lb. 12 oz. and his sister weighs 69 lb. 12 oz., how much do they weigh together?

7. The class in cooking had one piece of meat weighing 1 lb. 14 oz. and another weighing 2 lb. 9 oz. What was the total weight?

Add the following:

8.

lb.	oz.
25	12
24	4

9.

rd.	ft.
73	4½
26	12

10.

mi.	rd.
26	300
23	20

11.

mi.	rd.
37	280
42	50

12.

yd.	in.
41	25
41	11

13.

ft.	in.
60	9
10	4

14.

lb.	oz.
30	10
20	8

15.

lb.	oz.
41	8
16	8

16.

rd.	ft.
30	6½
40	10

17.

mi.	rd.
70	300
80	25

18.

yd.	in.
30	30
20	8

19.

yd.	in.
27	27
36	27

20.

da.	hr.
3	23
11	3
9	10

21.

wk.	da.
2	6
3	5
2	4

22.

bu.	pk.
44	2
17	3
16	2

23.

bu.	pk.
62	3½
18	3½
12	2

24.

ft.	in.
3	9
7	10
12	8

25.

sq. ft.	sq. in.
96	36
72	87
81	92

26.

cu. ft.	cu. in.
87	921
94	436
76	997

27.

lb.	oz.
147	7
962	12
321	8

28.

ft.	in.
29	11
16	8
37	6
16	4

29.

sq. ft.	sq. in.
39	47
46	83
92	54
33	2

30.

cu. ft.	cu. in.
37	692
49	875
89	981
75	100

31.

lb.	oz.
482	15
621	12
537	8
215	8

32.

gal.	qt.
29	3
37	2
16	1
19	3
36	2

33.

gal.	pt.
47	5
36	4
42	6
71	1
43	5

34.

ft.	in.
17	8
26	7
32	5
41	7
20	9

35.

hr.	min.	sec.
2	17	36
3	39	48
5	17	50
3	56	49
5	48	35

36.

gal.	qt.
36	3
48	1
29	2
43	2
51	3

37.

bu.	pk.
29	3
36	2
48	3
73	1
49	2

38.

ft.	in.
47	8
69	7
33	9
83	6
50	

39.

°	′	″
5	37	49
9	26	
2	52	46
	48	32
6	15	9

40.

qt.	pt.
17	1
36	1
49	1
27	
35	1

41.

pk.	qt.
19	3
17	6
23	4
19	7
13	5

42.

wk.	da.
12	4
32	5
16	6
10	5
13	3

43.

hr.	min.	sec.
17	23	32
23	15	48
4	7	15
16	19	49
43	6	33

44. Add 16° 27′ 32″, 37° 48′ 16″, 48° 29′ 36″, 28° 13′ 23″.

45. Add 2 A. 40 sq. rd., 3 A. 80 sq. rd., 4 A. 30 sq. rd., 10 sq. rd.

46. Add 5 sq. rd. 6 sq. yd., 7 sq. rd. 20¼ sq. yd., 6 sq. rd. 4 sq. yd.

47. Add 9 cu. yd. 18 cu. ft., 7 cu. yd. 12 cu. ft., 24 cu. ft. 920 cu. in., 808 cu. in.

48. Add 2½ lb., 4 lb. 4 oz., 3¾ lb., 5 lb. 8 oz., 6 lb. 9 oz., 5¼ lb., 2½ lb., 8 oz.

140. Subtraction of Compound Numbers. The general principles for subtracting compound numbers are the same as those for subtracting integers, except as explained in addition.

For example, from 22 hr. 16 min. 27 sec. required to subtract 14 hr. 25 min. 40 sec.

hr.	min.	sec.
22	16	27
14	25	40
7	50	47

Since we cannot subtract 40 sec. from 27 sec., we take 1 min. (60 sec.) of the 16 min. Then 60 sec. + 27 sec. = 87 sec., and 87 sec. − 40 sec. = 47 sec. Since we cannot subtract 25 min. from 15 min., we take 1 hr. (60 min.) of the 22 hr. Then 60 min. + 15 min. = 75 min., and 75 min. − 25 min. = 50 min. Then 21 hr. − 14 hr. = 7 hr. The difference is 7 hr. 50 min. 47 sec.

EXERCISE 78

1. The class made a lamp screen. Each wing was 1 ft. 6¾ in. high and 9¾ in. wide. It was how much higher than wide?

2. A girl tried some bluings in laundry work, using 2 gal. 1 qt. in one experiment and 2 qt. in another. How much more did she use in the first experiment than in the second?

3. If one boy is 5 ft. 1 in. in height and another is 4 ft. 10 in. in height, how much taller is the first boy?

Subtract:

4.

ft.	in.
58	10
9	6

5.

lb.	oz.
25	4
17	6

6.

rd.	ft.
35	3
26	7

7.

mi.	rd.
27	140
16	300

8. A lady buys 7½ yd. of cloth and uses 27 in. How much has she left?

9. A field is 42 rd. 8 ft. long and 24 rd. 12 ft. wide. The length of the field is how much more than the width?

10. A man starts out to travel 3½ mi. After he has gone 200 rd. how much farther has he to go?

Subtract:

11.		**12.**		**13.**		**14.**	
rd.	ft.	rd.	ft.	rd.	ft.	bu.	pk.
6	6½	6	6½	26	8½	42	1
	4		13	18	9¾	26	2

15.		**16.**		**17.**		**18.**	
mi.	rd.	mi.	rd.	mi.	rd.	gal.	qt.
10	60	10	0	76	7¾	39	0
3	20		20	29	8	17	2

19.		**20.**		**21.**		**22.**	
lb.	oz.	lb.	oz.	yd.	in.	qt.	pt.
37	6⅛	37	6⅛	53	28½	56	0
28	3	28	$7\frac{2}{8}$	47	32¾	29	1

23.		**24.**		**25.**		**26.**	
ft.	in.	yd.	ft.	lb.	oz.	gal.	qt.
29	7	12	2	173	0	42	2
16	10	11	1	69	10	29	3

27.		**28.**		**29.**		**30.**	
ft.	in.	bu.	qt.	lb.	oz.	qt.	pt.
82	5½	28	24	251	4	63	0
64	7.25	19	17	176	12	28	1

31.		**32.**		**33.**		**34.**	
sq. ft.	sq. in.	hr.	min.	cu. ft.	cu. in.	mi.	rd.
78	92	19	40	250	67	27	4
29	106	12	25	175	998	9	56

35.		**36.**		**37.**		**38.**	
sq. ft.	sq. in.	A.	sq. rd.	cu. ft.	cu. in.	min.	sec.
69	75.25	58	82	126	750	106	27
49	26½	42	120	78	1025	49	49

Subtract:

39.		40.		41.		42.	
ft.	in.	lb.	oz.	rd.	ft.	yd.	in.
93	7	47	8	32	4	27	4
61	8	26	9	26	5	19	16

43.		44.		45.		46.	
mi.	rd.	yd.	in.	ft.	in.	mi.	rd.
81	7	87	4	65½	0	246	30
65	98	69	8	48	9	129	75

47.		48.		49.		50.	
lb.	oz.	rd.	ft.	mi.	rd.	rd.	ft.
85	6	67	8¼	89	80	127	3
36½	0	39½	0	69½	0	99	12½

51.			52.			53.		
yr.	mo.	da.	yr.	mo.	da.	yr.	mo.	da.
1916	6	8	1919	7	3	1920	3	0
1914	7	9	1915	2	15	1917	9	16

In Exs. 51–53 reckon 1 mo. as 30 da.

54. The latitude of Albany is 42° 39′ 13″ N. (that is, north of the equator). How much does this lack of being 45°?

55. The latitude of Clinton, N.Y., is 43° 3′ 17″ N. This is how much more than that of Albany, as given in Ex. 54?

56. The latitude of New York City is 40° 45′ 23″ N. This is how much less than that of Albany?

57. The latitude of West Point is 41° 23′ 22″ N. How many degrees, minutes, and seconds is this from the north pole?

58. The longitude of Albany is 73° 44′ 48″ W. (that is, west of the prime meridian which passes through Greenwich, England). The longitude of New York City is 73° 58′ 25.5″ W. What is the difference in longitude?

141. Multiplication of Compound Numbers. Required to multiply 3 ft. 9 in. by 5.

ft.	in.
3	9
	5
18	9

Since 5 × 9 in. = 45 in., or 3 ft. 9 in., we write 9 in. under the inches, and reserve the 3 ft. to add to the product of 5 × 3 ft. Then 5 × 3 ft. = 15 ft., and 15 ft. + 3 ft. = 18 ft. Therefore the product is 18 ft. 9 in.

EXERCISE 79

Multiply:

1. 3 ft. 4 in. by 2.
2. 7 ft. 6 in. by 4.
3. 8 ft. 2 in. by 9.
4. 9 yd. 6 in. by 3.
5. 8 yd. 5 in. by 8.
6. 6 yd. 6 in. by 7.
7. 47 ft. 6 in. by 25.
8. 49 yd. 2 in. by 96.
9. 10 ft. 3 in. by 47.
10. 12 lb. 4 oz. by 92.
11. 40 yd. 3 in. by 68.
12. 37 ft. 6 in. by 7.
13. 49 ft. 11 in. by 15.
14. 64 ft. 10 in. by 16½.
15. 35 qt. 1 pt. by 17.
16. 62 gal. 3 qt. by 25.
17. 48 bu. 2 pk. by 62.
18. 15 hr. 27 min. by 6.
19. 621 mi. 20 ft. by 97.
20. 32 min. 48 sec. by 12.
21. 45 min. 15 sec. by 16.
22. 22 min. 30 sec. by 20.
23. 18 lb. 6 oz. by 6.
24. 7 rd. 9 ft. by 8.
25. 11 lb. 7 oz. by 4.
26. 9 gal. 3 qt. by 3.
27. 12 gal. 3 qt. by 2.
28. 10 mi. 1200 ft. by 5.
29. 27 lb. 3 oz. by 84.
30. 44 mi. 3 ft. by 89.
31. 273 ft. 2 in. by 35.
32. 309 lb. 5 oz. by 67.
33. 370 yd. 1 in. by 83.
34. 27 sq. ft. 60 sq. in. by 7.
35. 6 cu. ft. 700 cu. in. by 9.
36. 27 cu. ft. 200 cu. in. by 8.
37. 28 lb. 4 oz. by 35.
38. 48 min. 25 sec. by 16.
39. 55 min. 30 sec. by 20.
40. 18 bu. 3 pk. by 8.
41. 9 T. 1650 lb. by 4.
42. 404 mi. 70 ft. by 73.
43. 210 mi. 10 yd. by 50.
44. 175 mi. 10 rd. by 60

45. If the average weight of the 35 members of your class is 57 lb. 4 oz., what is the total weight of the class?

46. If the average length of your steps is 2 ft. 2 in., how far will you go in 425 steps?

47. If a wheel is 9 ft. 2 in. in circumference, how far will it travel in making 125 revolutions?

48. If one side of a square is 8 ft. 9 in., how far is it around the square; that is, what is the perimeter?

49. One side of a square is 7 ft. 9 in. What is the perimeter?

50. If a barrel will hold 31 gal. 2 qt., how much water will 15 barrels hold?

51. If one can of fruit weighs 1 lb. 12 oz., what will a dozen such cans weigh?

52. If a school has 6 recitations a day, each of 45 min., how many hours and minutes of recitations a day?

53. A year being 365 da. 5 hr. 48 min. 46 sec., how many days, hours, minutes, and seconds in 6 yr.?

54. If a cubic foot of water weighs 62 lb. 8 oz., what is the weight of the water in a tank that contains 48 cu. ft.?

55. What is the weight of the water in a tank that is 6 ft. long, 5 ft. wide, and 13 ft. deep?

56. What is the weight of the water in a tank that is 6 ft. 6 in. long, 5 ft. wide, and 12 ft. deep?

57. What is the weight of the water in a tank that is 6 ft. 6 in. long, 4 ft. 6 in. wide, and 8 ft. deep?

58. If the height of each story above the first or ground story in a city office building is 11 ft. 9 in., and that of the first story is 15 ft. 8 in., and there are nine stories in all, what is the height of the building?

142. Division of a Compound Number by an Abstract Number. Required to divide 7 hr. 5 min. 24 sec. by 2.

$$\begin{array}{r|rrr} & \text{hr.} & \text{min.} & \text{sec.} \\ 2 & 7 & 5 & 24 \\ \hline & 3 & 32 & 42 \end{array}$$

Arranging as in ordinary division, we have 7 hr. ÷ 2 = 3 hr., with remainder 1 hr., or 60 min. Then 60 min. + 5 min. = 65 min., and 65 min. ÷ 2 = 32 min., with remainder 1 min., or 60 sec. Then 60 sec. + 24 sec. = 84 sec., and 84 sec. ÷ 2 = 42 sec. Therefore the quotient is 3 hr. 32 min. 42 sec.

In most cases in practical business the dividend has only two denominations. Indeed, it is so much more common to write 2½ bu., 7½ lb., 28.25 A., instead of 2 bu. 2 pk., 7 lb. 8 oz., 28 A. 40 sq. rd., that compound numbers are not nearly as important as formerly.

143. Division of One Compound Number by Another. Required to divide 41 ft. 3 in. by 2 ft. 9 in.

Here it is advisable to reduce both numbers to the same denomination, and it makes no difference in principle whether we reduce to feet or to inches.

Reducing to inches, we have 41 ft. 3 in. = 495 in. and 2 ft. 9 in. = 33 in. Then 495 in. ÷ 33 in. = 15.

Or, reducing to feet, we have

$$41\tfrac{1}{4}\text{ ft.} \div 2\tfrac{3}{4}\text{ ft.} = \frac{\cancel{4}}{\cancel{11}} \text{ of } \frac{\overset{15}{\cancel{165}}}{\cancel{4}} = 15.$$

We aim to choose the denomination that makes the work the shorter.

EXERCISE 80

Divide:

1. 9 ft. 8 in. by 4.
2. 175 ft. 4 in. by 8.
3. 34 lb. 6 oz. by 11.
4. 29 yd. 18 in. by 6.
5. 47 mi. 288 ft. by 12.
6. 4 hr. 3 min. 30 sec. by 10.
7. 25 ft. 1 in. by 3 ft. 7 in.
8. 60 ft. 9 in. by 6 ft. 9 in.
9. 8 hr. 3 min. by 2 hr. 45 sec.
10. 59 yd. 21 in. by 5 yd. 15 in.
11. 240 lb. 12 oz. by 26 lb. 12 oz.
12. 21 mi. 1620 ft. by 1 mi. 345 ft.

13. How many planks, each 9 ft. 6 in. long, will it take to reach 76 ft.?

14. How many cans, each containing ⅞ qt., can be filled from 7 gal. of sirup?

15. If a train travels 20 mi. in 24 min. 20 sec., how long does it take to go a mile?

16. How many rods are there in 44 yd.? How many yards are there in 44 rd.?

17. From 9 gal. of sirup 40 cans are filled. How many pints does each can hold?

18. If a train moves 73 ft. 4 in. in a second, how many seconds will it take to go a mile?

19. How many lots, each containing 2 A. 20 sq. rd., can be cut from a lot containing 17 A.?

20. If a train is traveling at the rate of a mile in 72 sec., how many miles will it travel in 42 min.?

21. If a man walks at the rate of a mile in 19 min., how many miles can he walk in 1 hr. 54 min.?

22. If a man can build 30 ft. 8 in. of fence a day, how many days will it take him to build 214 ft. 8 in.?

23. A steamship changed its longitude 52° 30′ in 7 days. How many degrees of longitude did it sail per day?

24. If each tile has a surface area of 1 sq. ft. 12 sq. in., how many tiles will be required to cover a space containing 156 sq. ft.?

25. A real estate dealer has 3½ A. of land that he purposes to cut into five equal lots. How many square rods will there be in each lot?

26. A man is walking at the rate of a mile in 21 min., and a train is going at the rate of a mile in 72 sec. The train is traveling how many times as fast as the man?

EXERCISE 81

State the units used in measuring the following:

1. The length of dress goods; the width.
2. The quantity of a liquid.
3. The quantity of dry substances, like grain.
4. Sugar, in buying it; in cooking.
5. Rice, in buying it; in cooking.
6. Tea, in buying it; in making it for the table.
7. Cocoa, in buying it; in making it for the table.
8. Salt, in buying it; in cooking.
9. Berries, in buying them; in cooking.
10. Oranges, in large quantities; in small quantities.
11. Kerosene, in large quantities; in small quantities.
12. Water, in large quantities; in cooking.
13. The length of carpeting; the width.
14. The dimensions of a book cover; of a floor; of a field.
15. The area of a sheet of paper; of a floor; of a field.
16. The capacity of an aquarium; the contents of a grain elevator; the capacity of an oil tank.
17. Milk, when sold to a creamery; when sold to a dealer; when sold to a family.
18. Coffee, when sold in large quantities; when sold to families; when used for cooking.
19. Potatoes, when sold in large quantities; when sold in small quantities at a city grocery; when used in cooking.
20. Butter, when sold in large quantities; when sold in small quantities; when used in cooking.
21. Flour, when sold in large quantities; when sold in small quantities; when used in cooking.

EXERCISE 82

REVIEW QUESTIONS

1. What are denominate numbers? compound numbers?

2. What is meant by reduction descending? Illustrate.

3. What is meant by reduction ascending? Illustrate.

4. Write the common table of length. Make up and solve an example in addition of three compound numbers.

5. Write the common table of square measure. Make up and solve an example in subtraction in this table.

6. Write the common table of cubic measure. Make up and solve an example in multiplication in this table.

7. Write the common table of weight. Make up and solve an example in division in this table.

8. Write the common table of liquid measure. Make up and solve an example in division in this table.

9. Write the common table of dry measure. Make up an example involving this table, and solve it.

10. Why is it necessary to have some years of 366 days, and how often do they occur? What is the exception?

11. Write the tables of angle and arc measure. Make up an example involving each table, and solve it.

12. How does the addition of compound numbers differ from that of integers? Illustrate.

13. How does the subtraction of compound numbers differ from that of integers? Illustrate by the case of 42 – 39, and 4 ft. 2 in. – 3 ft. 9 in.

14. How does the multiplication of a compound number differ from that of an integer? Illustrate.

15. Illustrate the two kinds of division of compound nbers.

144. Solving Problems. We have now learned the operations of addition, subtraction, multiplication, and division with integers, common fractions, decimals, and compound numbers. We are therefore ready to consider more fully the methods of solving ordinary business problems involving denominate numbers.

145. Three Necessary Things. In solving a problem there is one thing that the teacher will ask you to do, and two things that may be required. These three are: (1) to work rapidly and accurately; that is, to take the shortest road to the answer, and to be certain the answer is correct; (2) to put neatly on paper not merely the operation but a brief explanation; (3) to give a brief analysis or oral explanation, showing that you understand the work.

For example, if 5 yd. of cloth cost $2.10, how much will 12 yd. cost?

(1) *The number work.*

$$\frac{12 \times \$\overset{0.42}{\cancel{2}.\cancel{1}\cancel{0}}}{\cancel{5}} \qquad \begin{array}{r} \$0.42 \\ \underline{12} \\ 84 \\ \underline{42} \\ \$5.04 \end{array}$$

(2) *The written explanation:*

5 yd. cost $2.10.
1 yd. cost $\frac{1}{5}$ of $2.10.
12 yd. cost 12 × $\frac{1}{5}$ of $2.10, or $5.04.

(3) *The oral analysis:*

Since 5 yd. cost $2.10, 1 yd. will cost $\frac{1}{5}$ of $2.10, and 12 yd. will cost 12 × $\frac{1}{5}$ of $2.10, or $5.04.

Teachers will not need to call for all this work with every example. Sometimes it will be necessary to emphasize the written explanation and sometimes the oral analysis, but the important thing is that the number work should be quickly and neatly, and, above all, *accurately* done.

146. Estimating the Answer. One of the best ways to avoid absurd answers is to *estimate the result* in advance.

For example, if 5 yd. cost $2.10, we know 12 yd. will cost nearly 2½ times as much, or somewhere near $5. When we solve, if we find such a result as $50.40, we see at once that there is a mistake, probably in the position of the decimal point. The correct result is $5.04.

Always estimate the answer in advance, write down the estimate, and compare it with the result. If there is a great difference, the work is probably wrong.

It is almost always possible to estimate an answer within a reasonable amount. The more one practices, the easier this becomes.

147. Accuracy. In any single operation the work is *right* or it is *wrong*. A business man will not excuse a bookkeeper who writes $9250.75 instead of $90,250.75. Only a zero is missing, but it means a difference of over $80,000.

A pupil may be excused if he does not understand the wording or the meaning of a problem, but for inaccurate numerical work he must be judged as he will be judged in business, — his work is right or else it is wrong. Accuracy is far more important than speed.

148. Checking the Work. *Always check the number work in some way. The first consideration in arithmetic is accuracy.*

One of the simplest checks is estimating the answer in advance, or seeing if it is reasonable after the work is performed.

149. Businesslike Work. Work as business men work, putting the additions and subtractions in columns except when a written analysis is required.

Much time is wasted over unbusinesslike analyses of problems that require nothing more than a rapid, accurate solution.

150. Neatness. Always have the work neatly done. Slovenly work leads to inaccuracy. A business man does not wish to employ any one whose letters, accounts, or computations have a slovenly appearance.

EXERCISE 83

1. A schoolroom is 32 ft. 8 in. long. Express the length in inches.

2. A room is 282 in. wide. Express the width in feet and inches; in feet and a fraction.

3. How much will a strip of carpet cost at 50¢ a yard, the strip being 16 ft. 6 in. long?

4. A wheel is 11 ft. in circumference. How many times will it turn in going $2\frac{1}{2}$ mi.?

5. If a lot is 197 ft. 9 in. long and $82\frac{1}{4}$ ft. wide, the length is how many rods greater than the width?

6. If a man is walking 88 yd. a minute, he is walking at the rate of how many miles per hour?

7. If a cubic inch of cast iron weighs $4\frac{5}{36}$ oz., how many pounds does a cubic foot weigh?

8. If a cubic foot of ice weighs 57 lb. 8 oz., how many cubic feet of ice will it take to weigh a ton?

9. The radius of the earth at the equator is estimated to be 20,926,202 ft. Express this in miles and feet.

10. If a pile of wood contains 2760 cu. ft., how many cords does it contain?

11. Express 2 ft. 6 in. as inches; as feet and a common fraction; as feet and a decimal; as a fraction of a yard; as a fraction of a rod.

12. A horse is 14 hands high. Express this in inches; in feet and inches; in feet and a common fraction; in feet and a decimal fraction.

13. A cubic foot of water weighs nearly 1000 oz. How many pounds is this? Express the result in pounds and ounces; in pounds and a common fraction of a pound; in pounds and a decimal fraction of a pound.

14. How many pounds does 5 cu. yd. 18 cu. ft. of earth weigh, if a cubic yard weighs 1 T. 825 lb.?

15. If the average weight of a certain kind of water pipe is 15 lb. 8 oz. per foot, how many feet of the pipe will it take to weigh a ton?

16. If the water in a certain harbor is 14 fathoms deep, a fathom being 6 ft., what is the depth in feet? If the depth is 165 ft., what is the depth in fathoms?

17. If a man takes 100 steps of 0.8 yd. each in a minute, how long will it take him to walk a distance of 16 mi.?

18. When sound travels 1116 ft. a second, how far distant is the lightning flash (which is seen practically instantaneously) when a man hears the thunder 9 sec. after he sees the flash?

19. If a cubic foot of ice weighs 57 lb. 8 oz., what will be the weight in tons of the mass of ice 1 ft. thick that covers a pond whose surface is 100,000 sq. yd.?

20. A train leaves Binghamton at 5 min. past 8, and reaches a place 59.6 mi. distant at 5 min. before 10. What is the average time per mile? Answer to two decimal places.

21. A train leaves Albany at 8.45 A.M., and reaches a place 284.74 mi. distant at 3.30 P.M. What is the average time per mile? Answer to two decimal places.

22. On a trip of three miles an automobile required 2 min. 34 sec. for the first mile, 2 min. 17 sec. for the second, and 1 min. 56 sec. for the third. How long did it take for the three miles?

23. When a man was three miles from his railroad station he had 20 min. 30 sec. before his train was due to leave. He then drove one mile in 6 min. 48 sec., and a second mile in 6 min. 54 sec. What is the longest time in which he can drive the last mile and still catch his train?

24. This farmer is pointing to a field that is 80 rd. long and 40 rd. wide. How many square rods does it contain? How many acres?

25. If a field is 80 rd. square, how many acres does it contain? How many acres will it contain if it is 160 rd. square?

26. A man thinks of replacing the fence around a field by a 4-strand barbed wire fence. The field is a rectangle 120 rd. by 30 rd. How many pounds of barbed wire will he need, allowing 1 lb. of barbed wire to the rod?

27. If an artesian well produces 3.265 cu. ft. of water per minute, and a cubic foot of water weighs 62.5 lb., what is the weight of the water produced in an hour?

28. If a man allows 350 cu. ft. of solidly stacked hay to the ton, how many tons of hay are there in a mow that contains 4375 cu. ft.?

29. If corn fodder loses $\frac{3}{5}$ of its weight in curing, how much will 28 T. of green fodder weigh after it is cured? How much will 45 T. weigh?

30. Taking the annual rainfall of New York as 37.52 in., what will be the weight of water that will fall on 1 sq. ft. of land per year? on 1 sq. rd.? on 1 A.?

31. If a farmer has 64 A. of wheat yielding 24½ bu. per acre, what will it cost him for threshing at 4¢ a bushel?

32. What is the total cost of 15½ bu. of wheat at 80¢, 17 bu. of potatoes at 28¢, 13 bu. of corn at 45¢, and 21 bu. of oats at 41¢?

33. If a man and team can seed 8 A. a day, how long will it take to seed a field of 96 A., and what will it cost at $6.12½ a day?

34. Supposing they are able to plow 2 A. a day, how much will it cost to plow a 76-acre field at $6.12½ a day?

35. If three fourths of the weight of potatoes is water, and if a bushel of potatoes weighs 60 lb., what is the weight of water in 275 bu. of potatoes?

36. If two twenty-fifths of the weight of barley is water, and if a bushel of barley weighs 48 lb., what is the weight of water in 275 bu. of barley?

37. When wheat is worth 92¢ a bushel, what is the value of the wheat grown on 75 A., if the average yield per acre is 22 bu.?

38. If wheat weighs 60 lb. per bushel, and the average yield is 24½ bu. per acre, what will be the weight of the wheat grown on a field of 42½ A.?

39. An experiment has shown that it requires a pull of 92 lb. to draw a ton over a good gravel road, and 2.35 times as much to draw the same load over a common dirt road. What pull is required for the dirt road?

40. It is estimated that a pine pillar of a certain length, and 4 in. square at the base, will support 12.1 T. of the weight of a barn floor. At this rate, how much will such a pillar 4 in. thick and 8 in. wide support? a pillar 4 in. thick and 10 in. wide? a pillar 4 in. thick and 12 in. wide?

151. Advantage of Cancellation. As we have already found, there is often an advantage in stating a problem in a single step, endeavoring then to simplify the work by cancellation.

For example, a man in rapid walking takes 120 steps a minute, averaging 26 in. each. How long will it take him to walk a mile?

Since he walks 26 in. in taking 1 step and takes 120 steps a minute, in 1 min. he will walk 120 × 26 in. Since in 1 mi. there are 5280 ft., or 5280 × 12 in., it will take him as many minutes to walk a mile as 5280 × 12 in. divided by 120 × 26 in. Therefore we have

$$\frac{5280 \times 12 \text{ in.}}{120 \times 26 \text{ in.}} = \frac{\overset{\overset{264}{\cancel{528}}}{\cancel{5280}} \times \cancel{12}}{\underset{\cancel{10}}{\cancel{120}} \times \underset{13}{\cancel{26}}} = 20\tfrac{4}{13}.$$

It will therefore take the man $20\frac{4}{13}$ minutes to walk 1 mile.

EXERCISE 84

1. A manufacturer ships 2500 lb. of brass bedsteads from New York to Chicago. Such freight takes the first-class rate, 75¢ per hundred pounds. The manufacturer agrees to pay two thirds of the freight. How much does he pay?

2. A manufacturer ships 1800 lb. of iron bedsteads, this kind of freight taking the second-class rate, 65¢ per hundred pounds. He agrees to pay one third of this freight. How much does he pay?

3. A manufacturer ships 650 lb. of furniture casters in boxes, this kind of freight taking the third-class rate, 50¢ per hundred pounds. If he pays three fourths of this freight, how much does he pay?

4. A manufacturer ships 900 lb. of crated bamboo bedsteads, this kind of freight taking double the first-class rate. If the first-class rate is 75¢ per hundred pounds, how much is the freight?

5. He ships 1200 lb. of crated bamboo cabinets, the rate being three times the first-class rate. If the first-class rate is 75¢ per hundred pounds, how much is the freight?

6. A dealer ships some bundles of green hides weighing 1900 lb., these taking the fourth-class rate, which is $\frac{7}{13}$ of the first-class rate. If the first-class rate is 75¢ per hundred pounds, how much is the freight?

7. A dealer ships 2200 lb. of dry hides, the rate being $1\frac{1}{2}$ times the first-class rate. If the first-class rate is 90¢ per hundred pounds, how much is the freight?

8. Car loads of green hides take the fifth-class rate. If the first-class rate is 80¢ per hundred pounds, and the fifth-class two fifths as much, and a dealer ships a car load of 36,000 lb., how much is the freight?

9. Wheat in car-load lots takes the sixth-class rate. If the first-class rate is 90¢ per hundred pounds, and the sixth-class one third as much, and a dealer ships a car-load lot of 40,000 lb., how much is the freight?

10. If a dealer sends by express 250 lb. of goods, the rate being $3.50 per hundred pounds, and he pays half of the transportation charges, how much does he pay?

11. A dealer sent an express package weighing 150 lb. and agreed to pay $\frac{2}{5}$ of the transportation charges. If the rate was $2.50 per hundred pounds, how much did he pay?

12. A canal boat carries 211.75 T. of freight, the rate being 20¢ per hundredweight. If $\frac{3}{7}$ of the freight is owned by one man, $\frac{2}{7}$ by another, and the rest by a third, what is the charge to be paid by each?

13. If a drayman in a city averages 3 loads of freight a day, and receives on an average $2.10 a load, how much will he earn in a month of 26 working days?

152. Unitary Analysis. A common form of analysis is illustrated in § 145, where we went from the cost of 5 yd. to the cost of 1 yd., and then to the cost of 12 yd.

An analysis that proceeds first to the unit and then to the required number of units is called *unitary analysis.*

In all such problems the words "at the same rate" are understood.

For example, if 50 bbl. of flour cost $235, how many barrels can be bought for $126.90?

If $235 buys 50 bbl., $1 will buy $\frac{50\text{ bbl.}}{235}$, and $126.90 will buy

$$126.90 \times \frac{\overset{10}{\cancel{50}}\text{ bbl.}}{\underset{47}{\cancel{235}}} = \frac{1269\text{ bbl.}}{47} = 27\text{ bbl.}$$

Sometimes the problem requires us to proceed to two or more units, as in some of the examples already given.

For example, if 16 horses eat 42 bu. of oats in 2 weeks, how many bushels will 32 horses eat in 3 weeks?

1. In 2 weeks 16 horses eat 42 bu.
2. In 1 week 16 horses eat $\frac{1}{2}$ of 42 bu.
3. In 1 week 1 horse eats $\frac{1}{16}$ of $\frac{1}{2}$ of 42 bu.
4. In 3 weeks 1 horse eats $3 \times \frac{1}{16}$ of $\frac{1}{2}$ of 42 bu.
5. In 3 weeks 32 horses eat $32 \times 3 \times \frac{1}{16}$ of $\frac{1}{2}$ of 42 bu.

Canceling, we find the result to be 126 bu.

EXERCISE 85

1. If 100 laborers can do a piece of work in 12 days, how many will be needed to do it in 8 days?

2. How many men will it take to do a piece of work in 168 days that 108 men can do in 266 days?

3. If a stable has on hand enough oats to last 25 horses 105 days, how long will the oats last 15 horses?

4. How long will it take 16 men to do a piece of work that 18 men can do in 24 days?

5. If a clock gains 1 min. 15 sec. in 24 hours, how much will it gain in 42 hours at the same rate?

6. If 80 ft. of water pipe weigh 1240 lb., how much will 4 rd. of the pipe weigh?

7. If a candle $5\frac{2}{3}$ in. long burns 3 hr. 24 min., how many hours will a candle 6.3 in. long burn?

8. If a certain weight of sheet iron $\frac{3}{8}$ in. thick covers 15 sq. ft., how many square feet would the same weight of sheet iron $\frac{1}{4}$ in. thick cover?

9. If a wheel 12 ft. 1 in. in circumference makes 3432 revolutions in going a certain distance, how many revolutions will a wheel 11 ft. 11 in. in circumference make?

10. If a wheel 10 ft. 2 in. in circumference makes 3036 revolutions in going a certain distance, what is the circumference of a wheel that makes 3660 revolutions in going the same distance?

11. At the rate of $2\frac{3}{4}$ mi. an hour, I can walk a certain distance in 2 hr. 30 min. How long will it take me to walk the same distance at the rate of $3\frac{1}{8}$ mi. an hour?

12. At the rate of $2\frac{7}{8}$ mi. an hour, I can walk a certain distance in 2 hr. 20 min. What is my rate per hour if I walk the same distance in 2 hr. 33 min. 20 sec.?

13. Using 24 electric lights, a man's electricity bill is $9 for 3 months. How much will it be for 4 months, if he uses half a dozen more lights, all lights running equally?

14. If 144 reams of paper are used in printing 7600 books, 84 sheets to a book, how many reams will be required to print 4900 books, containing 57 sheets of the same size?

15. If 27 sacks, each containing $3\frac{1}{3}$ bu., will hold a given quantity of grain, what must be the capacity of each sack that 18 sacks may hold the same quantity?

EXERCISE 86

PROBLEMS IN SHIPPING OUR FOOD

1. A dealer ships 75 bu. of blueberries to market, using cases of 24 qt. each. How many cases does he need?

2. A gardener ships 1272 bunches of asparagus in boxes of 2 dozen bunches each. How many boxes does he need?

3. A market gardener ships 960 bu. of early potatoes in 3-peck boxes. How many boxes does he need?

4. A Monroe County fruit grower ships 150 baskets of peaches to New York, each basket containing $\frac{1}{5}$ bu. If the dealers sell these by the quart, how many quarts will they sell?

5. A man ships 150 boxes of damson plums to a city dealer. The boxes hold $\frac{1}{3}$ bu. each. The plums are sold in the city by the quart. How many quarts are there?

6. A dealer shipped 1250 lambs to New York. The average weight was 81 lb., and he paid $4.75 per 100 lb. How much did he pay for the lot?

7. A Florida grower sends to the northern market 250 crates of strawberries, each containing 32 quart boxes. The berries retail at 40¢ a box. What is the total retail price?

8. A sheep raiser shipped a lot of lambs to a packing house, receiving $342 for the lot. The total weight was 7200 lb. At this rate, how much should he receive for another shipment of 3200 lb.?

9. A grape grower ships a ton of grapes to market. Of these $\frac{9}{10}$ were put into 9-pound baskets, and the rest into 10-pound baskets. How many baskets did he ship?

10. A gardener shipped 260 lb. of horse-radish to a city market, where it was sold for $9.85. If the transportation charges were 75¢, what was the net price per pound?

EXERCISE 87

PROBLEMS IN MANUFACTURING

1. If the output of rope in a single year in this country was 200,824,974 lb., how many tons were manufactured?

2. Our silk mills consumed 17,471,000 lb. of raw silk in a single year. How many tons were consumed?

3. If there were manufactured in the United States in a certain year 35,553,731 gross of buttons, how many buttons were manufactured?

4. If there were manufactured in a certain year in this country 136,887,782 gross of pins, how many pins were manufactured?

5. A tannery paid $390 for tanning materials, which was 0.13 of the total cost of the materials it used. What did it pay for all the materials?

6. If our cotton mills use 4,485,000 bales of cotton in one year, weighing 2,242,500,000 lb., what is the average weight per bale?

7. If the capital of a certain establishment is $75,500, and its annual business is 3.2 times its capital, what is its annual business?

8. If we have 1400 establishments engaged in manufacturing boots and shoes, and all but one twentieth are in the New England, North Central, and Middle Atlantic States, how many are in the other states?

9. A manufacturing establishment makes a certain article for which it pays 65¢ for material, 25¢ for labor, 24¢ for expenses and advertising, and 8¢ for putting the article on the market. It sells this article for $1.30. How much is its profit on 1728 such articles? How much is its profit on 60 doz. such articles?

EXERCISE 88

PROBLEMS IN HOME PURCHASES

1. If 16 lb. of a certain kind of biscuit can be bought for $1.44, how much will 10 lb. cost at the same rate?

2. A woman can buy pastry flour in 10-pound bags at 44¢, or in 24½-pound bags at 98¢. How much does she save a pound by buying the flour in the larger bags?

3. A dealer sells Ceylon tea at 68¢ a pound, and 5-pound tins at $3.14. If two families want 2½ lb. each, how much will each save if they buy a tin and divide it?

4. A dealer sells coffee in half-pound bags at 17¢, and in 5-pound cans at $1.64. If a woman wishes 5 lb., how much does she save by purchasing it by the can?

5. A woman can buy Dutch cocoa in ¼-pound boxes at 22¢, or in 5-pound cans at $2.78. If she wishes 5 lb., how much does she save by purchasing it by the can?

6. A woman can buy maple sirup at 44¢ a quart, or in gallon cans at $1.38. If she wishes 4 qt., how much does she save by purchasing it by the can?

7. A dealer sells large glass jars of comb honey at $5.06 a dozen, or 44¢ each. How much more will he receive by selling a gross by the jar than by the dozen?

8. A woman buys a pound each of mustard seed, allspice, and cayenne pepper, paying 60¢ for the three. The mustard seed costs 12¢ a pound, and the allspice costs the same as the pepper. How much does the allspice cost?

9. A woman bought a can each of cinnamon, nutmeg, and pepper, paying 39¢ for the three. The pepper cost 9¢, and the nutmeg cost twice as much as the cinnamon. How much did the cinnamon cost? How much did the cinnamon and nutmeg together cost?

EXERCISE 89

MISCELLANEOUS PROBLEMS

1. If a man walks at the rate of a mile in 18 min. 20 sec., how many miles can he walk in 1 hr. 31 min. 40 sec.?

2. When sailors measure the depth of the water they use as a measure a fathom of 6 ft. If the water is 120 fathoms deep, how many feet is this?

3. When we speak of a distance at sea being 480 mi., we mean nautical miles, or knots. A knot being nearly 1⅛ mi., how many miles does a ship sail in going 480 knots?

4. The more exact length of a knot being 6080.27 ft., how many miles in 3000 knots?

5. A sheet of paper 10 in. wide is to be cut into 4 leaves, each 8 in. long and 5 in. wide. What is the length of the sheet? Draw a plan.

6. A sheet of paper 10 in. wide is to be cut into 6 leaves, each 8 in. long and 5 in. wide. What is the length of the sheet? Draw a plan.

7. A ceiling is 24 ft. long and 18 ft. wide. It is to be calcimined at 9¢ a square yard. What will be the cost of calcimining it?

8. A field is 32 rd. long and 25 rd. wide. What is its area in acres?

9. A rectangular city lot is 106 ft. 7 in. long and 28 ft. 8 in. wide. Find the number of square feet in the lot.

10. A rectangular field containing 4 A. is 32 rd. long. How wide is it?

11. A rectangular field containing 4½ A. is 36 rd. long. How wide is it?

12. How many square rods in a rectangular field 27½ rd. long and 24 rd. wide? How many acres?

CHAPTER VIII

BILLS AND ACCOUNTS

153. Bill. A statement of the amount owed by one person to another is called a *bill.*

The person who owes the money is called the *debtor.*

The person to whom the money is owed is called the *creditor.*

The debtor or creditor may be a company of persons, instead of one person.

To *make the extensions in a bill* means to write the amount after each item. To *foot a bill* means to add the amounts of the several items and find the total amount.

To *receipt a bill* means to stamp or write the words "Paid" or "Received payment" at the foot, followed by the name of the creditor or his authorized agent.

154. Specimen of a Bill. The following is a receipted bill:

Ithaca, N.Y., Feb. 1, 1915

Mr. David Brownson, Newfield, N.Y.

Bought of **Charles Dunham**
1272 State St., Ithaca, N.Y.

Jan.	5	6 cans soup	@ 20¢	$1	20		
	20	10 lb. pearl barley	@ 9¢		90	$2	10

Received payment, Feb. 8, 1915

Chas. Dunham

Items quoted by the dozen or fraction of a dozen have prices given by the dozen. Thus, ½ doz. oranges @ 50¢ means that the price is 50¢ a dozen.

EXERCISE 90

Write and receipt bills for the following, inserting the place, date, and names, and completing each item:

1. 14 yd. flannel @ 45¢, 32 yd. sheeting @ 30¢, 4½ yd. velvet @ $1.60, 2½ yd. ribbon @ 40¢, 2 pr. gloves @ $1.75.

2. 6 lb. ham @ 19¢, 8 lb. bacon @ 22¢, 4 lb. sausage @ 24¢, 2 tins sliced smoked beef @ 25¢, 3 cans corned beef @ 13¢, 4 cans tomato soup @ 22¢, 3 jars extract of beef @ 40¢, 2 lb. tapioca @ 8¢.

3. 3 qt. Lima beans @ 15¢, 4 cans tomatoes @ 14¢, 6 cans French peas @ 22¢, 5 cans California asparagus @ 42¢, 2 cans imported asparagus @ 85¢, 4 cans sardines @ 18¢, 2 packages oatmeal @ 12¢.

4. 5 boxes Smyrna figs @ 60¢, 4 boxes stuffed dates @ 40¢, 6 lb. California prunes @ 20¢, 3 lb. Malaga raisins @ 22¢, 4 tins crystallized ginger @ 40¢, 6 jars marmalade @ 28¢, 6 cans peas @ 18¢.

5. 6 heads lettuce @ 6¢, 9½ lb. butter @ 32¢, 7½ gal. oil @ 18¢, 12 lb. raisins @ 12½¢, 6 lb. coffee @ 32¢, 4½ doz. eggs @ 36¢, ½ doz. oranges @ 60¢, 6 lb. powdered sugar @ 7½¢, 2 qt. lima beans @ 15¢.

6. 4 lb. pearl barley @ 9¢, 6 lb. cornstarch @ 8¢, 4 packages hominy @ 20¢, 6 packages oatmeal @ 12¢, 6 lb. rice @ 11¢, 5 lb. sago @ 10¢, 3 lb. tapioca @ 8¢.

7. 5 gal. molasses @ 45¢, 10 lb. coffee @ 18¢, 15 lb. sugar @ 7¢, 6 boxes crackers @ 8¢, 6½ doz. eggs @ 34¢, 9 lb. butter @ 33¢, 6½ lb. cheese @ 12¢.

8. 7 gro. toothbrushes @ $9.65, 8 doz. brushes @ $17.75, 2½ gro. nailbrushes @ $26.80, ½ doz. clothes brushes @ $2.92, 6 doz. boxes tooth powder @ $2.04, 4½ doz. atomizers @ $19.60.

9. 6 bedroom sets @ $48.75, 15 rockers @ $4.20, 9 tables @ $7.50, 16 chairs @ $2.10.

10. 75 gro. bone buttons @ 22¢, 675 yd. cashmere @ 85¢, 840 yd. cambric @ 17½¢, 560 yd. chambray @ 22½¢, 620 yd. flannel @ 37½¢, 950 yd. sheeting @ 24½¢.

11. 75 bbl. flour @ $5.23, 63 gal. molasses @ 39¢, 75 gal. fancy sirup @ 58¢, 3 doz. packages pancake flour @ $1.20, 260 lb. coffee @ 30¢.

12. 8 doz. hatchets @ $10.75, 6 doz. pr. hinges @ $4.35, 5 doz. carpenter's squares at $34.50, ½ gro. locks @ $42.50, 8 doz. files @ $6.25.

13. 575 yd. cotton flannel @ 38¢, 684 yd. cotton @ 7¢, 364 yd. buckram @ 9¢, 648 yd. sheeting @ 26¢, 327 yd. Java canvas @ 62¢.

14. 84 gro. bone buttons @ 18¢, 72 doz. pearl buttons @ 9¢, 694 yd. cambric @ 17¢, 364 yd. cashmere @ 82¢, 408 yd. chambray @ 21¢.

15. 4 gro. toothbrushes @ $9.35, 3 doz. atomizers @ $19.25, 8 doz. combs @ $1.95, 4 doz. brushes @ $18.37, ½ gro. nailbrushes @ $27.50.

16. 480 yd. matting @ 18¢, 375 yd. matting @ 19¢, 284 yd. carpet @ 48¢, 8 rugs @ $7.33, 4 doz. door mats @ $4.75, 240 yd. border @ $1.12½.

17. 9 dining sets @ $62.50, 16 rockers @ $5.35, 8 sideboards at $32.50, 6 card tables @ $8.75, 4 china closets @ $17.50, 6 dining tables @ $9.25.

18. 325 yd. carpet @ 48¢, 520 yd. matting @ 22¢, 16 rugs at $6.40, 4 rugs at $12.50, 3 doz. door mats @ $5.30, 328 yd. calico @ 6¢, 825 yd. sheeting @ 24¢.

19. 36 yd. carpet @ $1.12½, 15 yd. border @ $1.25, 15½ yd. cotton @ 38¢, 22 yd. cloth @ 37½¢, 7½ yd. silk @ $1.40, 2½ doz. buttons @ 48¢, 12½ yd. calico @ 8¢.

155. Accounts. A statement of the amount owed by one person to another, with any payments that may have been made, is called an *account.*

The words "bill" and "account" are often used for either a bill or an account.

156. Specimen of an Account. The following is an account:

Elmira, N.Y., March 30, 1916

Mr. Robert J. Spencer, Owego, N.Y.

To **Henderson Bros., Dr.**

Wholesale Stationers, 4056 Water St.

			Dr.		Cr.	
Mar.	*7*	*50 gro. pens @ 78¢*	*$39*			
		15 doz. notebooks @ $1.70	*25*	*50*		
	9	*Cash*			*$40*	
	12	*4 doz. rulers @ 1.32*	*5*	*28*		
	13	*Cash*			*10*	
	20	*30 gro. lead pencils @ 3.35*	*100*	*50*		
	25	*Cash to balance*			*120*	*28*
			$170	*28*	*$170*	*28*

The difference between the sum of the items of indebtedness and those representing payments is the *balance.*

EXERCISE 91

Make out accounts for the following, inserting the place, date, and names, and completing each item:

1. Feb. 4, 12 lb. ham @ 21¢, 6½ lb. butter @ 33¢, 8 doz. eggs @ 45¢; Feb. 7, 8 lb. sugar @ 6¾¢, 15 lb. raisins @ 15¢, 8 lb. prunes @ 19¢, 6 lb. tapioca @ 9¢. Payments: Feb. 5, $3; Feb. 9, $2; balance on Feb. 13.

2. March 2, 10 doz. brooms @ $3.72, 8 doz. mops @ $7.90, 12 doz. mop handles @ $2.45, 9 doz. laundry boards @ $5.25; March 7, 15 doz. clotheslines @ $5.50, 6 doz. cedar pails @ $2.70; March 12, 10 doz. feather dusters @ $3.20. Payments: March 3, $75; March 8, $50; balance on March 15.

3. April 3, 6 doz. wool dusters @ $4.80, 5 doz. dustpans @ $3.15, 2 doz. rattan carpet beaters @ $2.65, 5 waxing brushes @ $2.32, 3 doz. shoe brushes @ $6.35; April 8, 5 doz. scrub brushes @ $1.60, 9 floor brushes @ $1.44; April 12, 10 doz. whisk brooms @ $2.88. Payments: April 5, $25; April 10, $15; balance on April 15.

4. May 1, 8 doz. quarts ammonia @ $2.56, 75 lb. borax @ 12¢, 8 boxes sal soda @ $1.08; May 5, 15 doz. cans brick dust @ 67¢, 25 doz. cans emery powder @ $1.67, 12 boxes scouring soap @ $3.07; May 12, 35 boxes laundry soap @ $3.88, 8 doz. packages soap powder @ $1.88. Payments: May 3, $20; May 6, $25; balance on May 14.

5. June 7, 8 doz. bottles mixed pickles @ $2.67, 9 doz. bottles gherkins @ $3.07, 35 doz. packages gelatin @ $1.48; June 12, 9 doz. pints chowchow @ $3.12, 6 doz. bottles horse-radish @ $0.98, 4 doz. bottles mustard sauce @ $1.95; June 16, 8 doz. bottles salad dressing @ $2.58, 75 doz. cans tomatoes @ $2.48, 6 doz. bottles catsup @ $2.44, 5 doz. bottles chutney sauce @ $1.02. Payments: June 8, $50; June 14, $30; balance on June 20.

6. July 9, 6 gro. toothbrushes @ $9.68, 4 gro. nailbrushes @ $24.35, 16 doz. combs @ $1.88; July 15, 7 doz. whisk brooms @ $2.94, 5 doz. hairbrushes @ $9.60, 4 doz. bath brushes @ $6.48; July 20, 4 doz. hand mirrors @ $7.20, 7 doz. tubes cold cream @ $1.98. Payments: July 12, $50; July 16, $20; balance on July 21.

157. Cash Accounts. In keeping a cash account it is customary to write each item of the receipts on the left side (called the *debit* side), and each item of the expenditures on the right side (called the *credit* side), thus:

	1915					1915					
a	Mar.	4	Cash on hand	$125	50	Mar.	5	Groceries	$17	22	*f*
b		6	P. F. Roberts	30			6	Meat bill	6	90	*g*
c		7	R. S. Ball	72	25		7	Balance	203	63	*h*
d				227	75				227	75	*i*
e	Mar.	7	Balance	203	63						

Here on the left side *a* shows the balance on hand when this page of the account is begun; *b* and *c* are receipts from Messrs. Roberts and Ball; *d* is the sum of these three items on the day that the account is added, or balanced. On the right side *f* and *g* are bills paid. To find how much is now on hand we subtract the sum of $17.22 and $6.90, or $24.12, from $227.75, and we find that the remainder (in cash accounts called the *balance*) is $203.63, which is written on the right at *h*, and again on the left at *e*. To check the work, *f*, *g*, and *h* are added, and the sums at *d* and *i* must agree.

EXERCISE 92

Make out cash accounts and balance them as above:

1. Receipts: May 1, cash on hand, $36.50; May 2, R. S. Jacobs, $20; May 4, M. R. Randall, $32.60. Expenses: May 3, groceries, $7.86; meat, $3.50. Balanced on May 5.

2. Receipts: April 3, cash on hand, $25.30; April 4, J. B. Fuller, $30; April 7, R. H. Bates, $4.30. Expenses: April 4, groceries, $9.32. Balanced on April 8.

3. Receipts: Feb. 6, cash on hand, $135.60; Feb. 6, R. S. Roberts, $75.50; Feb. 7, M. L. Richards, $87.90; Feb. 8, R. P. Jasper, $42.25. Expenses: Feb. 6, rent, $75; Feb. 7, salaries, $60; Feb. 8, coal, $15.30; Feb. 8, account book, $2.25; Feb. 8, gas bill, $3.75. Balanced on Feb. 9.

Insert dates and items as in Exs. 1–3, and balance the following accounts:

4. Receipts: $125, $32.60, $28.90, $50; expenses: $35, $15.30, $28.40.

5. Receipts: $16.30, $15.50, $10.10, $3.50; expenses: $4.80, $3.45, $20, $4.75.

6. Receipts: $10.30, $5.25, $6.32, $7.20; expenses: $5.30, $2.40, $3.80, $1.50.

7. Receipts: $34.30, $15.30, $10.75, $6.32; expenses: $32.50, $8.20, $4.30, $3.

8. Receipts: $42.60, $18.50, $13.60, $2.80, $4; expenses: $15.40, $15, $32.75, $4.60.

9. Receipts: $75.50, $62.30, $4.72, $16, $25.30; expenses: $3.25, $80, $14.20, $15, $3.25.

10. Receipts: $62.30, $15.40, $14.75, $21.32, $5; expenses: $35.75, $10.42, $29.60, $28.40.

11. Receipts: $1250.50, $620, $300, $175, $16.25, $120.50, $32.75, $68.50; expenses: $600, $360, $166.67, $33.33, $240.

12. Receipts: $625.75, $130.50, $200, $75, $50, $15.20, $14.60, $33.80; expenses: $430, $250, $100, $32.75, $28.50.

13. Receipts: $128.50, $32.75, $68.75, $43.50, $28.90; expenses: $62.30, $28.50, $75.50, $38.40, $25.25, $20.30.

14. Receipts: $875.80, $260.50, $350, $400, $80, $25, $380, $125; expenses: $250, $375, $480.70, $200, $75.

15. Receipts: $1025.30, $535.40, $287.50, $126.75, $25, $324.80, $125.45, $135.42, $12.87; expenses: $10, $225, $430.75, $223.50, $121.60, $123.75, $2.35, $6.27, $4.68.

16. Receipts: $287.60, $821.75, $327.42, $196.80, $427, $236.90, $481.73, $87.96, $52.80; expenses: $327.50, $427.39, $179.86, $321.42, $83.96, $75, $1.24, $3.50.

EXERCISE 93

PROBLEMS WITHOUT NUMBERS

1. If you know the cost per dozen of certain articles, how do you find the cost of a given number of dozen? of each article?

2. How do you make the extensions in a bill? foot the bill? receipt the bill?

3. If you know the items representing purchases, and those representing payments on an account, how do you balance the account?

4. If you know the amount of indebtedness on an account, and the balance, how will you find the sum of the payments?

5. How do you keep a cash account? How do you balance it?

6. If you know the cost of each of several things, how do you find the cost of all of them?

7. If you know the sum of two numbers and one of them, how do you find the other?

8. If you know the cost of one thing, how do you find the cost of several things of the same kind?

9. If you know the cost of several things of the same kind, how do you find the cost of one?

10. Name some things that are sold by the dry quart; by the peck; by the bushel.

11. Name some things that are weighed by the ounce; by the pound; by the ton.

12. If you know the cost of an ounce of anything, how do you find the cost per pound?

13. If you know the cost of a pound of anything, how do you find the cost per ounce?

CHAPTER IX

FACTORS, MEASURES, MULTIPLES

158. Factors. Integral numbers whose product is a given number are called *factors* of that number.

Thus, since $2 \times 3 \times 5 \times 7 = 210$, some of the factors of 210 are 2, 3, 5, and 7. We also see that 210 has other factors, like 6, 10, 14, and so on. Although every number has as factors itself and 1, we do not usually speak of these as factors.

159. Prime Number. A number that has no factors is called a *prime number.*

As stated in § 158, this means that the number itself and 1 are excluded. Thus 2, 3, 5, and 7 are prime numbers.

160. Prime Factor. A factor that is a prime number is called a *prime factor.*

Thus 2, 3, and 5 are the prime factors of 30. Although 6 is a factor of 30, it is not a prime factor.

161. Composite Number. A number that is not a prime number is called a *composite number.*

Thus 4, 15, 100, and 250 are composite numbers.

162. Exact Divisor. A number that divides another without a remainder is called an *exact divisor* of that number.

163. Exact Division. When we have an exact divisor, the case is said to be one of *exact division.*

Thus 15 is *exactly divisible* by 5. Usually we speak of an exact divisor simply as a *divisor* of a number. Then again we often speak of an exact divisor as a *measure* of a number, because with it we may exactly measure the number, just as with a yardstick we may measure the length of a carpet that is 10 yd. long.

164. Divisible Numbers. When we speak of one number as being *divisible* by another we mean exactly divisible, that is, divisible without a remainder.

165. Even Number. A number that is divisible by 2 is called an *even number.*

166. Odd Number. A number that is not divisible by 2 is called an *odd number.*

167. Divisibility by 2. *A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8.*

Thus 36 must be divisible by 2 if 6 is; because 30, or any other number of tens, is divisible by 2, ten being 2 × 5.

Thus we may tell without dividing that 1,023,578 is divisible by 2.

168. Divisibility by 3. *A number is divisible by 3 if the sum of the digits is divisible by 3.*

We often use the word *digits* to mean the *value* of the symbols 0, 1, 2, and so on to 9, as well as the characters from 1 to 9.

Thus 54 is divisible by 3 since $5 + 4$ is divisible by 3.

Likewise, 534 is divisible by 3 because $5 + 3 + 4$, or 12, is divisible by 3.

169. Divisibility by 5. *A number is divisible by 5 if it ends in 0 or 5.*

For if a number ends in 0, it is tens, and 10 is divisible by 5. If it ends in 5, it is tens plus 5, and both are divisible by 5.

There is no easy test for divisibility by 7.

170. Finding Prime Factors. Find the prime factors of 420.

By § 167, 2 is a factor of 420 and of 210; by § 168, 3 is a factor of 105; by § 169, 5 is a factor of 35, the other factor being 7. Hence the prime factors of 420 are 2, 2, 3, 5, 7.

$$\begin{array}{r} 2\underline{)420} \\ 2\underline{)210} \\ 3\underline{)105} \\ 5\underline{)35} \\ 7 \end{array}$$

That is, *divide as often as possible by the smallest prime factor, and then by the next larger, and so on until a prime quotient is reached. The several divisors and the last quotient are the prime factors.*

171. Proving a Number Prime. Required to find if 239 is a prime number.

The last digit is not 0, 2, 4, 6, or 8, so 239 cannot contain the factor 2, and therefore cannot contain the factor 4, 6, 8, or any other multiple of 2. The sum of the digits does not contain the factor 3, and therefore 239 cannot contain the factor 3 or any multiple of 3. The last digit is not 0 or 5, and therefore 239 cannot contain the factor 5 or any multiple of 5. Therefore 2, 3, 4, 5, 6, 8, 9, 10, 12, 14, 15, 16, cannot be factors of 239. By trial we find that 7, 11, 13, 17, are not factors. We need not try any prime number higher than 17, as the quotient when 17 is tried is *less* than 17, and we have tried all prime factors below 17. Hence 239 is prime.

Try the various prime numbers, beginning with 2, to see if they are factors. If no prime factor is found before the quotient becomes less than the divisor, the number is prime.

EXERCISE 94

1. Select the even numbers: 432, 640, 287, 1024, 3036, 8292, 4427, 8329, 4878.

2. Select the numbers divisible by 3: 424, 636, 292, 444, 1002, 3027, 1104, 8822, 6777.

3. Select the numbers divisible by 5: 830, 272, 637, 4225, 3010, 7200, 6305, 9270, 4005.

4. Select the prime numbers: 53, 61, 97, 111, 121, 143, 187, 1005, 1001, 1203, 2112, 1331, 1337.

Find the prime factors of the following:

5. 120.
6. 210.
7. 150.
8. 220.
9 170.
10. 175.
11. 225.
12. 303.
13. 750.
14. 666.
15. 1001.
16. 1245.
17. 1331.
18. 3645.
19. 3432.
20. 1728.
21. 5280.
22. 6006.
23. 3737.
24. 9768.

172. Common Measure. A number that exactly divides each of two or more numbers is called a *common measure* of those numbers.

Thus 7 is a common measure of 28 and 42, since it exactly divides each of them, as a foot length exactly divides a 10-foot length and a 12-foot length that we are measuring. A common measure is also called a *common divisor*, or a *common factor*.

173. Greatest Common Measure. The greatest number that exactly divides each of two or more numbers is called the *greatest common measure* of those numbers.

Thus, while 7 is a common measure of 28 and 42, 14 is the *greatest* common measure. It is also called the *greatest common divisor*. The letters G.C.M. stand for the words Greatest Common Measure, and G.C.D. for Greatest Common Divisor.

174. Numbers Prime to Each Other. If two integers have no common measure, always excepting 1, they are said to be *prime to each other*.

Thus, 25 and 33 are prime to each other, although each has factors.

175. Finding the G.C.M. The greatest common measure of such numbers as we shall use in studying fractions is found easily by factoring.

For example, required the G.C.M. of 24, 30, and 36.

Finding the prime factors of each, we have the following:

$$\begin{array}{r} 2\underline{)24} \\ 2\underline{)12} \\ 2\underline{)6} \\ 3 \end{array} \qquad\qquad \begin{array}{r} 2\underline{)30} \\ 3\underline{)15} \\ 5 \end{array} \qquad\qquad \begin{array}{r} 2\underline{)36} \\ 2\underline{)18} \\ 3\underline{)9} \\ 3 \end{array}$$

The factor 2 occurs once in each of the numbers. The factor 3 occurs once in each of the numbers. No other factor occurs in *each one* of the numbers. Therefore the G.C.M. is 2 × 3, or 6.

This solution may be expressed as follows:

$$24 = 2 \times 2 \times 2 \times 3;\ \ 30 = 2 \times 3 \times 5;\ \ 36 = 2 \times 2 \times 3 \times 3.$$

Therefore the G.C.M. is 2 × 3, or 6.

Required the G.C.M. of 54, 81, and 108.

Now
$$\begin{aligned} 54 &= 2 \times 3 \times 3 \times 3; \\ 81 &= 3 \times 3 \times 3 \times 3; \\ 108 &= 2 \times 2 \times 3 \times 3 \times 3. \end{aligned}$$

Here 3 occurs three times as a factor in each number, and there is no other common factor. Hence the G.C.M. is $3 \times 3 \times 3$, or 27.

Hence, to find the G.C.M. of two or more numbers,

Separate the numbers into prime factors and find the product of the prime factors common to the numbers.

If one of the given numbers is a multiple of one of the others, it may be canceled, since it must contain all the factors of that other. Thus the G.C.M. of 48, 54, and 108 is the same as that of 48 and 54.

EXERCISE 95

Find the G.C.M. of the following:

1. 2, 8.
2. 4, 16.
3. 9, 33.
4. 12, 78.
5. 14, 49.
6. 15, 75.
7. 16, 56.
8. 17, 51.
9. 18, 81.
10. 19, 171.
11. 20, 175.
12. 21, 105.
13. 22, 121.
14. 23, 207.
15. 24, 246.
16. 6, 8, 42.
17. 9, 12, 33.
18. 8, 28, 84.
19. 9, 33, 81.
20. 10, 35, 85.
21. 12, 48, 68.
22. 14, 21, 77.
23. 16, 72, 96.
24. 18, 72, 81.
25. 20, 80, 95.
26. 24, 60, 96.
27. 30, 90, 96.
28. 32, 48, 96.
29. 36, 54, 90.
30. 39, 52, 91.
31. 63, 84, 126.
32. 33, 77, 121.
33. 30, 60, 250.
34. 32, 48, 128.
35. 51, 63, 132.
36. 36, 84, 132.
37. 60, 144, 576.
38. 38, 190, 760.
39. 32, 160, 808.
40. 33, 132, 143.
41. 50, 125, 250.
42. 64, 128, 256.
43. 64, 160, 640.
44. 63, 126, 333.
45. 75, 125, 375.

176. Finding the G.C.M. of Large Numbers. It is not often necessary to find the G.C.M. of large numbers that cannot be separated readily into factors. An example will illustrate a process often called the long-division method.

This method depends upon the principle that a common factor of two numbers is also a factor of their sum and of their difference.

Required the G.C.M. of 323 and 357.

```
323)357(1
    323
    ---
     34)323(9
        306
        ---
         17)34(2
            34
            --
```

Here we cannot easily factor the two numbers. We divide the larger number by the smaller to see if the smaller is the G.C.M. It is not because there is a remainder 34. We then divide the smaller number by this remainder and find there is a remainder 17. Hence 34 is not the G.C.M. Then we divide the last divisor 34 by the last remainder 17 and find that 17 is an exact divisor of 34. Then 17 is the G.C.M. required.

EXERCISE 96

Find the G.C.M. of the following:

1. 195 and 260.
2. 294 and 350.
3. 333 and 592.
4. 410 and 697.
5. 720 and 1728.
6. 1875 and 2500.
7. 1220 and 2013.
8. 1010 and 2121.
9. 2813 and 3973.
10. 2002 and 3010.
11. 7099 and 9389.
12. 2220 and 3293.
13. 2650 and 8003.
14. 1140 and 1767.
15. 3621 and 7100.
16. 3650 and 3796.

17. What is the largest number that will exactly divide 2378 and 3915?

18. What is the largest number that will exactly divide 551, 667, and 841?

177. Multiple. The product of two or more integers is called a *multiple* of each of them.

Thus 20 is a multiple of 2, 4, 5, and 10, because 2 × 2 × 5 = 4 × 5 = 2 × 10 = 20. This has already been defined (§ 36). We speak of 75, for example, as a multiple of 25, and of 51 as a multiple of 17, or we speak of each as a multiple of any other of its factors.

178. Common Multiple. A multiple of two or more numbers is called a *common multiple* of the numbers.

Thus 60 is a common multiple of 6 and 10, and so are 30, 90, 120, and so on, common multiples of 6 and 10.

179. Least Common Multiple. The smallest common multiple of two or more numbers is called their *least common multiple.* It is the least number that is exactly divisible by each of them.

Thus 60 is a common multiple of 6 and 10, but the *least* common multiple is 30.

The letters L.C.M. are used for the words Least Common Multiple.

Since the L.C.M. must contain each of the numbers, it must contain all of the prime factors of each, and this is the principle upon which the finding of the L.C.M. rests.

180. Finding the L.C.M. Required the L.C.M. of 24, 30, 36.

Resolving each number into its prime factors, we have

2)24	2)30	2)36
2)12	3)15	2)18
2)6	5	3)9
3		3

The L.C.M. must contain the factors of 24, which are 2, 2, 2, and 3. These are therefore taken as part of its factors.

It must contain the factors of 30, which are 2, 3, and 5; but we have already taken 2 and 3, so we need take only 5.

It must contain the factors of 36, which are 2, 2, 3, and 3; but we have already taken 2, 2, and 3, so we need take only one more 3.

Hence the factors of the L.C.M. are 2, 2, 2, 3, 5, 3. Their product is 360, and 360 is the L.C.M.

It is often convenient to adopt another arrangement of the work, as in finding the L.C.M. of 12, 18, 30, 45.

Here we arrange the numbers in a horizontal line, and divide by the smallest prime factor common to *two* or more of them.

$$\begin{array}{r|rrrr} 2 & 12 & 18 & 30 & 45 \\ \hline 3 & 6 & \cancel{9} & \cancel{15} & 45 \\ \hline & 2 & & & 15 \end{array}$$

We first divide by 2, and write the quotients and *the undivided numbers* as well in a line below. In the first line of quotients we cancel the 9 and the 15 because each is a factor of 45, and therefore 45 contains all the factors of 9 and of 15. We next divide by 3, and in the last line of quotients we have the numbers 2 and 15, which are prime to each other. Hence the L.C.M. is 2 × 3 × 2 × 15, or 180.

Required the L.C.M. of 5, 15, 30, 60.

$$\cancel{5} \quad \cancel{15} \quad \cancel{30} \quad 60$$

Here we cancel the 5 because it is a factor of 15. We cancel the 15 because it is a factor of 30, and cancel the 30 because it is a factor of 60. Therefore 60 contains all the factors of the other numbers and is itself the L.C.M.

181. General Directions for Finding the L.C.M. From the above examples the following directions may be observed:

Separate each number into its prime factors in any convenient manner. Find the product of these factors, using each factor the greatest number of times it occurs in any one of the given numbers.

EXERCISE 97

Find the L.C.M. of the following:

1. 22, 33.
2. 17, 51.
3. 16, 24.
4. 15, 35.
5. 27, 39.
6. 33, 55.
7. 3, 21, 35.
8. 4, 12, 38.
9. 6, 15, 39.
10. 8, 20, 48.
11. 9, 42, 49.
12. 12, 24, 144.
13. 6, 9, 12, 15.
14. 7, 14, 21, 35.
15. 8, 28, 36, 52.
16. 9, 21, 39, 93.
17. 10, 15, 25, 50.
18. 16, 24, 64, 96.

19. 6, 9, 12, 33.

20. 9, 27, 36, 63.

21. 20, 24, 30, 40.

22. 24, 36, 48, 60.

23. 11, 22, 33, 66.

24. 14, 28, 77, 91.

25. 20, 28, 56, 80.

26. 30, 40, 50, 60.

27. 32, 40, 48, 80.

28. 45, 63, 72, 99.

29. 4, 8, 12, 16, 32.

30. 5, 10, 20, 35, 45.

31. 6, 18, 30, 42, 66.

32. 7, 21, 63, 70, 84.

33. 8, 32, 64, 72, 96.

34. 9, 27, 54, 63, 90.

35. 10, 11, 12, 22, 33.

36. 11, 15, 33, 45, 55.

37. 12, 16, 24, 36, 72.

38. 13, 26, 52, 78, 104.

39. Find the least number of marbles that can be divided equally among 15, 20, or 25 boys.

40. Find the shortest distance that can be exactly measured by a 3-foot rule, or a 5-foot pole, or a 10-foot pole.

41. What is the smallest quantity of liquid that can be measured exactly by 2-quart, 4-quart, 6-quart, or 8-quart measures?

42. What is the smallest sum of money that can be counted out in 5-cent pieces, 10-cent pieces, 25-cent pieces, or 50-cent pieces?

43. What is the capacity of the smallest water tank that can be filled in an integral number of minutes by either of two pipes, if one discharges 32 gallons a minute and the other 48 gallons a minute?

44. Find the least number of oranges that, when arranged in groups of 6, 7, 8, or 9, has just 5 over in each case. (Find the L.C.M. and add 5. Test the result.)

45. Find the least number of marbles that, when arranged in groups of 8, 12, 15, 60, or 120, has just 7 over in each case.

182. Cancellation. As in § 97, we may apply our knowledge of divisibility of numbers, and shorten the work by canceling equal factors from both dividend and divisor.

For example, divide 11 × 27 × 30 by 3 × 9 × 15.

The common factor 3 is canceled from the 3 in the divisor and the 27 in the dividend. The common factor 9 is canceled from the 9 in the divisor and the 9 that remains uncanceled in the 27 of the dividend. The common factor 15 is canceled from the divisor and the 30 in the dividend. There remains for a dividend 11 × 1 × 2, and for a divisor 1 × 1 × 1. Hence the quotient is 22. This is much shorter than finding two products, 8910 and 405, and then the quotient 22.

$$\frac{11 \times \overset{\cancel{9}}{\cancel{27}} \times \overset{2}{\cancel{30}}}{\cancel{3} \times \cancel{9} \times \cancel{15}} = \frac{11 \times 1 \times 2}{1 \times 1 \times 1} = 22.$$

EXERCISE 98

Find the quotient of:

1. $\dfrac{6 \times 8 \times 99}{3 \times 4 \times 11}$.
2. $\dfrac{8 \times 12 \times 51}{4 \times 9 \times 17}$.
3. $\dfrac{9 \times 30 \times 38}{6 \times 10 \times 19}$.
4. $\dfrac{12 \times 21 \times 57}{9 \times 14 \times 19}$.
5. $\dfrac{13 \times 20 \times 91}{13 \times 13 \times 14}$.
6. $\dfrac{12 \times 32 \times 78}{9 \times 16 \times 26}$.
7. $\dfrac{15 \times 25 \times 64}{10 \times 80 \times 10}$.
8. $\dfrac{18 \times 27 \times 96}{12 \times 36 \times 54}$.
9. $\dfrac{4 \times 5 \times 6 \times 8}{2 \times 3 \times 4 \times 5}$.
10. $\dfrac{8 \times 16 \times 28 \times 92}{2 \times 32 \times 23 \times 14}$.
11. $\dfrac{36 \times 19 \times 29 \times 39}{3 \times 38 \times 58 \times 13}$.
12. $\dfrac{27 \times 35 \times 42 \times 72}{5 \times 9 \times 14 \times 18}$.
13. $\dfrac{34 \times 35 \times 36 \times 37}{5 \times 17 \times 9 \times 74}$.
14. $\dfrac{24 \times 34 \times 44 \times 54}{3 \times 8 \times 17 \times 33}$.
15. $\dfrac{15 \times 18 \times 48 \times 75}{25 \times 30 \times 36 \times 24}$.
16. $\dfrac{24 \times 63 \times 82 \times 91}{41 \times 13 \times 21 \times 12}$.

17. What is the quotient of 35 times the product of 48 and 119, divided by 14 times the product of 21 and 17?

18. If a dividend is the product of 68, 91, and 95, and the divisor is the product of all the prime numbers between 12 and 20, what is the quotient?

19. If we divide the product of 16, twice 16, and three times 16, by the product of the square of 2 taken three times as a factor, what is the quotient?

20. How many times is the product of all the prime numbers between 20 and 30 contained in the product of 203 and 230?

21. A man worked 4 weeks, 6 days a week, for a market gardener, and received for his pay 24 crates of berries valued at $2 a crate. How much did he earn a day?

22. How many dozen quart bottles of imported pickles at $6 a dozen must be given in exchange for 3 cases of olives, each containing 2 dozen bottles at $3 a dozen?

23. A grocer exchanged 16 boxes of soap, each containing 60 cakes at 4 cents a cake, for a certain number of cases of preserved fruit, 2 dozen jars to a case at 40 cents a jar. How many cases did he get?

24. A man exchanged 9 firkins of butter weighing 56 pounds each at 30 cents a pound for several rolls of matting, 40 yards to the roll worth 42 cents a yard. How many rolls did he get?

25. If 24 men, working 9 hours a day, can do a piece of work in 12 days, how many days will it take 18 men, working 8 hours a day, to do the work?

26. If 15 men can do a piece of work in 16 days, working 9 hours a day, how many days will it take 20 men working 12 hours a day to do the work?

27. A clerk working 48 weeks in a store, at $12 a week, receives the same amount that another clerk receives in 36 weeks. What is the weekly wages of the other clerk?

28. A dealer sells a shipment of 36 horses for $2700. He sells another shipment of 25 horses for the same amount. What is the average price per horse in each of the two shipments?

29. We find the volume of a rectangular box by taking the product of its three dimensions expressed in the same unit of measure. The dimensions of one box, expressed as abstract numbers, are 4, 5, and 6, and two dimensions of another box of the same volume are 3 and 10. What is the other dimension of the second box?

30. Expressed as abstract numbers, the dimensions of one box are 8, 12, and 15, and two dimensions of another box of the same volume are 6 and 30. What is the other dimension of the second box?

31. Expressing the dimensions as abstract numbers, a cellar is 20 long, 14 wide, and 6 deep. How deep must another cellar be that is 16 long and 15 wide, in order that the capacity of both cellars may be the same?

32. How many times as much earth must be removed from a cellar excavation 175 feet long, 80 feet wide, and 15 feet deep, as from a cellar excavation 150 feet long, 40 feet wide, and 5 feet deep?

33. How many times as much earth must be removed from a cellar excavation 160 feet long, 40 feet wide, and 9 feet deep, as from a cellar excavation 60 feet long, 40 feet wide, and 6 feet deep?

34. How many times as much will a box contain whose dimensions are 8, 27, and 35, as one whose dimensions are 7, 12, and 18?

CHAPTER X

COMMON FRACTIONS

183. Fraction. A number that shows what part or what number of parts of a unit is taken is called a *fraction.*

Fractions have already been studied in the Fourth Year. They are now reviewed and are studied more carefully.

184. Common Fraction. A fraction expressed by two numbers, one under the other with a line between them, is called a *common fraction.*

185. Terms of a Fraction. In a common fraction the number above the line is called the *numerator;* the number below the line is called the *denominator;* the two together are called the *terms* of the fraction.

In the fraction $\frac{3}{4}$ the numerator is 3 and the denominator is 4. The denominator (namer) shows into how many equal parts the unit has been divided, and the numerator (numberer) shows how many of these equal parts have been taken.

186. Unit Fraction. A fraction whose numerator is one is called a *unit fraction.*

Thus $\frac{1}{4}$ is a unit fraction. It is also called a *fractional unit.*

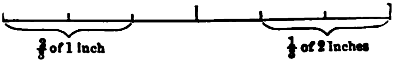
187. Proper Fraction. A fraction whose numerator is less than the denominator is called a *proper fraction.*

For example, $\frac{3}{4}$ is a proper fraction.

188. Improper Fraction. A fraction whose numerator is not less than the denominator is called an *improper fraction.*

For example, $\frac{4}{4}$ and $\frac{5}{3}$ are improper fractions.

189. Fraction as an Expressed Division. In this figure we see that $\frac{2}{3}$ of 1 inch equals $\frac{1}{3}$ of 2 inches, or the result of dividing 2 inches by 3; that is, $\frac{2}{3}$ is the same as $\frac{1}{3}$ of 2, or 2 divided by 3. Therefore,



A fraction is an expressed division, the numerator being the dividend and the denominator the divisor.

190. Reduction of a Fraction to Lowest Terms. Since we may divide both terms of a fraction by the same number without changing the value of the fraction, we may continue this division until the terms are prime to each other.

If the terms of a fraction are prime to each other, the fraction is expressed in *lowest terms.*

For example, $\frac{14}{17}$ is expressed in lowest terms, but $\frac{8}{12}$ is not.

Therefore, *to reduce a fraction to lowest terms, cancel all the factors common to both numerator and denominator.*

For example, reduce $\frac{24}{30}$ to lowest terms.

Canceling 2, we have $\frac{12}{15}$; canceling 3, we have $\frac{4}{5}$.

EXERCISE 99

Reduce to lowest terms:

1. $\frac{24}{96}$.
2. $\frac{36}{84}$.
3. $\frac{72}{88}$.
4. $\frac{48}{84}$.
5. $\frac{54}{99}$.
6. $\frac{23}{69}$.
7. $\frac{21}{63}$.
8. $\frac{12}{36}$.
9. $\frac{32}{48}$.
10. $\frac{18}{60}$.
11. $\frac{17}{51}$.
12. $\frac{19}{57}$.
13. $\frac{23}{92}$.
14. $\frac{42}{56}$.
15. $\frac{35}{63}$.
16. $\frac{48}{64}$.
17. $\frac{36}{144}$.
18. $\frac{48}{144}$.
19. $\frac{64}{156}$.
20. $\frac{72}{156}$.
21. $\frac{84}{168}$.
22. $\frac{64}{192}$.
23. $\frac{64}{160}$.
24. $\frac{33}{144}$.
25. $\frac{96}{144}$.
26. $\frac{72}{256}$.
27. $\frac{64}{448}$.
28. $\frac{78}{273}$.
29. $\frac{111}{120}$.
30. $\frac{121}{132}$.
31. $\frac{132}{144}$.
32. $\frac{144}{156}$.

191. Reduction of a Fraction to Higher Terms. By multiplying both terms of a fraction by the same number we may reduce a fraction to *higher terms.*

For example, reduce $\frac{11}{12}$ to sixtieths.

Here 60 ÷ 12 = 5, so that the denominator must be multiplied by 5 to make 60. Therefore we multiply both terms by 5, and $\frac{11}{12} = \frac{55}{60}$.

Therefore, *to reduce a fraction to a fraction having a higher denominator, divide the required denominator by the given denominator and multiply both terms of the fraction by the quotient.*

192. Reduction of an Integer to an Improper Fraction. An integer may be expressed as an improper fraction with any required denominator.

Reduce 3 to an improper fraction with denominator 7.

We may simply think of 3 as equal to $\frac{3}{1}$. We have, then, the same case as in § 191. Multiplying both terms by 7, we have $\frac{3}{1} = \frac{21}{7}$.

Therefore, *to reduce an integer to a fraction with a given denominator, multiply it by the given denominator and under this product write the denominator.*

EXERCISE 100

Reduce as indicated:

To 60ths:	1. $\frac{1}{2}$.	2. $\frac{8}{15}$.	3. $\frac{9}{20}$.	4. $\frac{11}{30}$.
To 80ths:	5. $\frac{1}{2}$.	6. $\frac{5}{16}$.	7. $\frac{7}{20}$.	8. $\frac{9}{40}$.
To 144ths:	9. $\frac{1}{2}$.	10. $\frac{1}{12}$.	11. $\frac{5}{24}$.	12. $\frac{7}{72}$.
To 156ths:	13. $\frac{1}{2}$.	14. $\frac{2}{39}$.	15. $\frac{5}{78}$.	16. $\frac{7}{78}$.
To 256ths:	17. $\frac{1}{2}$.	18. $\frac{3}{16}$.	19. $\frac{5}{32}$.	20. $\frac{7}{64}$.
To 288ths:	21. $\frac{1}{2}$.	22. $\frac{1}{12}$.	23. $\frac{5}{24}$.	24. $\frac{7}{48}$.
To 320ths:	25. $\frac{1}{2}$.	26. $\frac{3}{16}$.	27. $\frac{5}{32}$.	28. $\frac{9}{64}$.
To 100ths:	29. $\frac{1}{2}$.	30. $\frac{3}{10}$.	31. $\frac{4}{25}$.	32. $\frac{9}{50}$.

193. Mixed Number. A number composed of an integer and a fraction taken together is called a *mixed number.*

Thus $2\frac{3}{4}$ is a mixed number. It is read "two and three fourths."

194. Reduction of an Improper Fraction. An improper fraction may always be reduced to an integer or a mixed number.

For example, required to reduce $\frac{72}{12}$ to an integer or a mixed number.

Dividing both terms by 12, we have $\frac{6}{1}$, which equals 6.

Required to reduce $\frac{36}{16}$ to an integer or a mixed number.

Since $\frac{36}{16}$ is an expressed division, we divide 36 by 16 and the quotient is $2\frac{4}{16}$, or $2\frac{1}{4}$.

Therefore, *to reduce an improper fraction to an integer or a mixed number, divide the numerator by the denominator.*

EXERCISE 101

Reduce to integers or mixed numbers:

1. $\frac{36}{12}$.
2. $\frac{38}{12}$.
3. $\frac{40}{12}$.
4. $\frac{60}{12}$.
5. $\frac{72}{12}$.
6. $\frac{36}{16}$.
7. $\frac{40}{16}$.
8. $\frac{72}{16}$.
9. $\frac{80}{16}$.
10. $\frac{96}{16}$.
11. $\frac{36}{24}$.
12. $\frac{72}{24}$.
13. $\frac{96}{24}$.
14. $\frac{72}{18}$.
15. $\frac{96}{32}$.
16. $\frac{96}{48}$.
17. $\frac{72}{48}$.
18. $\frac{56}{48}$.
19. $\frac{84}{48}$.
20. $\frac{240}{48}$.
21. $\frac{60}{50}$.
22. $\frac{75}{50}$.
23. $\frac{90}{50}$.
24. $\frac{95}{50}$.
25. $\frac{350}{50}$.
26. $\frac{84}{64}$.
27. $\frac{72}{64}$.
28. $\frac{80}{64}$.
29. $\frac{88}{64}$.
30. $\frac{448}{64}$.
31. $\frac{80}{72}$.
32. $\frac{84}{72}$.
33. $\frac{96}{72}$.
34. $\frac{144}{72}$.
35. $\frac{288}{72}$.
36. $\frac{192}{96}$.
37. $\frac{198}{96}$.
38. $\frac{288}{96}$.
39. $\frac{480}{96}$.
40. $\frac{576}{96}$.
41. $\frac{250}{100}$.
42. $\frac{375}{100}$.
43. $\frac{425}{100}$.
44. $\frac{550}{100}$.
45. $\frac{750}{100}$.

46. If the numerator of a fraction is equal to the denominator, to what integer is the fraction equal?

47. If the numerator of a fraction is $2\frac{1}{2}$ times the denominator, to what mixed number is the fraction equal? To what integer is it equal if the denominator is $\frac{1}{3}$ of the numerator?

195. Reduction of a Mixed Number. A mixed number may always be reduced to an improper fraction.

For example, required to reduce $2\frac{3}{4}$ to an improper fraction.

Since 1 equals 4 fourths ($\frac{4}{4}$), 2 equals 2 × 4 fourths, or 8 fourths ($\frac{8}{4}$); and 8 fourths and 3 fourths are 11 fourths ($\frac{11}{4}$).

Therefore, *to reduce a mixed number to an improper fraction, multiply the denominator by the integral part of the mixed number, and to the product add the numerator; under this sum write the denominator.*

EXERCISE 102

Reduce to improper fractions:

1. $3\frac{7}{8}$.
2. $4\frac{7}{8}$.
3. $7\frac{5}{8}$.
4. $6\frac{7}{8}$.
5. $9\frac{5}{8}$.
6. $7\frac{3}{16}$.
7. $5\frac{5}{16}$.
8. $8\frac{9}{16}$.
9. $9\frac{7}{16}$.
10. $6\frac{11}{16}$.
11. $2\frac{5}{24}$.
12. $5\frac{7}{24}$.
13. $8\frac{11}{24}$.
14. $2\frac{13}{24}$.
15. $6\frac{17}{24}$.
16. $7\frac{3}{32}$.
17. $9\frac{5}{32}$.
18. $3\frac{7}{32}$.
19. $6\frac{9}{32}$.
20. $2\frac{11}{32}$.
21. $8\frac{5}{36}$.
22. $6\frac{7}{36}$.
23. $4\frac{11}{36}$.
24. $7\frac{13}{36}$.
25. $2\frac{17}{36}$.
26. $5\frac{5}{48}$.
27. $8\frac{7}{48}$.
28. $2\frac{11}{48}$.
29. $6\frac{17}{48}$.
30. $1\frac{23}{48}$.
31. $1\frac{7}{64}$.
32. $5\frac{11}{64}$.
33. $7\frac{17}{64}$.
34. $3\frac{29}{64}$.
35. $6\frac{37}{64}$.
36. $1\frac{5}{72}$.
37. $6\frac{7}{72}$.
38. $5\frac{23}{72}$.
39. $35\frac{35}{72}$.
40. $72\frac{41}{72}$.

41. How many eighths ($\frac{1}{8}$) are there in $\frac{7}{8}$? in 3? in $3\frac{7}{8}$?

42. How many sixteenths ($\frac{1}{16}$) are there in $\frac{5}{16}$? in 5? in $5\frac{5}{16}$?

43. How many thirty-seconds ($\frac{1}{32}$) are there in $\frac{7}{32}$? in 3? in $3\frac{7}{32}$?

44. How many tins, $\frac{1}{8}$ of a pound each, can be filled from $\frac{7}{8}$ of a pound of pepper? from 2 pounds? from $2\frac{7}{8}$ pounds?

196. Common Denominator. If two or more fractions have the same denominator they are said to have a *common denominator.*

Thus $\frac{3}{8}$ and $\frac{5}{8}$ have the common denominator 8. The letters L.C.D. stand for the words Least Common Denominator.

197. Similar Fractions. Fractions that have a common denominator are called *similar fractions.* It is always possible to reduce fractions to similar fractions.

For example, required to reduce $\frac{2}{3}$ and $\frac{3}{4}$ to similar fractions.

The L.C.M. of the denominators is 12. Reducing $\frac{2}{3}$ to twelfths (§ 191) we have $\frac{8}{12}$, and reducing $\frac{3}{4}$ to twelfths we have $\frac{9}{12}$.

The L.C.D. of several fractions is the L.C.M. of the given denominators.

Hence, *to reduce fractions to similar fractions, reduce them to fractions whose common denominator is the L.C.M. of the given denominators.*

EXERCISE 103

Reduce to similar fractions, using the L.C.D.:

1. $\frac{3}{8}, \frac{5}{12}$.
2. $\frac{5}{6}, \frac{7}{8}$.
3. $\frac{5}{8}, \frac{3}{32}$.
4. $\frac{7}{8}, \frac{4}{5}$.
5. $\frac{5}{12}, \frac{7}{16}$.
6. $\frac{7}{12}, \frac{9}{16}$.
7. $\frac{11}{12}, \frac{15}{16}$.
8. $\frac{15}{16}, \frac{19}{24}$.
9. $\frac{13}{16}, \frac{29}{32}$.
10. $\frac{13}{24}, \frac{5}{32}$.
11. $\frac{7}{32}, \frac{11}{48}$.
12. $\frac{15}{32}, \frac{29}{48}$.
13. $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}$.
14. $\frac{1}{3}, \frac{1}{4}, \frac{5}{8}$.
15. $\frac{2}{3}, \frac{1}{4}, \frac{3}{8}$.
16. $\frac{2}{3}, \frac{3}{4}, \frac{5}{8}$.
17. $\frac{1}{5}, \frac{1}{8}, \frac{1}{12}$.
18. $\frac{2}{5}, \frac{3}{8}, \frac{5}{12}$.
19. $\frac{3}{5}, \frac{7}{8}, \frac{7}{12}$.
20. $\frac{1}{8}, \frac{1}{10}, \frac{1}{12}$.
21. $\frac{3}{8}, \frac{7}{10}, \frac{7}{12}$.
22. $\frac{3}{8}; \frac{3}{10}, \frac{5}{12}$.
23. $\frac{5}{8}, \frac{5}{16}, \frac{7}{24}$.
24. $\frac{7}{8}, \frac{7}{16}, \frac{7}{24}$.
25. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}$.
26. $\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}$.
27. $\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{1}{8}$.
28. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{3}{8}$.
29. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{7}{8}$.
30. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{3}{16}$.
31. $\frac{3}{4}, \frac{7}{8}, \frac{5}{16}, \frac{3}{32}$.
32. $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{24}$.
33. $\frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{5}{24}$.
34. $\frac{5}{8}, \frac{1}{16}, \frac{3}{32}, \frac{1}{64}$.
35. $\frac{7}{8}, \frac{5}{16}: \frac{5}{32}, \frac{7}{64}$.
36. $\frac{7}{8}, \frac{1}{24}, \frac{1}{32}, \frac{5}{64}$.

198. Addition of Fractions. If we add 3 marbles and 5 pencils, we have as a sum neither marbles nor pencils, but just 8 things, of two different kinds. So if we add $\frac{3}{4}$ and $\frac{5}{6}$ we shall have as a sum neither fourths nor sixths, but just 8 parts of a unit, of two different sizes.

Therefore, to add fractions that are not similar and obtain a definite sum, we must reduce them to similar fractions.

Required the sum of $\frac{3}{4}$ and $\frac{5}{6}$.

Here the L.C.M. of 4 and 6 is 12. Therefore the L.C.D. is 12.

Then $\frac{3}{4} = \frac{9}{12}$,

and $\frac{5}{6} = \frac{10}{12}$.

Now just as \$9 + \$10 = \$19, 9 twelfths + 10 twelfths = 19 twelfths.

That is, $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12} = 1\frac{7}{12}$.

Therefore, *to add fractions, reduce them to similar fractions having the least common denominator, if they are not similar. Write the sum of the numerators over the least common denominator, and reduce the result to lowest terms.*

EXERCISE 104

Find the sum of:

1. $\frac{1}{2}$, $\frac{1}{4}$.
2. $\frac{1}{2}$, $\frac{3}{4}$.
3. $\frac{1}{2}$, $\frac{1}{8}$.
4. $\frac{1}{2}$, $\frac{5}{8}$.
5. $\frac{1}{2}$, $\frac{7}{8}$.
6. $\frac{1}{4}$, $\frac{1}{8}$.
7. $\frac{1}{4}$, $\frac{3}{8}$.
8. $\frac{1}{4}$, $\frac{5}{8}$.
9. $\frac{1}{4}$, $\frac{7}{8}$.
10. $\frac{3}{4}$, $\frac{3}{8}$.
11. $\frac{3}{4}$, $\frac{5}{8}$.
12. $\frac{3}{4}$, $\frac{7}{8}$.
13. $\frac{1}{4}$, $\frac{1}{16}$.
14. $\frac{3}{4}$, $\frac{7}{16}$.
15. $\frac{3}{8}$, $\frac{5}{16}$.
16. $\frac{7}{8}$, $\frac{11}{16}$.
17. $\frac{1}{2}$, $\frac{3}{32}$.
18. $\frac{3}{4}$, $\frac{3}{32}$.
19. $\frac{5}{8}$, $\frac{7}{32}$.
20. $\frac{7}{8}$, $\frac{13}{32}$.
21. $\frac{1}{4}$, $\frac{5}{24}$.
22. $\frac{3}{4}$, $\frac{11}{24}$.
23. $\frac{3}{8}$, $\frac{7}{24}$.
24. $\frac{5}{8}$, $\frac{11}{24}$.
25. $\frac{3}{16}$, $\frac{1}{24}$.
26. $\frac{5}{16}$, $\frac{5}{24}$.
27. $\frac{7}{16}$, $\frac{11}{24}$.
28. $\frac{9}{16}$, $\frac{13}{24}$.
29. $\frac{1}{2}$, $\frac{1}{48}$.
30. $\frac{3}{4}$, $\frac{1}{48}$.
31. $\frac{3}{4}$, $\frac{15}{28}$.
32. $\frac{3}{8}$, $\frac{13}{48}$.
33. $\frac{5}{8}$, $\frac{17}{48}$.
34. $\frac{7}{8}$, $\frac{23}{48}$.
35. $\frac{1}{16}$, $\frac{17}{48}$.
36. $\frac{7}{16}$, $\frac{31}{48}$.
37. $\frac{1}{8}$, $\frac{5}{64}$.
38. $\frac{5}{12}$, $\frac{1}{64}$.
39. $\frac{7}{12}$, $\frac{9}{64}$.
40. $\frac{15}{16}$, $\frac{63}{64}$.

199. Addition of Mixed Numbers. In adding mixed numbers we may proceed as in the following example:

Required the sum of $3\frac{4}{5}$, $5\frac{7}{8}$, and $16\frac{3}{4}$.

The L.C.D. is 2 × 2 × 2 × 5, or 40. Reducing the fractions to similar fractions with denominator 40,

$$\begin{array}{lr}
\text{we have} & 3\tfrac{4}{5} = 3\tfrac{32}{40}, \\
 & 5\tfrac{7}{8} = 5\tfrac{35}{40}, \\
\text{and} & 16\tfrac{3}{4} = \underline{16\tfrac{30}{40}}. \\
\quad\text{Therefore} & \text{the sum} = \overline{24\tfrac{97}{40}} = 24 + 2\tfrac{17}{40} = 26\tfrac{17}{40}.
\end{array}$$

Therefore, *to add mixed numbers, add separately the integers and the fractions and find the sum of the results.*

EXERCISE 105

Find the sum of:

1. $2\frac{1}{2}$, $3\frac{5}{8}$.
2. $2\frac{3}{4}$, $7\frac{7}{8}$.
3. $4\frac{1}{4}$, $6\frac{5}{8}$.
4. $3\frac{1}{2}$, $5\frac{1}{3}$.
5. $4\frac{1}{2}$, $6\frac{2}{3}$.
6. $5\frac{1}{4}$, $7\frac{1}{3}$.
7. $27\frac{3}{4}$, $42\frac{3}{8}$.
8. $26\frac{4}{5}$, $37\frac{1}{3}$.
9. $29\frac{3}{5}$, $32\frac{2}{3}$.
10. $46\frac{1}{8}$, $35\frac{3}{16}$.
11. $27\frac{2}{3}$, $62\frac{1}{8}$.
12. $37\frac{7}{8}$, $71\frac{5}{16}$.
13. $1\frac{5}{8}$, $3\frac{1}{3}$, $4\frac{3}{4}$.
14. $1\frac{2}{3}$, $3\frac{4}{5}$, $5\frac{2}{15}$.
15. $2\frac{1}{5}$, $4\frac{1}{8}$, $6\frac{3}{10}$.
16. $3\frac{1}{8}$, $6\frac{3}{4}$, $5\frac{3}{16}$.
17. $4\frac{2}{3}$, $3\frac{1}{6}$, $7\frac{1}{2}$.
18. $3\frac{7}{8}$, $4\frac{3}{4}$, $5\frac{5}{16}$.

19. In a pile of four books the first is $\frac{7}{8}$ of an inch thick, the second $1\frac{1}{16}$ in., the third $\frac{3}{4}$ of an inch, and the fourth $2\frac{1}{3}$ in. What is the height of the pile?

20. A merchant had four pieces of cloth of the same kind. The first piece contained $84\frac{3}{8}$ yd., the second $112\frac{5}{8}$ yd., the third $132\frac{3}{4}$ yd., and the fourth $148\frac{1}{2}$ yd. How many yards of cloth had he in all?

21. Five cans of sirup were measured very carefully and the first was found to contain $2\frac{1}{4}$ qt., the second $2\frac{3}{8}$ qt., the third $2\frac{7}{16}$ qt., the fourth $2\frac{5}{32}$ qt., and the fifth $2\frac{2}{3}$ qt. How much sirup was there in all?

200. Subtraction of Fractions. In subtraction we reduce the fractions to similar fractions with the L.C.D. as in addition.

For example, required to take $\frac{3}{8}$ from $\frac{2}{3}$.

The L.C.D. is 24. Reducing to twenty-fourths,

we have $\frac{2}{3} = \frac{16}{24}$,

and $\frac{3}{8} = \frac{9}{24}$.

Therefore the difference is $\frac{16}{24} - \frac{9}{24}$, or $\frac{7}{24}$.

Therefore, *to subtract one fraction from another, reduce them to similar fractions having the least common denominator, if they are not similar. Write the difference of the numerators over the least common denominator, and reduce the result to lowest terms.*

EXERCISE 106

Subtract as indicated:

1. $\frac{1}{2} - \frac{3}{8}$.
2. $\frac{3}{4} - \frac{3}{8}$.
3. $\frac{7}{8} - \frac{3}{4}$.
4. $\frac{7}{8} - \frac{1}{2}$.
5. $\frac{5}{8} - \frac{1}{4}$.
6. $\frac{1}{2} - \frac{5}{16}$.
7. $\frac{1}{2} - \frac{3}{16}$.
8. $\frac{1}{4} - \frac{1}{16}$.
9. $\frac{3}{4} - \frac{5}{16}$.
10. $\frac{5}{8} - \frac{5}{16}$.
11. $\frac{7}{8} - \frac{9}{16}$.
12. $\frac{1}{2} - \frac{3}{32}$.
13. $\frac{1}{4} - \frac{5}{32}$.
14. $\frac{29}{32} - \frac{3}{4}$.
15. $\frac{29}{32} - \frac{7}{8}$.
16. $\frac{1}{2} - \frac{1}{64}$.
17. $\frac{1}{2} - \frac{15}{64}$.
18. $\frac{1}{2} - \frac{23}{64}$.
19. $\frac{1}{4} - \frac{3}{64}$.
20. $\frac{3}{4} - \frac{31}{64}$.
21. $\frac{3}{4} - \frac{35}{64}$.
22. $\frac{57}{64} - \frac{3}{4}$.
23. $\frac{1}{8} - \frac{3}{64}$.
24. $\frac{3}{8} - \frac{5}{64}$.
25. $\frac{5}{8} - \frac{21}{64}$.
26. $\frac{7}{8} - \frac{51}{64}$.
27. $\frac{3}{16} - \frac{3}{64}$.
28. $\frac{5}{16} - \frac{7}{64}$.
29. $\frac{7}{16} - \frac{13}{48}$.
30. $\frac{9}{16} - \frac{17}{48}$.
31. $\frac{1}{2} - \frac{5}{72}$.
32. $\frac{1}{2} - \frac{11}{72}$.
33. $\frac{1}{4} - \frac{7}{72}$.
34. $\frac{3}{4} - \frac{35}{72}$.
35. $\frac{1}{8} - \frac{1}{72}$.
36. $\frac{3}{8} - \frac{23}{72}$.
37. $\frac{5}{8} - \frac{41}{72}$.
38. $\frac{7}{8} - \frac{61}{72}$.
39. $\frac{1}{16} - \frac{1}{72}$.
40. $\frac{3}{16} - \frac{5}{72}$.
41. $\frac{5}{16} - \frac{7}{72}$.
42. $\frac{7}{12} - \frac{1}{144}$.
43. $\frac{11}{12} - \frac{37}{144}$.
44. $\frac{7}{10} - \frac{37}{100}$.
45. $\frac{9}{10} - \frac{113}{1000}$.

201. Subtraction of Mixed Numbers. In subtracting mixed numbers we proceed as in the following examples:

(1) Required to subtract $4\frac{2}{3}$ from $7\frac{7}{8}$.

The L.C.D. is 3 × 8, or 24. Reducing to twenty-fourths,

we have $7\frac{7}{8} = 7\frac{21}{24}$,
and $4\frac{2}{3} = 4\frac{16}{24}$.

Therefore the difference is $7\frac{21}{24} - 4\frac{16}{24}$, or $3\frac{5}{24}$.

(2) Required to subtract $5\frac{3}{4}$ from $8\frac{1}{6}$.

The L.C.D. is 2 × 2 × 3, or 12. Reducing to twelfths,

we have $8\frac{1}{6} = 8\frac{2}{12}$,
and $5\frac{3}{4} = 5\frac{9}{12}$.

Since we cannot take 9 twelfths from 2 twelfths, we change $8\frac{2}{12}$ to $7 + 1\frac{2}{12}$, or $7\frac{14}{12}$. Then $7\frac{14}{12} - 5\frac{9}{12} = 2\frac{5}{12}$.

Therefore, *to subtract one mixed number from another, reduce the fractions to similar fractions; then subtract the fractions and integers separately.*

EXERCISE 107

Subtract as indicated:

1. $7\frac{7}{8} - 2\frac{1}{4}$.
2. $9\frac{7}{8} - 3\frac{3}{4}$.
3. $6\frac{5}{8} - 2\frac{1}{2}$.
4. $8\frac{5}{8} - 2\frac{1}{4}$.
5. $7\frac{3}{16} - 5\frac{1}{8}$.
6. $11\frac{1}{8} - 2\frac{1}{4}$.
7. $10\frac{3}{8} - 3\frac{3}{4}$.
8. $15\frac{5}{8} - 7\frac{3}{4}$.
9. $12\frac{3}{4} - 6\frac{7}{8}$.
10. $14\frac{1}{4} - 9\frac{3}{8}$.
11. $12\frac{1}{3} - 8\frac{1}{4}$.
12. $15\frac{1}{4} - 7\frac{1}{3}$.
13. $16\frac{1}{3} - 2\frac{3}{5}$.
14. $19\frac{2}{3} - 3\frac{4}{5}$.
15. $31\frac{5}{6} - 7\frac{7}{12}$.

16. From a piece of silk containing $126\frac{1}{8}$ yd. there is cut $23\frac{3}{4}$ yd., $36\frac{5}{8}$ yd., and $24\frac{1}{2}$ yd. How much is left?

17. A man had a farm of $275\frac{1}{8}$ acres. He sold $45\frac{3}{4}$ acres to one man, $27\frac{7}{8}$ acres to another, and $16\frac{1}{5}$ acres to a third. How much had he left?

18. A train that reached Rochester at $9\frac{1}{4}$ o'clock A.M. left New York $7\frac{1}{2}$ hours earlier. When did it leave New York?

202. Multiplication of a Fraction by an Integer. Just as

2 times 2 feet are 4 feet,
so 2 times 2 fifths ($\frac{2}{5}$) are 4 fifths ($\frac{4}{5}$).

In the same way, $3 \times \frac{1}{8} = \frac{3}{8}$, $3 \times \frac{2}{7} = \frac{6}{7}$, $5 \times \frac{3}{8} = \frac{15}{8} = 1\frac{7}{8}$.

If we have to find $6 \times \frac{3}{4}$, it is better to indicate the multiplication first and shorten the actual work by cancellation, thus:

$$6 \times \frac{3}{4} = \frac{\overset{3}{\cancel{6}} \times 3}{\underset{2}{\cancel{4}}} = \frac{9}{2} = 4\frac{1}{2}.$$

Therefore, *to multiply a fraction by an integer, multiply the numerator by the integer and write the product over the denominator.*

Before actually multiplying, indicate the multiplication and cancel if possible. Reduce the result to lowest terms.

EXERCISE 108

Multiply:

1. $6 \times \frac{3}{5}$.
2. $8 \times \frac{3}{4}$.
3. $15 \times \frac{2}{3}$.
4. $18 \times \frac{3}{4}$.
5. $22 \times \frac{3}{8}$.
6. $26 \times \frac{5}{8}$.
7. $32 \times \frac{7}{8}$.
8. $35 \times \frac{3}{16}$.
9. $42 \times \frac{5}{16}$.
10. $56 \times \frac{7}{16}$.
11. $120 \times \frac{9}{16}$.
12. $128 \times \frac{11}{16}$.
13. $160 \times \frac{3}{32}$.
14. $176 \times \frac{5}{32}$.
15. $192 \times \frac{7}{32}$.
16. $208 \times \frac{9}{32}$.
17. $224 \times \frac{11}{32}$.
18. $240 \times \frac{15}{32}$.
19. $256 \times \frac{27}{32}$.
20. $272 \times \frac{31}{32}$.
21. $126 \times \frac{5}{12}$.
22. $140 \times \frac{7}{12}$.
23. $150 \times \frac{11}{12}$.
24. $216 \times \frac{5}{24}$.
25. $228 \times \frac{7}{24}$.
26. $252 \times \frac{11}{24}$.
27. $264 \times \frac{13}{24}$.
28. $325 \times \frac{9}{10}$.
29. $432 \times \frac{7}{48}$.
30. $500 \times \frac{11}{48}$.
31. If a book costs $\$\frac{3}{4}$, how much will 17 books cost?
32. If a train travels $\frac{5}{6}$ of a mile in a minute, how far will it travel in 132 minutes?

203. To find a Fractional Part of an Integer. One of the most important operations with fractions is to find a fractional part of a number. To take a fractional part of a number is called *multiplication by a fraction.*

That is, to take $\frac{1}{3}$ of 6 is called the multiplication of 6 by $\frac{1}{3}$. We indicate the operation by the expression: $\frac{1}{3}$ of $6 = 2$, or $\frac{1}{3} \times 6 = 2$, the symbol × being read *of* in the latter case. Special emphasis should be given to this important case.

Therefore, *to multiply an integer by a fraction, multiply the integer by the numerator of the fraction and write the product over the denominator.*

It is advisable to cancel before the actual multiplication, if possible. Thus, in taking $\frac{4}{15}$ of 12, the work should be arranged as follows:

$$\frac{4}{15} \text{ of } 12 = \frac{4 \times \overset{4}{\cancel{12}}}{\underset{5}{\cancel{15}}} = \frac{16}{5} = 3\frac{1}{5}.$$

EXERCISE 109

Multiply:

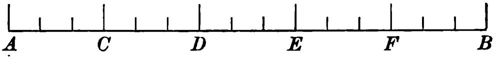
1. $\frac{2}{3}$ of 18.
2. $\frac{3}{4}$ of 24.
3. $\frac{2}{5}$ of 35.
4. $\frac{3}{5}$ of 48.
5. $\frac{4}{5}$ of 75.
6. $\frac{5}{6}$ of 93.
7. $\frac{3}{8}$ of 24.
8. $\frac{5}{12}$ of 72.
9. $\frac{7}{12}$ of 80.
10. $\frac{11}{12}$ of 96.
11. $\frac{3}{16}$ of 72.
12. $\frac{5}{16}$ of 80.
13. $\frac{7}{16}$ of 96.
14. $\frac{9}{16}$ of 100.
15. $\frac{5}{24}$ of 72.
16. $\frac{7}{24}$ of 80.
17. $\frac{11}{24}$ of 84.
18. $\frac{13}{24}$ of 96.
19. $\frac{17}{24}$ of 100.
20. $\frac{5}{32}$ of 100.
21. $\frac{7}{32}$ of 200.

22. At 72¢ a yard, how much will $\frac{5}{8}$ of a yard of cloth cost? $\frac{3}{4}$ of a yard?

23. At 96¢ a yard, how much will $\frac{7}{8}$ of a yard of cloth cost? $\frac{5}{8}$ of a yard?

24. At 88¢ a yard, how much will $\frac{3}{4}$ of a yard of cloth cost? $\frac{3}{8}$ of a yard? $\frac{5}{8}$ of a yard?

204. To find a Fractional Part of a Fraction. Suppose it is required to find $\frac{2}{3}$ of $\frac{4}{5}$.



In this figure let the line AB be divided into five equal parts at the points C, D, E, and F. Then AF is $\frac{4}{5}$ of AB.

Let each of the 5 parts be divided into 3 equal parts. Then each of these parts evidently is $\frac{1}{15}$ of AB.

That is, $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$ of the whole.
Then $\frac{1}{3}$ of $\frac{4}{5}$ must be 4 times as much, or $\frac{4}{15}$.
Then $\frac{2}{3}$ of $\frac{4}{5}$ must be twice $\frac{4}{15}$, or $\frac{8}{15}$.

Therefore, *to multiply a fraction by a fraction, find the product of the numerators for the required numerator and the product of the denominators for the required denominator.*

This case also demands special emphasis.

Always cancel as far as possible. For example:

$$\frac{11}{12}\text{ of }\frac{15}{22}=\frac{\cancel{11}\times\overset{5}{\cancel{15}}}{\underset{4}{\cancel{12}}\times\underset{2}{\cancel{22}}}=\frac{5}{8}.$$

EXERCISE 110

Multiply:

1. $\frac{3}{4}$ of $\frac{8}{9}$.
2. $\frac{2}{3}$ of $\frac{9}{10}$.
3. $\frac{2}{3}$ of $\frac{15}{16}$.
4. $\frac{2}{3}$ of $\frac{11}{24}$.
5. $\frac{2}{3}$ of $\frac{25}{32}$.
6. $\frac{3}{4}$ of $\frac{5}{24}$.
7. $\frac{1}{8}$ of $\frac{4}{5}$.
8. $\frac{3}{8}$ of $\frac{7}{8}$.
9. $\frac{7}{8}$ of $\frac{4}{7}$.
10. $\frac{5}{8}$ of $\frac{3}{10}$.
11. $\frac{3}{16}$ of $\frac{8}{9}$.
12. $\frac{7}{16}$ of $\frac{8}{21}$.
13. $\frac{1}{32}$ of $\frac{4}{5}$.
14. $\frac{1}{48}$ of $\frac{24}{25}$.
15. $\frac{7}{48}$ of $\frac{16}{21}$.
16. $\frac{3}{5}$ of $\frac{5}{48}$.
17. $\frac{4}{5}$ of $\frac{25}{48}$.
18. $\frac{5}{9}$ of $\frac{81}{100}$.
19. How much is half of three quarters of a yard?
20. How much is $\frac{2}{3}$ of $\frac{5}{8}$ of an acre?
21. How much is a quarter of $\frac{7}{8}$ of a mile?
22. How much is an eighth of $\frac{5}{16}$ of an inch?

Find the products of the following:

23. $\frac{5}{8} \times \frac{24}{25}$.
24. $\frac{3}{8} \times \frac{16}{21}$.
25. $\frac{7}{8} \times \frac{32}{35}$.
26. $\frac{9}{16} \times \frac{4}{9}$.
27. $\frac{7}{16} \times \frac{5}{7}$.
28. $\frac{5}{16} \times \frac{32}{45}$.
29. $\frac{13}{16} \times \frac{2}{3}$.
30. $\frac{15}{16} \times \frac{2}{5}$.
31. $\frac{7}{12} \times \frac{4}{9}$.
32. $\frac{5}{12} \times \frac{3}{5}$.
33. $\frac{11}{12} \times \frac{4}{5}$.
34. $\frac{11}{16} \times \frac{4}{11}$.
35. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.
36. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$.
37. $\frac{2}{3} \times \frac{2}{3} \times \frac{3}{4}$.
38. $\frac{3}{5} \times \frac{3}{4} \times \frac{1}{8}$.
39. $\frac{3}{4} \times \frac{3}{8} \times \frac{4}{5}$.
40. $\frac{5}{8} \times \frac{7}{8} \times \frac{3}{8}$.

41. The perimeter of (distance around) a square being $\frac{7}{8}$ of an inch, what is the length of each side?

42. The perimeter of a triangle being $\frac{7}{8}$ of an inch and the sides being equal, what is the length of each side?

43. If the distance to a certain house is $\frac{3}{8}$ of a mile, what fraction of a mile has a boy gone when he has covered $\frac{5}{16}$ of the distance?

44. If it takes a train $\frac{3}{4}$ of an hour to reach a certain station, what fraction of an hour will it take to cover $\frac{3}{5}$ of the distance?

45. A remnant of lace cost $\$\frac{5}{8}$. How much did $\frac{4}{5}$ of the remnant cost? How much did $\frac{3}{5}$ of it cost?

46. If $\frac{3}{4}$ of a yard of lace is needed to trim two neck bands, how much lace will be needed to trim one neck band?

47. If an automobile travels a mile in $\frac{58}{60}$ of a minute in a certain race, how long would it take to travel $\frac{3}{4}$ of a mile at the same rate?

48. If a woman pays $\$\frac{3}{4}$ for a yard of ribbon, how much would she pay for $\frac{2}{3}$ of a yard? for $\frac{1}{3}$ of a yard? for $\frac{1}{2}$ of a yard?

49. A milliner used $\frac{7}{8}$ of a yard of velvet for the upper brim and the crown of a hat, and $\frac{3}{5}$ as much for the lower brim. How much velvet did she use for the hat?

50. A piece of goods $\frac{7}{8}$ of a yard long is $\frac{2}{3}$ as long after tucking. How long is it after tucking?

51. A piece of cloth $\frac{5}{8}$ of a yard long loses $\frac{2}{5}$ of its length in tucking. How long is it then?

52. A woman has cut some goods $\frac{1}{4}$ of a yard wide for a ruffle. She uses $\frac{1}{4}$ of this width for a hem and $\frac{2}{3}$ of the rest for tucking. How wide is the ruffle when it is finished?

53. If I pay $2 for $\frac{3}{4}$ of a yard of a certain kind of velvet, what part of a yard will I buy with $1? What part with $1.50, which is $\frac{3}{4}$ of $2?

54. A girl in a shop makes $\frac{5}{8}$ of a yard of a certain kind of chiffon appliqué in an hour. How much does she make in 20 minutes? in 45 minutes?

55. If it takes $\frac{3}{4}$ of an hour to solve a page of examples, how long would it take to solve $\frac{3}{5}$ of them at the same rate? How many minutes?

56. A man who owned $\frac{5}{9}$ of the stock in a certain company sold $\frac{3}{5}$ of what he owned. What part of the stock of the company did he then own?

57. In making the plans for a house an architect wishes to draw a line that shall be $\frac{2}{3}$ as long as one that is $\frac{15}{16}$ of an inch long. How long shall he draw it?

58. In 40 minutes the minute hand of a clock makes $\frac{2}{3}$ of a revolution. What part of a revolution will it make in 32 minutes? (32 minutes = $\frac{32}{40}$ of 40 minutes.)

59. In 55 minutes the minute hand of a clock makes $\frac{11}{12}$ of a revolution. What part of a revolution will it make in 33 minutes?

60. A boy finds that when the front wheel of his cart has made one revolution the cart has moved $\frac{5}{6}$ of a yard. How far has it gone when the wheel has made $\frac{3}{4}$ of a revolution?

205. Multiplication of a Mixed Number by an Integer. Since $3 \times 5 = 15$, and $3 \times \frac{5}{6} = \frac{5}{2} = 2\frac{1}{2}$, therefore $3 \times 5\frac{5}{6}$ equals the sum of these products, that is, $15 + 2\frac{1}{2}$, or $17\frac{1}{2}$.

Therefore, *to multiply a mixed number by an integer, multiply the whole part and the fractional part separately by the integer and add the products.*

EXERCISE 111

Multiply:

1. $2 \times 17\frac{2}{3}$.
2. $3 \times 26\frac{1}{4}$.
3. $3 \times 18\frac{3}{4}$.
4. $4 \times 35\frac{3}{4}$.
5. $2 \times 43\frac{3}{8}$.
6. $3 \times 25\frac{5}{8}$.
7. $6 \times 56\frac{7}{8}$.
8. $4 \times 33\frac{5}{12}$.
9. $3 \times 25\frac{7}{12}$.
10. $5 \times 48\frac{11}{12}$.
11. $2 \times 16\frac{3}{16}$.
12. $3 \times 17\frac{5}{16}$.
13. $4 \times 25\frac{7}{16}$.
14. $8 \times 39\frac{11}{16}$.
15. $8 \times 58\frac{15}{16}$.
16. $2 \times 45\frac{3}{32}$.
17. $4 \times 74\frac{5}{32}$.
18. $9 \times 86\frac{7}{32}$.
19. $8 \times 69\frac{9}{32}$.
20. $6 \times 97\frac{11}{32}$.
21. $4 \times \$32\frac{3}{4}$.
22. $7 \times \$21\frac{1}{4}$.
23. $6 \times \$35\frac{1}{2}$.
24. $8 \times \$27\frac{3}{4}$.
25. $12 \times 117\frac{1}{2}$.
26. $15 \times 222\frac{1}{5}$.
27. $12 \times 305\frac{1}{8}$.
28. $16 \times 211\frac{7}{8}$.
29. $25 \times 107\frac{1}{5}$.
30. $35 \times 121\frac{3}{10}$.

31. At $\$2\frac{1}{4}$ a yard, how much will 9 yards of cloth cost?

32. How many feet in the perimeter of a square $26\frac{3}{16}$ feet on a side?

33. A dealer sells 16 notebooks at $3\frac{3}{4}$ cents each. How much does he receive?

34. A rod being $16\frac{1}{2}$ feet long, how many feet in the perimeter of a lot that is 26 rods long and 16 rods wide?

35. In a motor race one of the cars averaged $1\frac{3}{8}$ miles a minute. How many miles did it travel in 7 minutes?

36. If a boy lives $1\frac{3}{4}$ miles from this school, and has a cousin living 25 times as far from here, how far from here does the cousin live?

206. Multiplication of an Integer by a Mixed Number. Since $2 \times 9 = 18$, and $\frac{3}{4}$ of $9 = \frac{27}{4} = 6\frac{3}{4}$, therefore $2\frac{3}{4} \times 9$ equals the sum of these products, or $18 + 6\frac{3}{4}$, or $24\frac{3}{4}$.

The symbol × after a mixed number is read *times*, as with integers.

Therefore, *to multiply an integer by a mixed number, multiply the integer by the whole part and the fractional part separately and add the products.*

In practice, to take the product of an integer and a mixed number we use as the multiplier whichever is the more convenient. For example, it is shorter to multiply $249\frac{3}{4}$ by 2 than to multiply 2 by $249\frac{3}{4}$. But if it is required to multiply $4\frac{7}{8}$ by 648, it is easier to change the order and multiply 648 by $4\frac{7}{8}$. These two examples may be arranged thus:

$$\begin{array}{r} 249\frac{3}{4} \\ \underline{2} \\ 498\frac{3}{2} = 499\frac{1}{2} \end{array} \qquad\qquad \begin{array}{rl} 648 & \\ \underline{4\frac{7}{8}} & \\ 81 & (\frac{1}{8} \text{ of } 648) \\ \underline{7} & \\ 567 & (\frac{7}{8} \text{ of } 648) \\ \underline{2592} & (4 \times 648) \\ 3159 & \end{array}$$

EXERCISE 112

Multiply:

1. $3\frac{4}{5} \times 75$.
2. $4\frac{2}{3} \times 69$.
3. $5\frac{1}{4} \times 72$.
4. $7\frac{3}{4} \times 84$.
5. $6\frac{1}{4} \times 31$.
6. $8\frac{3}{4} \times 39$.
7. $7\frac{3}{8} \times 48$.
8. $9\frac{3}{8} \times 50$.
9. $5\frac{5}{8} \times 64$.
10. $7\frac{5}{8} \times 70$.
11. $3\frac{7}{8} \times 96$.
12. $5\frac{7}{8} \times 91$.
13. $2\frac{1}{16} \times 32$.
14. $3\frac{3}{16} \times 64$.
15. $5\frac{5}{16} \times 80$.
16. $9\frac{7}{16} \times 96$.
17. $5\frac{3}{16} \times 38$.
18. $4\frac{5}{16} \times 60$.
19. $9\frac{7}{16} \times 88$.
20 $7\frac{9}{16} \times 62$.
21. $3\frac{3}{32} \times 160$.
22. $4\frac{5}{32} \times 640$.
23. $7\frac{7}{32} \times 176$.
24. $8\frac{9}{32} \times 656$.
25. $9\frac{11}{32} \times 352$.
26. $12\frac{13}{32} \times 288$.
27. $15\frac{15}{32} \times 256$.
28. $20\frac{5}{48} \times 240$.
29. $300\frac{7}{48} \times 528$.
30. $500\frac{9}{64} \times 384$.

207. Multiplication of Two Mixed Numbers. We may multiply one mixed number by another by the method explained in § 206 or by that in § 204.

$$\begin{array}{rl}
9\frac{3}{4} & \\
2\frac{1}{3} & \\
\hline
3\frac{1}{4} & (\frac{1}{3} \text{ of } 9\frac{3}{4}) \\
19\frac{2}{4} & (2 \times 9\frac{3}{4}) \\
\hline
22\frac{3}{4} &
\end{array}$$

For example, we may multiply $9\frac{3}{4}$ by $2\frac{1}{3}$ as here shown. But it is usually easier to reduce both to improper fractions and multiply as in § 204. Thus in the case of $6\frac{2}{5} \times 4\frac{11}{16}$ we have $6\frac{2}{5} = \frac{32}{5}$, and $4\frac{11}{16} = \frac{75}{16}$. Then

$$6\frac{2}{5} \times 4\frac{11}{16} = \frac{\overset{2}{\cancel{32}} \times \overset{15}{\cancel{75}}}{\cancel{5} \times \cancel{16}} = 30.$$

Therefore, *to multiply a mixed number by a mixed number, reduce each to an improper fraction and multiply.*

In the case of a fraction and a mixed number, reduce the mixed number to an improper fraction.

EXERCISE 113

Multiply:

1. $2\frac{1}{2} \times 3\frac{3}{8}$.
2. $3\frac{1}{3} \times 4\frac{3}{4}$.
3. $2\frac{2}{3} \times 3\frac{1}{4}$.
4. $5\frac{3}{5} \times 7\frac{3}{4}$.
5. $4\frac{1}{3} \times 6\frac{3}{8}$.
6. $2\frac{1}{2} \times 31\frac{1}{2}$.
7. $3\frac{1}{2} \times 25\frac{1}{3}$.
8. $7\frac{1}{2} \times 33\frac{1}{3}$.
9. $3\frac{3}{4} \times 66\frac{2}{3}$.
10. $5\frac{1}{5} \times 29\frac{3}{8}$.
11. $33\frac{1}{3} \times 66\frac{2}{3}$.
12. $33\frac{1}{3} \times 120\frac{3}{5}$.
13. $16\frac{2}{3} \times 132\frac{3}{5}$.
14. $16\frac{2}{3} \times 336\frac{6}{7}$.
15. $21\frac{3}{4} \times 441\frac{1}{3}$.

16. At the rate of $\$66\frac{2}{3}$ a month, how much rent will a man pay for a city apartment in 6 months?

17. If a train travels at the rate of $48\frac{3}{4}$ miles per hour, how far will it travel in $2\frac{1}{2}$ hours?

18. Some blotting paper is $\frac{3}{64}$ of an inch thick. What will be the height of a pile of $2\frac{3}{4}$ dozen sheets?

19. If an arithmetic is $1\frac{1}{8}$ inches thick and a reader is $\frac{15}{16}$ of an inch thick, what is the height of a pile of 7 arithmetics and 9 readers?

208. Division of a Fraction by an Integer. There are two cases to be considered:

(1) When the numerator of the fraction is divisible by the divisor.

In this case, just as 8 ft. ÷ 2 = 4 ft., and \$8 ÷ 2 = \$4, so 8 ninths ($\frac{8}{9}$) ÷ 2 = 4 ninths ($\frac{4}{9}$). In the same way, $\frac{9}{10} \div 3 = \frac{3}{10}$, and $\frac{15}{16} \div 5 = \frac{3}{16}$.

(2) When the numerator of the fraction is not divisible by the divisor.

Take the case of $\frac{3}{5} \div 2$. This is the same as $\frac{1}{2}$ of $\frac{3}{5}$, which equals $\frac{3}{10}$. In the same way, $\frac{5}{8} \div 3 = \frac{1}{3}$ of $\frac{5}{8} = \frac{5}{24}$.

Therefore, *to divide a fraction by an integer, divide the numerator by the integer if possible, and write the quotient over the denominator.*

If it is impossible to divide the numerator by the integer, multiply the denominator by the integer and write the product under the numerator.

It is easier to indicate this multiplication and then cancel if possible. Thus

$$\frac{15}{16} \div 6 = \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{6}} \times 16} = \frac{5}{32}.$$

EXERCISE 114

Find the quotient of:

1. $\frac{35}{48} \div 7$.
2. $\frac{27}{32} \div 9$.
3. $\frac{51}{64} \div 17$.
4. $\frac{3}{16} \div 6$.
5. $\frac{15}{16} \div 10$.
6. $\frac{45}{32} \div 15$.
7. $\frac{15}{16} \div 25$.
8. $\frac{15}{32} \div 20$.
9. $\frac{19}{24} \div 57$.
10. $\frac{17}{48} \div 51$.
11. $\frac{34}{100} \div 51$.
12. $\frac{38}{100} \div 57$.
13. $\frac{25}{32} \div 100$.
14. $\frac{25}{64} \div 125$.
15. $\frac{21}{24} \div 126$.
16. $\frac{23}{24} \div 161$.
17. $\frac{27}{100} \div 170$.
18. $\frac{37}{100} \div 333$.
19. $\frac{33}{1000} \div 77$.
20. $\frac{51}{1000} \div 170$.
21. $\frac{143}{1000} \div 130$.

209. Division of a Mixed Number by an Integer. A mixed number may be divided by an integer by first reducing it to an improper fraction and then proceeding as in § 208.

For example, $3\frac{5}{8} \div 7 = \frac{29}{8} \div 7 = \frac{29}{56}$.

In case the mixed number is long, however, it is easier to divide as with integers, continuing the division until a remainder is found that is smaller than the divisor.

For example, $177\frac{1}{2} \div 5 = 35$ with remainder $2\frac{1}{2}$. Dividing the remainder, we have $2\frac{1}{2} \div 5 = \frac{5}{2} \div 5 = \frac{1}{2}$. Hence the entire quotient is $35\frac{1}{2}$.

Therefore, *to divide a mixed number by an integer, divide as with integers until a remainder is found that is less than the divisor; then divide this remainder by the divisor.*

We may reduce to a decimal fraction before dividing.

EXERCISE 115

Find the quotient of:

1. $3\frac{3}{4} \div 9$.
2. $4\frac{3}{5} \div 7$.
3. $5\frac{2}{3} \div 6$.
4. $6\frac{7}{8} \div 5$.
5. $8\frac{2}{3} \div 4$.
6. $7\frac{3}{8} \div 59$.
7. $23\frac{2}{3} \div 8$.
8. $28\frac{3}{4} \div 7$.
9. $32\frac{5}{8} \div 9$.
10. $41\frac{5}{8} \div 6$.
11. $34\frac{1}{16} \div 7$.
12. $78\frac{3}{16} \div 9$.
13. $142\frac{1}{2} \div 5$.
14. $166\frac{2}{3} \div 10$.
15. $133\frac{1}{3} \div 20$.
16. $275\frac{3}{4} \div 12$.
17. $266\frac{7}{8} \div 15$.
18. $345\frac{5}{16} \div 25$.

19. If 9 books cost $\$6\frac{3}{4}$, or \$6.75, what is the average cost?

20. If 7 yards of cloth cost $\$8\frac{3}{4}$, or \$8.75, what does it cost per yard?

21. If 19 yards of basket-weave burlap cost $\$21\frac{3}{8}$, or $\$21.37\frac{1}{2}$, what does it cost per yard?

22. If 15 dozen pairs of buttonhole scissors cost $\$67\frac{1}{2}$, or \$67.50, what do they cost per dozen?

210. Dividing by a Unit Fraction. Since there are 4 fourths of an inch in 1 inch, therefore 1 inch $\div \frac{1}{4}$ of an inch $= 4$, and 2 inches $\div \frac{1}{4}$ of an inch $= 8$. In the same way, $5 \div \frac{1}{4} = 20$, $7 \div \frac{1}{4} = 28$, and so on.

And if $5 \div \frac{1}{4} = 20$, then $\frac{5}{8} \div \frac{1}{4}$ must equal $\frac{1}{8}$ as much, or $\frac{20}{8}$, or $2\frac{1}{2}$.

Therefore, *to divide by a unit fraction, multiply by the denominator.*

That is,

$$\begin{aligned} 15 \div \tfrac{1}{8} &= 8 \times 15 = 120, \\ 27 \div \tfrac{1}{4} &= 4 \times 27 = 108, \\ \tfrac{3}{8} \div \tfrac{1}{16} &= 16 \times \tfrac{3}{8} = 6, \\ 2\tfrac{1}{2} \div \tfrac{1}{32} &= 32 \times 2\tfrac{1}{2} = 80. \end{aligned}$$

EXERCISE 116

Find the quotient of:

1. $28 \div \frac{1}{2}$.
2. $37 \div \frac{1}{3}$.
3. $49 \div \frac{1}{4}$.
4. $56 \div \frac{1}{5}$.
5. $93 \div \frac{1}{7}$.
6. $\frac{2}{3} \div \frac{1}{9}$.
7. $\frac{3}{4} \div \frac{1}{16}$.
8. $\frac{4}{5} \div \frac{1}{35}$.
9. $\frac{5}{6} \div \frac{1}{72}$.
10. $\frac{7}{8} \div \frac{1}{25}$.
11. $2\frac{1}{2} \div \frac{1}{2}$.
12. $3\frac{1}{3} \div \frac{1}{6}$.
13. $5\frac{1}{4} \div \frac{1}{8}$.
14. $6\frac{3}{4} \div \frac{1}{12}$.
15. $25\frac{7}{8} \div \frac{1}{32}$.

16. If a sheet of veneer is $\frac{1}{16}$ of an inch thick, how many sheets pressed together will have a thickness of $1\frac{1}{8}$ inches?

17. If a sheet of blotting paper is $\frac{1}{32}$ of an inch thick, how many sheets are there in a pile that is $15\frac{5}{8}$ inches thick?

18. If each book is $\frac{1}{12}$ of a foot thick, how many books are there in a single pile of books $8\frac{1}{3}$ feet high?

19. If a bicycle is traveling at the rate of a mile in $\frac{1}{8}$ of an hour, how many miles will it travel in $3\frac{3}{4}$ hours?

20. How many weights of $\frac{1}{16}$ of a pound (or 1 ounce) each will it take to weigh $17\frac{3}{4}$ pounds?

21. If a wagon wheel makes $\frac{1}{12}$ of a revolution in going 1 foot, how many feet will it go in making $17\frac{3}{4}$ revolutions?

211. Dividing by a Fraction. We have found (§ 210) that when we have to divide by $\frac{1}{3}$ we may multiply by 3. We also know that when we multiply the divisor by 2 we divide the quotient by 2.

That is, $6 \div \frac{1}{3} = 3 \times 6$ (§ 210),

and $6 \div \frac{2}{3} =$ half as much (§ 65) $= \frac{3 \times 6}{2} = 9.$

That is, $6 \div \frac{2}{3} = \frac{3}{2}$ of 6.

Therefore, *to divide by a common fraction, multiply by the inverted divisor.*

That is, $15 \div \frac{3}{5} = \frac{5}{\cancel{3}}$ of $\overset{5}{\cancel{15}} = 25$. Similarly, $\frac{3}{8} \div \frac{5}{6} = \frac{\overset{3}{\cancel{6}}}{5}$ of $\frac{3}{\underset{4}{\cancel{8}}} = \frac{9}{20}.$

An inverted fraction is also called the *reciprocal* of the fraction. That is, the reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$. The product of a fraction and its reciprocal is always one; thus, $\frac{5}{6}$ of $\frac{6}{5} = 1$.

EXERCISE 117

Find the quotient of:

1. $3 \div \frac{2}{3}$.
2. $5 \div \frac{3}{4}$.
3. $7 \div \frac{4}{5}$.
4. $9 \div \frac{3}{5}$.
5. $8 \div \frac{3}{8}$.
6. $5 \div \frac{5}{8}$.
7. $8 \div \frac{7}{8}$.
8. $9 \div \frac{5}{6}$.
9. $8 \div \frac{2}{5}$.
10. $9 \div \frac{3}{7}$.
11. $15 \div \frac{2}{3}$.
12. $15 \div \frac{3}{4}$.
13. $16 \div \frac{4}{5}$.
14. $21 \div \frac{3}{5}$.
15. $24 \div \frac{2}{5}$.
16. $30 \div \frac{3}{8}$.
17. $40 \div \frac{5}{8}$.
18. $20 \div \frac{7}{8}$.
19. $32 \div \frac{5}{6}$.
20. $75 \div \frac{5}{9}$.
21. $\frac{2}{3} \div \frac{3}{4}$.
22. $\frac{3}{4} \div \frac{2}{3}$.
23. $\frac{3}{8} \div \frac{5}{8}$.
24. $\frac{7}{8} \div \frac{5}{8}$.
25. $\frac{3}{8} \div \frac{3}{4}$.
26. $\frac{3}{4} \div \frac{7}{8}$.
27. $\frac{5}{12} \div \frac{3}{4}$.
28. $\frac{7}{12} \div \frac{2}{3}$.
29. $\frac{3}{8} \div \frac{7}{12}$.
30. $\frac{5}{8} \div 1\frac{1}{2}$.
31. $2\frac{1}{3} \div \frac{3}{4}$.
32. $3\frac{2}{3} \div \frac{3}{4}$.
33. $5\frac{1}{8} \div \frac{1}{2}$.
34. $5\frac{3}{8} \div \frac{3}{4}$.
35. $7\frac{5}{8} \div \frac{3}{8}$.
36. $8\frac{5}{8} \div \frac{5}{8}$.
37. $6\frac{7}{8} \div \frac{5}{8}$.
38. $7\frac{3}{16} \div \frac{7}{8}$.
39. $9\frac{5}{16} \div \frac{3}{4}$.
40. $8\frac{5}{32} \div \frac{7}{16}$.

212. Dividing by a Mixed Number. To divide by a mixed number it is generally best to reduce the divisor to an improper fraction and proceed as in § 211.

For example, to divide 10 by $3\frac{1}{3}$ is the same as to divide 10 by $\frac{10}{3}$, or to take $\frac{3}{10}$ of 10. But $\frac{3}{10}$ of $10 = 3$; hence $10 \div 3\frac{1}{3} = 3$.

Therefore, *to divide by a mixed number, reduce the divisor to an improper fraction and multiply the dividend by the reciprocal of the divisor.*

Thus $$17\frac{1}{2} \div 3\frac{1}{4} = \frac{35}{2} \div \frac{13}{4} = \frac{\overset{2}{\cancel{4}}}{13} \text{ of } \frac{35}{\cancel{2}} = \frac{70}{13} = 5\frac{5}{13}.$$

EXERCISE 118

Find the quotient of:

1. $3 \div 7\frac{1}{4}$.
2. $5 \div 4\frac{3}{4}$.
3. $9 \div 6\frac{1}{2}$.
4. $7 \div 5\frac{1}{8}$.
5. $5 \div 4\frac{3}{8}$.
6. $5 \div 6\frac{7}{8}$.
7. $8 \div 3\frac{4}{5}$.
8. $25 \div 3\frac{1}{3}$.
9. $62 \div 6\frac{2}{3}$.
10. $48 \div 12\frac{1}{2}$.
11. $72 \div 16\frac{2}{3}$.
12. $26 \div 33\frac{1}{3}$.
13. $48 \div 66\frac{2}{3}$.
14. $50 \div 37\frac{1}{2}$.
15. $\frac{3}{4} \div 2\frac{1}{2}$.
16. $\frac{3}{8} \div 3\frac{1}{3}$.
17. $\frac{4}{5} \div 2\frac{2}{3}$.
18. $\frac{3}{8} \div 5\frac{1}{4}$.
19. $\frac{5}{8} \div 7\frac{3}{4}$.
20. $\frac{3}{16} \div 6\frac{2}{3}$.
21. $\frac{5}{16} \div 16\frac{2}{3}$.
22. $12\frac{1}{2} \div 6\frac{1}{4}$.
23. $12\frac{1}{2} \div 3\frac{1}{3}$.
24. $12\frac{1}{2} \div 6\frac{2}{3}$.
25. $16\frac{2}{3} \div 12\frac{1}{2}$.
26. $33\frac{1}{3} \div 66\frac{2}{3}$.
27. $37\frac{1}{2} \div 6\frac{1}{4}$.

28. Find the cost per ton if $3\frac{1}{4}$ tons of hay cost \$$37\frac{3}{8}$.

29. How many strips of cloth, each $2\frac{1}{2}$ yards long, can be cut from a piece that is 45 yards long?

30. A dealer paid \$$247\frac{1}{2}$ for $7\frac{1}{2}$ dozen pairs of shoes. How much did they cost per dozen? How much per pair?

31. A dealer paid \$$224\frac{1}{4}$ for $3\frac{1}{4}$ dozen pairs of shears. How much did they cost per dozen? How much per pair?

213. To find what Fraction One Number is of Another. For example, 3 is what fraction of 4?

Since $1 = \frac{1}{4}$ of 4,
therefore $3 = 3 \times \frac{1}{4}$ of 4, or $\frac{3}{4}$ of 4.

The result therefore equals $3 \div 4$.

As a further example, $\frac{3}{7}$ is what part of $\frac{3}{2}$?

Since $1 = \frac{2}{3}$ of $\frac{3}{2}$,
therefore $\frac{3}{7} = \frac{3}{7}$ of $\frac{2}{3}$ of the second fraction
$= \frac{2}{7}$ of the second fraction.

The result therefore equals $\frac{3}{7} \div \frac{3}{2}$.

Therefore, *to find what fraction one number is of a second, divide the first number by the second.*

The result may be a mixed number. It may also be integral, as when we find what fraction 33 is of $16\frac{1}{2}$, the answer being 2.

Special emphasis should be given to this important case, and the class should be asked to suggest original problems based upon it.

EXERCISE 119

Find what fraction the first number is of the second:

1. 5, 15.
2. 19, 57.
3. 15, 35.
4. 29, 87.
5. 17, 85.
6. $16\frac{2}{3}$, 50.
7. 17, $8\frac{1}{2}$.
8. 39, $9\frac{3}{4}$.
9. $9\frac{2}{5}$, 47.
10. $7\frac{3}{8}$, 59.
11. $5\frac{1}{16}$, 91.
12. $3\frac{3}{32}$, 99.
13. $\frac{2}{3}$, $\frac{3}{5}$.
14. $\frac{3}{4}$, $\frac{5}{8}$.
15. $\frac{7}{8}$, $\frac{5}{16}$.
16. $\frac{5}{12}$, $\frac{3}{16}$.
17. $\frac{7}{16}$, $\frac{5}{32}$.
18. $\frac{9}{32}$, $\frac{5}{48}$.
19. $3\frac{2}{3}$, $5\frac{1}{2}$.
20. $4\frac{1}{4}$, $5\frac{3}{8}$.
21. $6\frac{2}{3}$, $3\frac{1}{3}$.
22. $8\frac{1}{3}$, $16\frac{2}{3}$.
23. $33\frac{1}{3}$, 100.
24. $66\frac{2}{3}$, 100.

Find what part of a dollar is represented by the following:

25. 10 cents.
26. 20 cents.
27. 50 cents.
28. 25 cents.
29. 75 cents.
30. $12\frac{1}{2}$ cents.
31. $33\frac{1}{3}$ cents.
32. $66\frac{2}{3}$ cents.
33. $6\frac{1}{4}$ cents.
34. $37\frac{1}{2}$ cents.
35. $62\frac{1}{2}$ cents.
36. $87\frac{1}{2}$ cents.

37. If a man has to drive $17\frac{1}{2}$ mi., what part of the distance has he gone when he has driven $3\frac{1}{2}$ mi.?

38. If 2 lb. of flour are produced from $2\frac{1}{2}$ lb. of wheat, the weight of the flour is what part of the weight of the wheat?

39. A cubic inch of copper weighs $5\frac{3}{20}$ oz. This is what part of the weight of a piece of copper wire weighing $10\frac{3}{10}$ oz.?

40. Two men are working on the same job for the same wages. One works 15 days and the other works 21 days. If the total wages are $99, how much does each man receive?

41. Two men are working on the same job for the same wages. One works $4\frac{7}{8}$ days and the other $5\frac{3}{4}$ days. What part of the total wages should each receive?

42. From a barrel containing 30 gal. of oil $1\frac{7}{8}$ gal. leaked out. What fractional part of all the oil remained?

43. If a boy is making a 25-mile trip on his bicycle, what fractional part of his journey has been completed when he has gone $18\frac{3}{4}$ mi.?

44. What part of a barrel ($31\frac{1}{2}$ gal.) is 1.23 gal.?

45. If a man by working 8 hr. can finish $\frac{3}{8}$ of a piece of work, what part of the work can he finish in 3 hr.?

46. In $6\frac{7}{8}$ lb. of beefsteak there are $4\frac{1}{3}$ lb. of water. What part of the beefsteak is water?

47. If a piece of bronze weighing $7\frac{1}{2}$ lb. contains $5\frac{1}{4}$ lb. of copper, what part of the bronze is copper?

48. If a mass of lead and zinc weighing $16\frac{1}{8}$ lb. contains $10\frac{3}{4}$ lb. of lead, what fractional part of the whole mass is lead?

49. If a book has 540 pages, what part is finished when 216 pages have been read?

50. In Ex. 49 what part of the book still remains to be finished? Add the answers to Exs. 49 and 50.

214. To find the Whole when a Fractional Part is Given. For example, if $\frac{3}{8}$ of a yard of cloth costs 30 cents, what does 1 yard cost?

Since $\frac{3}{8}$ of a yard costs 30 cents,
then $\frac{1}{8}$ of a yard costs $\frac{1}{3}$ of 30 cents, or 10 cents,
and $\frac{8}{8}$ of a yard, or a whole yard, costs 8 × 10 cents, or 80 cents.

A short way to obtain this result is evidently to multiply 30 cents by $\frac{8}{3}$, that is, by the reciprocal (§ 211) of the fraction. Special emphasis should be given to this important case.

Therefore, *to find the whole when a fractional part is given, multiply the given part by the reciprocal of the fraction.*

EXERCISE 120

1. $16 is $\frac{4}{7}$ of what sum of money?

2. $45 is $\frac{5}{8}$ of what sum of money?

3. 84 mi. is $\frac{7}{9}$ of how many miles?

4. $17\frac{1}{5}$ ft. is $\frac{4}{5}$ of how many feet?

5. $243\frac{2}{5}$ lb. is $\frac{4}{5}$ of how many pounds?

6. If $\frac{5}{8}$ of an acre costs $35, what does 1 acre cost?

7. If $\frac{7}{8}$ of a roll of carpeting costs $84, what is the cost of the whole roll?

8. A dealer sold 12 yd. of cloth, which was $\frac{4}{15}$ of the whole piece. How many yards were there in the piece?

9. A dealer sold $28\frac{1}{2}$ yd. of cloth, which was $\frac{2}{3}$ of the whole piece. How many yards were there in the piece?

10. $75 is $\frac{1}{4}$ more than (that is, $\frac{5}{4}$ of) what sum of money?

11. $75 is $\frac{1}{4}$ less than (that is, $\frac{3}{4}$ of) what sum of money?

12. 119 ft. is $\frac{1}{16}$ more than what length?

13. 155 ft. is $\frac{1}{32}$ less than what length?

14. $1107 is $\frac{1}{8}$ more than what sum of money?

15. $1001 is $\frac{1}{8}$ less than what sum of money?

Given the following whole numbers, which are the indicated fractional parts of certain other numbers, find the other numbers:

16. 48, $\frac{1}{2}$.
17. 48, $\frac{3}{4}$.
18. 56, $\frac{7}{8}$.
19. 72, $\frac{8}{9}$.
20. 96, $\frac{8}{9}$.
21. 99, $\frac{9}{10}$.
22. 27, $\frac{9}{16}$.
23. 102, $\frac{1}{2}$.
24. 102, $\frac{3}{4}$.
25. 120, $\frac{3}{8}$.
26. 125, $\frac{5}{6}$.
27. 130, $\frac{5}{8}$.
28. 140, $\frac{7}{8}$.
29. 168, $\frac{8}{9}$.
30. 225, $\frac{5}{6}$.
31. 222, $\frac{6}{7}$.
32. 235, $\frac{5}{9}$.
33. 333, $\frac{9}{10}$.
34. 1200, $\frac{6}{7}$.
35. 1440, $\frac{12}{13}$.
36. 1760, $\frac{16}{25}$.

37. If 45 qt. will fill 48 bottles, each bottle requires what part of a quart?

38. What part of $8\frac{3}{4}$ qt. of sirup will it take to fill a bottle containing $\frac{7}{8}$ qt.?

39. What part of $9\frac{5}{16}$ qt. is a cupful, the cup holding $\frac{3}{16}$ qt.?

40. What part of 28 in. is $\frac{1}{2}$ in.? $\frac{1}{4}$ in.? $\frac{3}{4}$ in.? $\frac{7}{8}$ in.? 7 in.? $5\frac{3}{8}$ in.? $4\frac{2}{3}$ in.?

41. What part of $7\frac{1}{4}$ in. is $1\frac{7}{8}$ in.?

42. What part of 675 lb. of butter is used to fill a jar containing $18\frac{3}{4}$ lb.?

43. A man is traveling $9\frac{3}{4}$ mi. What part of the distance has he covered when he has gone $3\frac{1}{4}$ mi.?

44. If a man had to travel 69 mi. and has already traveled $7\frac{2}{3}$ mi., what part of his journey is finished?

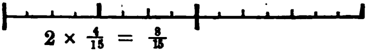
45. A boy had 50 mi. to ride on his bicycle. He has already ridden $6\frac{3}{4}$ mi. What part of his journey is finished?

46. A figure having a certain number of equal sides has a perimeter of $34\frac{4}{5}$ in. Each side is $5\frac{4}{5}$ in. long. This is what part of the perimeter? How many sides has the figure?

215. Multiplying and Dividing the Terms of a Fraction. The following important principles should now be reviewed:

Multiplying the numerator by any number multiplies the fraction by that number.

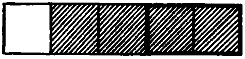
That is, $\frac{2 \times 4}{15} = 2 \times \frac{4}{15}$.



This is seen by studying the accompanying figure.

Dividing the numerator by any number divides the fraction by that number.

That is, $\frac{4 \div 2}{5} = \frac{4}{5} \div 2$.

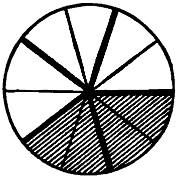


Show by these squares that $\frac{4}{5}$ of the rectangle divided by 2 is equal to $\frac{4 \div 2}{5}$ of the rectangle, or $\frac{2}{5}$ of the rectangle.

Multiplying the denominator by any number divides the fraction by that number.

That is, $\frac{3}{2 \times 5} = \frac{3}{5} \div 2$.

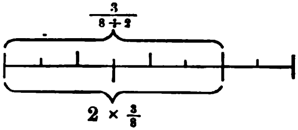
Show by this circle that $\frac{3}{2 \times 5}$, or $\frac{3}{10}$, equals $\frac{3}{5} \div 2$.



Dividing the denominator by any number multiplies the fraction by that number.

That is, $\frac{3}{8 \div 2} = \frac{3}{4} = 2 \times \frac{3}{8}$.

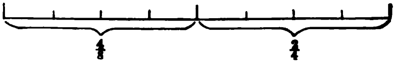
Show by this line that $\frac{3}{8 \div 2}$, or $\frac{3}{4}$, equals $2 \times \frac{3}{8}$.



Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

That is, $\frac{2}{4} = \frac{2 \times 2}{2 \times 4} = \frac{4}{8}$;

$\frac{4}{8} = \frac{4 \div 2}{8 \div 2} = \frac{2}{4}$, and both $\frac{4}{8}$ and $\frac{2}{4}$ equal $\frac{1}{2}$.



EXERCISE 121

DRILL WORK IN FRACTIONS

Find the sum of:

1. $\frac{1}{2}$, $\frac{3}{8}$.
2. $\frac{1}{4}$, $\frac{3}{16}$.
3. $\frac{1}{4}$, $\frac{5}{16}$.
4. $\frac{1}{4}$, $\frac{9}{16}$.
5. $\frac{3}{4}$, $\frac{13}{16}$.
6. $\frac{1}{8}$, $\frac{3}{16}$.
7. $\frac{3}{8}$, $\frac{7}{16}$.
8. $\frac{3}{8}$, $\frac{15}{16}$.
9. $\frac{5}{8}$, $\frac{15}{16}$.
10. $\frac{7}{8}$, $\frac{13}{16}$.
11. $\frac{1}{2}$, $\frac{7}{32}$.
12. $\frac{1}{2}$, $\frac{15}{32}$.
13. $\frac{1}{4}$, $\frac{25}{32}$.
14. $\frac{3}{4}$, $\frac{31}{32}$.
15. $\frac{3}{8}$, $\frac{5}{32}$.
16. $\frac{3}{4}$, $\frac{7}{24}$.
17. $\frac{3}{4}$, $\frac{17}{24}$.
18. $\frac{5}{8}$, $\frac{5}{24}$.
19. $\frac{7}{8}$, $\frac{23}{24}$.
20. $\frac{5}{16}$, $\frac{1}{64}$.

21. To the sum of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$, add the sum of $\frac{3}{5}$, $\frac{7}{15}$, and $\frac{2}{3}$.

22. To the sum of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{3}{16}$, add the sum of $\frac{1}{2}$, $\frac{7}{8}$, and $\frac{9}{16}$.

Find the sum of:

23. $8\frac{3}{4}$, $5\frac{1}{3}$.
24. $6\frac{3}{4}$, $8\frac{2}{3}$.
25. $2\frac{1}{3}$, $7\frac{1}{8}$.
26. $6\frac{1}{3}$, $5\frac{3}{8}$.
27. $8\frac{2}{3}$, $9\frac{5}{8}$.
28. $7\frac{2}{3}$, $8\frac{7}{8}$.
29. $3\frac{3}{4}$, $5\frac{5}{12}$.
30. $4\frac{3}{8}$, $6\frac{5}{16}$.
31. $62\frac{5}{6}$, $29\frac{3}{5}$.
32. $75\frac{7}{8}$, $62\frac{15}{16}$.
33. $92\frac{5}{6}$, $75\frac{4}{15}$.
34. $73\frac{2}{3}$, $68\frac{7}{15}$.
35. $52\frac{4}{5}$, $75\frac{13}{15}$.
36. $48\frac{9}{10}$, $73\frac{11}{12}$.
37. $52\frac{1}{5}$, $61\frac{7}{20}$.
38. $37\frac{4}{15}$, $32\frac{11}{20}$.
39. $5\frac{4}{5}$, $6\frac{2}{3}$, $4\frac{2}{15}$.
40. $6\frac{7}{8}$, $3\frac{1}{2}$, $5\frac{15}{16}$.
41. $4\frac{1}{3}$, $3\frac{3}{5}$, $5\frac{1}{30}$.
42. $2\frac{7}{8}$, $5\frac{13}{16}$, $6\frac{5}{32}$.
43. $3\frac{5}{6}$, $2\frac{5}{12}$, $6\frac{7}{24}$.
44. $4\frac{7}{8}$, $3\frac{2}{3}$, $5\frac{11}{24}$.
45. $3\frac{1}{16}$, $5\frac{3}{32}$, $7\frac{11}{64}$.
46. $9\frac{2}{15}$, $13\frac{13}{30}$, $15\frac{17}{20}$.

Subtract as indicated:

47. $\frac{3}{4} - \frac{1}{8}$.
48. $\frac{5}{8} - \frac{1}{2}$.
49. $\frac{5}{8} - \frac{3}{16}$.
50. $\frac{7}{8} - \frac{5}{16}$.
51. $\frac{1}{2} - \frac{5}{32}$.
52. $\frac{7}{8} - \frac{5}{32}$.
53. $\frac{15}{32} - \frac{3}{64}$.
54. $32 - 4\frac{7}{8}$.
55. $63 - 5\frac{3}{4}$.
56. $72 - 8\frac{9}{10}$.
57. $48 - 3\frac{7}{32}$.
58. $67 - 5\frac{3}{16}$.
59. $71 - 6\frac{5}{24}$.
60. $80 - 9\frac{5}{48}$.
61. $9\frac{7}{16} - 7\frac{3}{8}$.
62. $8\frac{9}{16} - 3\frac{3}{8}$.
63. $6\frac{11}{16} - 4\frac{5}{8}$.
64. $7\frac{15}{16} - 3\frac{7}{8}$.
65. $9\frac{31}{32} - 6\frac{3}{4}$.
66. $8\frac{29}{32} - 5\frac{3}{8}$.
67. $23\frac{1}{8} - 8\frac{3}{16}$.

Find the value of:

68. $9\frac{3}{16} - 7\frac{11}{32}$.

69. $42\frac{5}{6} - 8\frac{5}{12}$.

70. $39\frac{3}{5} - 2\frac{1}{12}$.

71. $48\frac{4}{5} - 6\frac{5}{12}$.

72. $73\frac{1}{3} - 17\frac{7}{12}$.

73. $69\frac{2}{3} - 48\frac{3}{16}$.

74. $75\frac{1}{16} + \frac{3}{16} - 9\frac{3}{32}$.

75. $\frac{1}{2} + \frac{3}{4} + \frac{1}{4} + \frac{1}{2} - \frac{1}{3}$.

76. $\frac{2}{3} + \frac{3}{5} + \frac{1}{3} + \frac{4}{5} - \frac{1}{2}$.

77. $\frac{3}{4} + \frac{7}{8} - \frac{2}{3}$.

78. $\frac{4}{5} + \frac{5}{8} - \frac{3}{4}$.

79. $\frac{3}{16} + \frac{7}{8} - \frac{3}{4}$.

80. $\frac{1}{2} + \frac{7}{8} - \frac{1}{4} - 1$.

81. $\frac{7}{8} + \frac{3}{4} - \frac{1}{2} - 1$.

82. $\frac{3}{5} + \frac{1}{15} + 2 - \frac{1}{3}$.

83. $\frac{2}{3} + \frac{1}{6} + 5\frac{1}{2} - \frac{1}{2}$.

84. $20 + 1\frac{3}{5} - 2\frac{1}{3} - 6$.

85. $30 - 3\frac{3}{4} - 1\frac{7}{8} - 5$.

86. If we add $12\frac{1}{2}$ to $16\frac{2}{3}$, and then subtract $9\frac{1}{6}$ from the sum, what is the result?

Find the value of:

87. $\frac{5}{8}$ of 68.

88. $\frac{7}{8}$ of 96.

89. $\frac{5}{8}$ of 90.

90. $\frac{11}{16}$ of 110.

91. $\frac{13}{16}$ of 120.

92. $\frac{15}{16}$ of 240.

93. $\frac{5}{64}$ of 144.

94. $\frac{7}{64}$ of 128.

95. $\frac{15}{64}$ of 192.

Multiply as indicated:

96. $75 \times \frac{4}{5}$.

97. $64 \times \frac{7}{16}$.

98. $96 \times \frac{5}{12}$.

99. $81 \times \frac{5}{9}$.

100. $\frac{3}{4}$ of $\frac{3}{4}$.

101. $\frac{5}{8}$ of $\frac{3}{5}$.

102. $\frac{7}{16}$ of $\frac{8}{9}$.

103. $\frac{7}{15}$ of $\frac{3}{20}$.

104. $\frac{3}{32}$ of $\frac{5}{9}$.

105. $\frac{5}{32}$ of $\frac{3}{10}$.

106. $\frac{3}{64}$ of $\frac{8}{9}$.

107. $\frac{5}{48}$ of $\frac{12}{25}$.

Find the following products:

108. $5\frac{3}{5} \times 6\frac{7}{8}$.

109. $6\frac{2}{7} \times 7\frac{7}{8}$.

110. $6\frac{1}{8} \times 9\frac{5}{7}$.

111. $7\frac{3}{5} \times 7\frac{7}{8}$.

112. $9\frac{2}{3} \times 8\frac{5}{8}$.

113. $52 \times 607\frac{3}{4}$.

114. $7\frac{3}{4} \times 63\frac{7}{8}$.

115. $4\frac{2}{9} \times 28\frac{1}{8}$.

116. $8\frac{5}{7} \times 31\frac{1}{16}$.

117. $5\frac{3}{7} \times 48\frac{11}{16}$.

118. $4\frac{4}{5} \times 51\frac{9}{16}$.

119. $28 \times 635\frac{1}{4}$.

120. $63 \times 429\frac{3}{7}$.

121. $48\frac{2}{3} \times 48\frac{3}{16}$.

122. $63\frac{1}{5} \times 127\frac{1}{32}$.

123. $35\frac{1}{5} \times 228\frac{9}{32}$.

124. $61\frac{1}{24} \times 240\frac{24}{25}$.

125. $37\frac{7}{24} \times 480\frac{48}{100}$.

Perform the divisions indicated:

126. $\frac{4}{5} \div \frac{2}{3}$.

127. $\frac{2}{3} \div \frac{3}{7}$.

128. $\frac{6}{7} \div \frac{3}{4}$.

129. $\frac{5}{7} \div 2\frac{1}{5}$.

130. $9\frac{2}{3} \div 3\frac{1}{3}$.

131. $29\frac{1}{5} \div \frac{1}{3}$.

132. $4\frac{7}{8} \div 2\frac{2}{3}$.

133. $\frac{2}{3} \div \frac{4}{5}$.

134. $\frac{7}{8} \div \frac{4}{5}$.

135. $\frac{3}{4} \div \frac{6}{7}$.

136. $\frac{2}{3} \div 4\frac{2}{3}$.

137. $10\frac{4}{5} \div 3\frac{1}{2}$.

138. $3\frac{9}{10} \div 2\frac{1}{3}$.

139. $4\frac{6}{7} \div 1\frac{3}{14}$.

140. $\frac{3}{7} \div \frac{2}{3}$.

141. $\frac{4}{5} \div \frac{7}{8}$.

142. $\frac{3}{4} \div 5\frac{1}{2}$.

143. $7\frac{6}{7} \div 1\frac{3}{4}$.

144. $5\frac{5}{7} \div 1\frac{1}{7}$.

145. $2\frac{4}{5} \div 3\frac{1}{2}$.

146. $16\frac{2}{7} \div 2\frac{1}{5}$.

Find the value of:

147. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.

148. $\frac{3}{8}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of 1 cu. ft.

149. $\frac{1}{2}$ of 5 times 8 sq. ft.

150. $\frac{7}{8}$ of 4 times 16 sq. in.

151. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of 1 cu. ft.

152. $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of 1 cu. in.

153. $\frac{3}{5}$ of 15 times 48 sq. ft.

154. $\frac{3}{7}$ of 13 times 14 sq. in.

155. Multiply the sum of $\frac{2}{3}$ and $\frac{5}{6}$ by the sum of $\frac{3}{4}$ and $\frac{7}{8}$.

156. Divide 7 by the sum of $\frac{3}{4}$ and $\frac{5}{16}$.

157. Divide the sum of $\frac{4}{9}$ and $\frac{5}{6}$ by 79.

158. Divide the product of $\frac{2}{3}$ and $\frac{4}{5}$ by the product of $\frac{3}{8}$ and $\frac{8}{9}$.

In the following, the second number is what part of the first number?

159. $\frac{22}{7}$, $\frac{6}{11}$.

160. 82, $9\frac{1}{9}$.

161. 243, 27.

162. $16\frac{5}{8}$, $2\frac{3}{8}$.

163. $66\frac{2}{3}$, $33\frac{1}{3}$.

164. $41\frac{7}{8}$, $21\frac{2}{5}$.

165. $33\frac{1}{3}$, $16\frac{2}{3}$.

166. $3\frac{1}{3}$, $1\frac{2}{3}$.

167. $41\frac{3}{5}$, $5\frac{1}{5}$.

168. 100, $37\frac{1}{2}$.

169. $42\frac{1}{2}$, $14\frac{1}{6}$.

170. 225, $37\frac{1}{2}$.

171. $37\frac{1}{2}$, $33\frac{1}{3}$.

172. 427, $47\frac{4}{9}$.

173. 17, $8\frac{1}{2}$.

174. $66\frac{2}{3}$, $16\frac{2}{3}$.

175. $45\frac{3}{5}$, $7\frac{3}{5}$.

176. $62\frac{1}{8}$, $7\frac{7}{8}$.

177. $22\frac{1}{2}$, $16\frac{2}{3}$.

178. 421, $60\frac{1}{7}$.

179. 325, $29\frac{6}{11}$.

216. Practical Uses of Cancellation. We have often seen that cancellation is useful in simplifying the work. It is of particular value in connection with mixed numbers.

What is the product of $\frac{1}{15}$, $45\frac{1}{2}$, $33\frac{1}{3}$, and $16\frac{7}{8}$?

Reducing the mixed numbers to improper fractions, $45\frac{1}{2} = \frac{91}{2}$, $33\frac{1}{3} = \frac{100}{3}$, $16\frac{7}{8} = \frac{135}{8}$. Proceeding as in § 182, and canceling, the product is $1706\frac{1}{4}$.

$$\frac{1 \times 91 \times \overset{25}{\cancel{100}} \times \overset{\overset{3}{\cancel{9}}}{\cancel{135}}}{\cancel{15} \times 2 \times \cancel{3} \times \underset{2}{\cancel{8}}} = \frac{6825}{4} = 1706\frac{1}{4}$$

EXERCISE 122

Multiply:

1. $20\frac{1}{2} \times 32\frac{1}{5} \times 16\frac{7}{8}$.
2. $15\frac{2}{3} \times 16\frac{7}{8} \times 33\frac{1}{3}$.
3. $27\frac{1}{5} \times 24\frac{1}{2} \times 37\frac{1}{2}$.
4. $42\frac{2}{3} \times 66\frac{2}{3} \times 75\frac{1}{16}$.
5. $35\frac{1}{16} \times 22\frac{1}{32} \times 12\frac{4}{5}$.
6. $7\frac{3}{5} \times 2\frac{1}{7} \times 5\frac{1}{4} \times 21\frac{1}{3}$.
7. $2\frac{1}{2} \times 3\frac{3}{4} \times 4\frac{4}{5} \times 5\frac{5}{6}$.
8. $3\frac{1}{2} \times 4\frac{2}{3} \times 5\frac{3}{4} \times 6\frac{4}{5} \times 7\frac{5}{6}$.
9. $8\frac{2}{3} \times 7\frac{1}{2} \times 5\frac{7}{16} \times 1\frac{3}{13}$.
10. $2\frac{2}{3} \times 3\frac{4}{5} \times 5\frac{6}{7} \times 6\frac{7}{8} \times 7\frac{1}{7}$.

11. Divide the product of 4, 7, 19, 21, and 64 by the product of 14, 38, 28, and 3.

12. Divide the product of 3, 9, 15, 38, 42, and 75 by the product of 19, 7, 3, 5, 27, 60, and 8.

13. Divide the product of 2, 3, 4, 5, 6, 7, 8, and 9 by the product of 10, 12, 14, 15, and 16.

14. How many times as much will it cost to transport 12 boxes of merchandise weighing 300 lb. each 200 mi., as to transport 6 boxes weighing 400 lb. each 75 mi., at the same rate per pound for each mile?

15. How many times as much will it cost to transport 3 boxes of merchandise weighing 400 lb. each a given distance, as to transport 2 boxes weighing 300 lb. each the same distance, at the same rate per pound?

16. A bushel contains 2150½ cu. in. The capacity of an orange box, 12 in. by 12 in. by 26⅝ in., is how many cubic inches more than 1¾ bu.?

17. A bushel contains 2150½ cu. in. In shipping Georgia peaches a crate 8 in. by 12½ in. by 22 in. is used. This contains how many cubic inches more than 1 bu.?

18. A bushel contains 2150½ cu. in. In New Jersey a bushel cranberry box is 12 in. by 8¾ in. by 22 in. How does this differ in capacity from the bushel?

19. California apples and pears for local use are shipped in a 50-pound box 22 in. by 10 in. by 12 in. How does this differ in capacity from the bushel of 2150½ cu. in.?

20. A California cantaloupe crate is 38 in. long, 16 in. wide, and 15 in. deep. This is how many cubic inches?

21. The California orange box is sometimes 26½ in. by 11¼ in. by 11¼ in., and sometimes 22 in. by 7¾ in. by 17½ in. Which holds the more, and how many cubic inches more?

22. Dried fruits are shipped in 50-pound or 25-pound boxes. The 50-pound box is 15¼ in. by 9 in. by 9 in. How many cubic inches in the box?

23. A 25-pound box for shipping dried fruit is sometimes 13¾ in. by 9¾ in. by 5¾ in., and sometimes 16 in. by 9 in. by 5 in. Which box holds the more, and how much more?

24. For shipping fruit out of the state, the sizes of some of the boxes used in California are as follows:

Plums,	18½ in. by 11¾ in. by 4 in.
Cherries,	16½ in. by 10⅜ in. by 2½ in.
Pears,	18½ in. by 11¾ in. by 8½ in.
Peaches,	18½ in. by 11¾ in. by 4⅞ in.

Find the number of cubic inches in each box.

EXERCISE 123

DRILL WORK

Add as indicated:

1. $47\frac{1}{2} + 93\frac{1}{3}$.
2. $62\frac{1}{5} + 35\frac{1}{3}$.
3. $67\frac{4}{9} + 29\frac{1}{6}$.
4. $39\frac{9}{10} + 28\frac{3}{4}$.
5. $72\frac{1}{4} + 83\frac{1}{6}$.
6. $85\frac{4}{5} + 16\frac{2}{3}$.
7. $89\frac{3}{7} + 23\frac{1}{2}$.
8. $87\frac{3}{5} + 62\frac{7}{10}$.
9. $29\frac{3}{4} + 47\frac{1}{5}$.
10. $87\frac{5}{6} + 19\frac{2}{3}$.
11. $67\frac{4}{5} + 49\frac{1}{2}$.
12. $37\frac{4}{7} + 18\frac{1}{14}$.

Subtract as indicated:

13. $79\frac{4}{5} - 22\frac{1}{2}$.
14. $92\frac{1}{4} - 37\frac{2}{5}$.
15. $72\frac{2}{5} - 56\frac{1}{8}$.
16. $137\frac{1}{2} - 86\frac{2}{3}$.
17. $30\frac{2}{3} - 19\frac{7}{8}$.
18. $41\frac{3}{8} - 16\frac{5}{6}$.
19. $48\frac{3}{4} - 23\frac{7}{8}$.
20. $82\frac{1}{2} - 68\frac{7}{8}$.
21. $72 - 46\frac{2}{3}$.

Multiply as indicated:

22. $27 \times 14\frac{2}{3}$.
23. $18 \times 96\frac{3}{4}$.
24. $32\frac{2}{5} \times 85$.
25. $12\frac{1}{2} \times 12\frac{1}{2}$.
26. $15\frac{3}{4} \times 8\frac{4}{5}$.
27. $16 \times 417\frac{3}{4}$.
28. $32 \times 92\frac{1}{2}$.
29. $30\frac{1}{2} \times 68$.
30. $41\frac{3}{4} \times 96$.
31. $16\frac{1}{3} \times 2\frac{1}{5}$.
32. $32 \times 35\frac{1}{4}$.
33. $21 \times 220\frac{3}{7}$.
34. $67 \times 74\frac{3}{8}$.
35. $40\frac{1}{3} \times 96$.
36. $75 \times 60\frac{4}{5}$.
37. $3\frac{1}{4} \times 6\frac{4}{5}$.
38. $32 \times 40\frac{3}{4}$.
39. $\frac{2}{3}$ of $\frac{7}{8}$ of $1\frac{5}{7}$.

Find the following:

40. $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$.
41. $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of 1 ft.
42. $\frac{1}{4}$ of 15 times 8 ft.
43. $\frac{3}{8}$ of 9 times 16 in.
44. $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of 2 ft.
45. $\frac{7}{8}$ of $\frac{3}{5}$ of $\frac{2}{3}$ of 5 in.
46. $\frac{4}{5}$ of 25 times 48 ft.
47. $\frac{3}{16}$ of 12 times 24 in.

Find the number of which:

48. 23 is $\frac{1}{3}$.
49. 75 is $\frac{3}{8}$.
50. 222 is $\frac{6}{7}$.
51. \$150 is $\frac{3}{5}$.
52. 38 is $\frac{2}{3}$.
53. 45 is $\frac{9}{10}$.
54. 63 is $\frac{7}{8}$.
55. 125 is $\frac{25}{32}$.
56. \$2.25 is $\frac{5}{8}$.

Divide, using cancellation:

57. 2 × 3 × 6 by 5 × 9 × 4.

58. 3 × 14 × 20 by 15 × 21 × 7.

59. 4 × 15 × 27 by 9 × 5 × 24.

60. 7 × 21 × 77 × 6 by 49 × 33 × 42.

61. 20 × 25 × 32 × 48 by 27 × 40 × 15 × 12.

62. 32 × 27 × 44 × 39 by 55 × 72 × 26 × 8 × 3.

63. 125 × 108 × 51 × 8 by 40 × 45 × 84 × 12 × 3.

Divide as indicated:

64. $\frac{4}{5} \div \frac{2}{5}$.
65. $\frac{3}{14} \div \frac{3}{7}$.
66. $\frac{3}{7} \div \frac{3}{8}$.
67. $\frac{7}{8} \div \frac{2}{3}$.
68. $\frac{5}{12} \div \frac{7}{8}$.
69. $\frac{5}{64} \div \frac{7}{32}$.
70. $\frac{7}{48} \div \frac{7}{12}$.
71. $\frac{2}{3} \div \frac{5}{8}$.
72. $\frac{5}{8} \div \frac{3}{5}$.
73. $\frac{3}{4} \div \frac{4}{9}$.
74. $\frac{5}{6} \div \frac{3}{4}$.
75. $\frac{3}{8} \div \frac{5}{12}$.
76. $\frac{9}{24} \div \frac{5}{16}$.
77. $\frac{3}{5} \div \frac{3}{7}$.
78. $\frac{4}{5} \div \frac{5}{4}$.
79. $\frac{3}{4} \div 2\frac{1}{2}$.
80. $\frac{9}{10} \div \frac{4}{5}$.
81. $\frac{7}{16} \div \frac{3}{32}$.
82. $\frac{3}{40} \div \frac{6}{25}$.
83. $\frac{3}{4} \div 1\frac{1}{16}$.
84. $5\frac{1}{4} \div 19\frac{3}{8}$.
85. $7\frac{3}{10} \div 1\frac{4}{5}$.
86. $1\frac{2}{3} \div 3\frac{2}{3}$.
87. $40\frac{3}{5} \div 33\frac{1}{2}$.
88. $13\frac{9}{10} \div 2\frac{2}{3}$.
89. $14\frac{5}{7} \div 2\frac{3}{7}$.
90. $\frac{4}{7} \div 2\frac{1}{5}$.
91. $6\frac{2}{3} \div 3\frac{1}{3}$.
92. $9\frac{1}{5} \div 1\frac{1}{3}$.
93. $3\frac{7}{8} \div 4\frac{2}{3}$.
94. $4\frac{5}{9} \div 12\frac{1}{3}$.
95. $10\frac{7}{36} \div 9\frac{1}{4}$.
96. $2\frac{7}{16} \div 3\frac{1}{8}$.
97. $3\frac{9}{20} \div 2\frac{3}{10}$.
98. $4\frac{9}{10} \div 12\frac{5}{6}$.
99. $5\frac{6}{7} \div 2\frac{3}{4}$.
100. $15\frac{5}{7} \div 3\frac{2}{7}$.
101. $12\frac{4}{5} \div 2\frac{1}{5}$.
102. $46\frac{3}{7} \div 5\frac{1}{7}$.

103. At 12½¢ each, find the cost of 147 balls.

104. At 16⅔¢ each, find the cost of 114 notebooks.

105. At 33⅓¢ each, find the cost of 102 handkerchiefs.

106. At $1.33⅓ a volume, find the cost of 165 books.

107. At $1.16⅔ a yard, find the cost of 142 yd. of lace.

108. At $1.12½ each, find the cost of 112 yd. of velvet.

EXERCISE 124

Miscellaneous Problems

1. At 32¢ a gallon, what will 12½ gal. of vinegar cost?

In each of the following examples estimate the approximate answer and write it down. After the problem is solved compare the approximate answer with the real answer. This is one of the best checks against absurd results. Thus 3½ × 22¢ is a reasonable approximation for 3⅝ × 22¢ in Ex. 2.

2. At 22¢ a pound, what will 3⅝ lb. of steak cost?

3. At 18¢ a gallon, what will 6¼ gal. of oil cost?

4. At 26¢ a pound, what will 6⅞ lb. of butter cost?

5. At 12½¢ a yard, what will 8¾ yd. of ribbon cost?

6. At $2.50 a yard, what will 7⅝ yd. of velvet cost?

7. If 6 yd. of silk cost $6.75, what is the price per yard?

8. If 8 doz. men's hose cost $55, what is the price per dozen?

9. A man drives 39.6 mi. in 5½ hr. What is the rate per hour?

10. If 48 doz. boys' hose cost $57.60, what is the price per dozen?

11. At 14¢ a gross, how many gross of buttons can be bought for $3.22?

12. A dealer buys 200 lb. of raisins for $19. How much do they cost a pound?

13. A grocer pays $10.08 a gross for canned soup. How much does he pay per can?

14. If 20 electric lights cost 10¢ per hour, how long can we burn 4 such lights for 20¢?

15. A dealer pays $18.83 for 7 rocking chairs. How much does he pay for each?

16. A dealer sells 17 pails of mackerel for $40.80. What is the price per pail?

17. A man pays $1110 a year for the rent of a city house. How much does he pay a month?

18. A grocer buys a 50-gallon barrel of Porto Rico molasses for $11.50. How much does it cost per gallon?

19. A grocer sells a half chest of tea, 140 lb. to the chest, for $19.60. What is the price per pound?

20. A merchant sold a 40-yard piece of cloth for $45, gaining 12½¢ a yard. How much did the piece cost him?

21. A grocer buys 33⅓ doz. 1-pound cans of cocoa at 43¢ each and sells them at 58¢ each. How much does he gain?

22. A dry-goods dealer sold 75 yd. of cloth for $112.50, thereby gaining $28.50. How much did it cost him per yard?

23. If a family uses 3 bbl. of flour in a year, and can buy the flour at $5.50 a barrel or at 3¢ a pound, how much will be saved by purchasing by the barrel? (Allow 196 lb. to the barrel.)

24. If a family uses 1¼ gal. of kerosene in 5 weeks, and the kerosene costs 15¢ a gallon, how much will be used in 52 weeks, and how much will it all cost?

25. If gas costs $1 per 1000 cu. ft., and a family uses 2320 cu. ft. in October and 2475 cu. ft. in November, how much is the gas bill for the two months?

26. At the beginning of the month the gas meter registers 14,260, and at the end of the month 17,140. How much is the gas bill for the month, at $1.25 per 1000 cu. ft.?

27. A grocer buys six 50-pound boxes of dried apples at 4½¢ a pound and sells them at 6¢ a pound. How much does he gain in all? What fractional part of the cost does he gain?

28. A salesman receives $15 a week. He works 49 weeks a year. His expenses are $396 a year. How much does he save a year? What fractional part of his income does he save?

29. I bought 6 lb. of prunes @ 12½¢, 8 lb. of dried apricots @ 12½¢, 2½ lb. of raisins @ 16¢, 1½ lb. of figs @ 22¢, and ½ lb. of English walnuts @ 18¢, and gave the grocer a $5 bill. How much change should I receive?

30. If 1 cu. ft. of water weighs 62 lb. 8 oz., what part of 1 oz. does 1 cu. in. of water weigh?

31. A gallon being 231 cu. in., what does 1 gal. of water weigh? 1 qt. of water?

32. A certain kind of stone is 2⅞ times as heavy as water. What does 2½ cu. ft. of this stone weigh?

33. Another kind of stone is 3⅛ times as heavy as water. What does 3¾ cu. ft. of this stone weigh?

34. If good milk is 1.03 times as heavy as water, what is the weight of a can of such milk containing 3½ cu. ft., the can itself weighing 6 lb. 4 oz.?

35. Ice is 0.92 as heavy as water. How much does 1 cu. ft. weigh? How much will a block of ice weigh that is 3½ ft. long, 2½ ft. wide, and 14 in. thick?

36. A water tank for fire purposes is placed on top of a factory. It is 16½ ft. long, 8¼ ft. wide, and 4¾ ft. deep. When filled within 2 in. of the top, what is the weight of the water?

37. I bought 8 cans of peaches @ $3.75 a dozen, 15 cans of tomatoes @ $1.37½ a dozen, 9 cans of soup @ $1.50 a dozen, 3½ lb. of raisins @ 18¢ a pound, 2½ lb. of nuts @ 17¢ a pound, 1½ gal. of sirup @ 60¢ a gallon, 1½ doz. cakes of soap @ 42¢ a dozen. How much was my bill? Write the bill.

38. If a boy rides 23⅜ mi. on a bicycle in 2¾ hr., what is his rate per hour?

39. In buying 2½ doz. cans of peaches, how much is gained by buying @ $3.74 a dozen over buying at the rate of 3 cans for a dollar?

40. What is the cost of 6¼ lb. of mackerel @ 16¢, 3½ lb. of codfish @ 12¢, and 4 cans of salmon at the rate of 2 cans for a quarter?

41. I bought ½ gal. of olive oil @ $3.30 a gallon, but ¼ of what I bought leaked out. What rate per gallon do I pay for what is left?

42. In buying 48 cans of tomatoes, how much is gained by buying two cases of 2 doz. each, @ $2.89 a case, over buying at the rate of 3 cans for 40¢?

43. I bought 3 gal. of best maple sirup @ $1.12½ a gallon, and 2½ gal. of New Orleans molasses @ 62¢ a gallon, and gave the grocer a $5 bill. How much change did I receive? (Neglect fractions less than ½¢, although in a bill a fraction of a cent is usually called a cent.)

44. I bought 8 lb. of prunes @ 12½¢, 4 lb. of figs @ 19¢, 2½ lb. of candles @ 12½¢, 12 lb. of sugar @ 6½¢, and 2½ lb. of English walnuts @ 18¢, and gave the grocer a $5 bill. How much change did I receive?

45. I bought 10 lb. of white beans @ 6¼¢, 3½ lb. of split peas @ 7¢, 3 cans of soup @ 12½¢, and 2½ lb. of paraffin candles @ 12½¢. How much was my bill?

46. If we buy 12 lb. of sugar at 5¾¢, 3½ lb. of tea at 60¢, and 12 lb. of coffee at 37½¢, how much is the bill?

47. If 12 cups of flour make 4 loaves of bread, how many loaves will a barrel of flour make? Allow 3 cups of flour to the pound, and 196 lb. of flour to the barrel.

48. If a recipe will provide enough for 5 persons, and you wish to provide for 8 persons, what part must you add to the recipe? Instead of 10 teaspoons you would need how many?

49. Providing 1½ potatoes for each person at a meal, how many potatoes would you need for 8 persons?

50. If there are 40 potatoes in a peck, and you use 12 of them, what fractional part of a peck do you use?

51. If a family uses ⅖ of a peck of potatoes in one day, how many pecks will it use in a month of 30 days?

52. Allowing 4 cups to the quart, a quart of cranberries divided among 16 pupils in a class would allow what part of a cup to each member?

53. Cranberries are cooked with half as many cups of sugar as of berries, and half as many cups of water as of sugar. If you wish to cook 3 pt. of cranberries, how much sugar will you use? How much water?

54. If flour costs $6.25 a barrel (196 lb.), or 3½¢ a pound when bought by the pound, how much is saved on 196 lb. of flour in buying by the barrel?

55. If a family uses 12 cups of flour in each baking of bread, and there are 2 bakings a week, and we allow ¼ lb. to the cup, how many pounds of flour would be used in a year?

56. If we pay 25¢ for a box of domino sugar weighing 3½ lb., how much do we pay per pound?

57. A gallon of water contains 231 cu. in., and 1 cu. ft. of water weighs 62½ lb. If the water in a tank weighs 14,437½ lb., how many gallons are there?

On pages 226–230 many industrial problems are included, requiring the use of denominate numbers and fractions, as suggested by the New York State Syllabus. If further work of this nature is needed at this time, pages 99–142 may be reviewed in whole or in part.

CHAPTER XI

PERCENTAGE

217. Per Cent. Another name for hundredths is *per cent.*

Thus 0.01, or $\frac{1}{100}$, is the same as 1 per cent; $0.00\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{100}$, is the same as $\frac{1}{2}$ per cent; 0.06 is the same as 6 per cent.

That part of arithmetic which treats of per cent is called *percentage.*

218. Symbol for Per Cent. The symbol for per cent is written thus: %.

We may read 0.06 either "6 hundredths" or "6 per cent." In the same way, we may think of 6% as either "6 per cent" or "6 hundredths," although it is read "6 per cent."

The expression 800% means $\frac{800}{100}$ and equals the whole number 8; 225% equals the mixed number 2.25, or $2\frac{1}{4}$; $\frac{1}{2}\%$ means $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$, and is read either "$\frac{1}{2}$ per cent" or, quite commonly, "$\frac{1}{2}$ of 1%."

219. Relation to Fractions. Since 6% means $\frac{6}{100}$, which equals 0.06, or $\frac{3}{50}$, we see that we may express per cent as a decimal fraction or as a common fraction.

It is often convenient to use one form, and often another. Thus, if we are multiplying by 13.5%, it is more convenient to think of the multiplier as 0.135; but if we are multiplying by $33\frac{1}{3}\%$, it is better to think of it as $\frac{1}{3}$ instead of $0.33\frac{1}{3}$.

220. Per Cents as Common Fractions. Since $62\frac{1}{2}\% = \dfrac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}$, therefore

To express per cent as a common fraction, write the number indicating the per cent for the numerator and 100 for the denominator, and reduce this fraction to lowest terms.

For example, 50% means 0.50, or $\frac{50}{100}$, and this equals $\frac{1}{2}$. Likewise, 25% means 0.25, or $\frac{25}{100}$, and this equals $\frac{1}{4}$.

EXERCISE 125

Reduce to a common fraction, integer, or mixed number:

1. 10%.
2. 20%.
3. 30%.
4. 40%.
5. 50%.
6. 4%.
7. 5%.
8. 6%.
9. 8%.
10. 9%.
11. 100%.
12. 200%.
13. 300%.
14. 150%.
15. 275%.
16. 33⅓%.
17. 37½%.
18. 66⅔%.
19. 62½%.
20. 16⅔%.

21. How much is ⅕ of 35? 20% of 35? 20% of 350?
22. How much is ¼ of 40? 25% of 40? 25% of 400?
23. How much is ½ of 66? 50% of 66? 50% of 660?
24. How much is ¾ of 48? 75% of 48? 75% of 480?
25. How much is ⅘ of 90? 80% of 90? 80% of 900?

221. Per Cents as Decimals. Since 25.5% and 0.255 have the same value, therefore

To express as a decimal a number written with the per cent sign, omit the sign and move the decimal point two places to the left.

Thus 1½% = 0.01½, or 0.015; 125% = 1.25; 0.6% = 0.006.

EXERCISE 126

Express as decimals:

1. 25%.
2. 30%.
3. 50%.
4. 75%.
5. 80%.
6. 12½%.
7. 33⅓%.
8. 37½%.
9. 66⅔%.
10. 87½%.
11. 125%.
12. 150%.
13. 175%.
14. 250%.
15. 375%.
16. 37.5%.
17. 16.5%.
18. 34.8%.
19. 62.5%.
20. 78.3%.

21. Express ½ as a decimal. Express ½% as a decimal.
22. Express ¼ as a decimal. Express ¼% as a decimal.

222. Decimals as Per Cents. Since per cent means hundredths, to express a decimal as per cent we have to consider only how many hundredths the decimal represents.

Thus $0.3 = 0.30 = 30$ hundredths $= 30\%$.
$0.375 = 0.37\frac{1}{2} = 37\frac{1}{2}$ hundredths $= 37\frac{1}{2}\%$, or 37.5%.
$1.25 = \frac{125}{100} = 125\%$.

Therefore, *to express a decimal as per cent, write the per cent sign after the number of hundredths.*

EXERCISE 127

Express as per cents:

1. 0.25. 3. 0.5. 5. 1.50. 7. 0.375. 9. $0.3\frac{1}{3}$.
2. 0.75. 4. 0.8. 6. 2.5. 8. 0.625. 10. $0.6\frac{2}{3}$.

11. A pint is what per cent of a quart? of a gallon?

12. A dime is what per cent of a dollar? of a cent?

223. Common Fractions as Per Cents. Since per cent means hundredths, therefore

To express a common fraction as per cent, reduce it to hundredths, omit the denominator, and write the numerator followed by the per cent sign.

This is easily done by reducing the common fraction to a decimal and proceeding as in § 222.

Thus $\frac{4}{5} = 0.80 = 80\%$; $\frac{5}{4} = 1\frac{1}{4} = 1.25 = 125\%$.

EXERCISE 128

Express as per cents:

1. $\frac{1}{2}$. 3. $\frac{3}{4}$. 5. $\frac{3}{5}$. 7. $\frac{3}{8}$. 9. $3\frac{3}{4}$.
2. $\frac{1}{4}$. 4. $\frac{1}{5}$. 6. $\frac{1}{8}$. 8. $2\frac{5}{8}$. 10. $6\frac{7}{8}$.

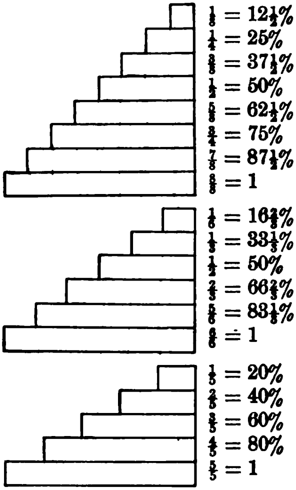
11. A foot is what per cent of a yard? of 2 yards?

12. An ounce is what per cent of a pound? of 2 pounds?

224. Important Per Cents. Certain per cents are used so frequently that their equivalent common fractions should be remembered. These are as follows:

$50\% = \frac{1}{2}$
$25\% = \frac{1}{4}$
$75\% = \frac{3}{4}$
$12\frac{1}{2}\% = \frac{1}{8}$
$6\frac{1}{4}\% = \frac{1}{16}$
$3\frac{1}{8}\% = \frac{1}{32}$
$37\frac{1}{2}\% = \frac{3}{8}$
$62\frac{1}{2}\% = \frac{5}{8}$
$87\frac{1}{2}\% = \frac{7}{8}$
$16\frac{2}{3}\% = \frac{1}{6}$
$33\frac{1}{3}\% = \frac{1}{3}$
$66\frac{2}{3}\% = \frac{2}{3}$

$83\frac{1}{3}\% = \frac{5}{6}$
$20\% = \frac{1}{5}$
$40\% = \frac{2}{5}$
$60\% = \frac{3}{5}$
$80\% = \frac{4}{5}$
$5\% = \frac{1}{20}$
$8\frac{1}{3}\% = \frac{1}{12}$
$10\% = \frac{1}{10}$
$30\% = \frac{3}{10}$
$70\% = \frac{7}{10}$
$90\% = \frac{9}{10}$
$100\% = 1$



To take $87\frac{1}{2}\%$ of 648 is, therefore, the same as to take $\frac{7}{8}$ of 648.

The similarity of these per cents to the corresponding decimal fractions given on page 87 should be noted.

EXERCISE 129

Find:

1. 50% of $274.
2. 25% of $372.
3. $12\frac{1}{2}\%$ of $512.
4. $6\frac{1}{4}\%$ of $3376.
5. $37\frac{1}{2}\%$ of $17.76.
6. $62\frac{1}{2}\%$ of $11.92.
7. $87\frac{1}{2}\%$ of $34.48.
8. $3\frac{1}{8}\%$ of $739.20.
9. $66\frac{2}{3}\%$ of $80.07.
10. $83\frac{1}{3}\%$ of $71.10.
11. 20% of $735.15.
12. 40% of $611.25.
13. 60% of $734.15.
14. $112\frac{1}{2}\%$ of 72; of 88.
15. $137\frac{1}{2}\%$ of 96; of 88.
16. $116\frac{2}{3}\%$ of 72; of 96.

17. How much is $\frac{1}{8}$ of $2640? 12½% of $2640?

18. How much is $\frac{3}{8}$ of $3360? 37½% of $3360?

19. How much is $\frac{5}{8}$ of $4320? 62½% of $4320?

20. How much is $\frac{7}{8}$ of $5768? 87½% of $5768?

21. An inch is what fraction of a foot? what per cent of a foot?

22. A pint is what fraction of a gallon? what per cent of a gallon?

23. Five cents is what fraction of a dollar? what per cent of a dollar?

24. If you wish to find 33⅓% of a number, what fractional part of the number do you wish to find?

25. If a man spends one fifth of his income for rent, what per cent does he spend for this purpose?

26. What per cent of the perimeter is one side of a square? What per cent are two sides? three sides?

27. If there are 30 pupils in a class and 50% are boys, what fraction are boys? How many boys are there?

28. If there are 450 pupils in a school and 52% are boys, what decimal fraction of the pupils are boys? How many boys are there? How many girls are there?

29. If a city has a population of 18,400, and 22½% are pupils in school, what decimal fraction of the population is in school? How many are in school?

30. If a shop employs 1680 workers, of whom 37½% are men, 50% are women, and the rest are boys, how many are there of each?

31. A merchant has $9600 invested in his business. The first year he made a profit of 6¼%, the second year 12½%, and the third year 16⅔%. How much did he make in the three years?

225. Terms used in Percentage. The number of which some per cent is to be taken is called the *base*.

The number of hundredths of the base is called the *rate*.

For example, in 25% of $300, $300 is the base and 25% is the rate.

Sometimes 25 is called the *rate per cent*, 25% being called the *rate*, but these two terms are commonly used to mean the same thing.

The result found by taking a certain per cent of the base is called the *percentage*.

Therefore, *the percentage is the product of the base and the rate.*

226. To find Some Per Cent of a Number. Required to find 23¾% of 275.

$$23\tfrac{3}{4}\% = 0.23\tfrac{3}{4}.$$

Multiplying by 0.23¾, we have 65.31¼.

If we wish, we may write 23¾% as 0.2375 and then multiply. We may also, if we choose, write the product 65.3125.

$$\begin{array}{r} 275 \\ \underline{.23\tfrac{3}{4}} \\ 206\tfrac{1}{4} \\ 825 \\ \underline{550} \\ 65.31\tfrac{1}{4} \end{array}$$

Therefore, *to find a required per cent of a number, multiply the number by the given rate.*

EXERCISE 130

Find the following:

1. 10% of 80.
2. 25% of 48.
3. 75% of 96.
4. 15% of 60.
5. 40% of 40.
6. 6% of $75.
7. 4% of $50.
8. 5% of $80.
9. 3% of $90.
10. 4½% of $80.
11. 33⅓% of 66.
12. 66⅔% of 99.
13. 37½% of 48.
14. 62½% of 168.
15. 16⅔% of 360.

16. In a school of 300 pupils 11% are in the sixth grade. How many pupils are in the sixth grade?

17. In a box of 192 oranges 6¼% have spoiled. How many have spoiled?

18. From a barrel of flour weighing 196 lb., 25% has been sold. How many pounds have been sold?

19. A contractor agreed to build a stone wall 72 ft. long. He has already built 20% of it. How many feet has he still to build?

20. A trolley car has a run of 56 mi. a day. When it has run 15% of this distance, how many miles has it run?

21. How much will 48% of a cubic foot of steel weigh if a cubic foot weighs 490 lb.?

22. If a steel car when full carries 96,000 lb. of coal, how much does it carry when it is 62½% full?

23. If out of 240 problems in arithmetic you solved 93¾% without any errors, how many did you solve correctly?

24. If a locomotive weighing 116 tons can exert a pull equal to 22% of its weight, how many tons of pull can it exert?

25. A ball team has played 32 games this season and has lost 37½% of them. How many games has it won?

26. A cubic foot of water weighs 62½ lb. Ice is 92% as heavy ~~as water~~. What does 25¾ cu. ft. of ice weigh?

27. The diameter of a circle is about 31⅚% as long as the circumference. What is the diameter of a circle whose circumference is 15 ft.?

28. A meter is 9 13/36% longer than a yard. How many inches in a meter?

29. A knot, used at sea, is 15⅕% longer than a mile. How many feet are there in a knot?

30. A sidewalk 112 ft. long and 4 ft. 2 in. wide is to be made 12½% longer and 20% wider. How many feet will it be extended? How much wider will it be? By how many square feet will its area be increased?

227. The Per Cent One Number is of Another. Because the percentage is the product of the base and the rate (§ 225), therefore

The rate equals the percentage divided by the base.

For example, what per cent of 5 is 4?

Since 4 is the product of 5 by some per cent, therefore 4 ÷ 5, or 80%, equals the required per cent.

EXERCISE 131

Find what per cent the second number is of the first:

1. 5, 3.
2. 10, 5.
3. 12, 4.
4. 25, 5.
5. 75, 15.
6. 100, 6.
7. 150, 25.
8. 175, 25.
9. 225, 75.
10. 250, 50.
11. $125, $6.25.
12. $150, $9.00.
13. $180, $9.00.
14. $250, $7.50.
15. $475, $23.75.

16. A man bought a horse for $150 and sold it for $175. How much did he gain? What per cent of the cost did he gain?

17. A man bought a farm for $6500 and sold it for $7150. How much did he gain? What per cent of the cost did he gain?

18. A boy weighed 75 lb. a year ago and he now weighs 82½ lb. What per cent of 75 lb. has he gained?

19. A village had a population of 1200 five years ago. It now has a population of 1332. What has been the per cent of increase?

20. A poultry raiser has 231 chickens this year. Last year he had 220 chickens. What is the per cent of gain this year? The year before he had 200 chickens. What was the per cent of gain last year?

21. A man saved $750 more this year than last year. Last year he saved $2000. What is the per cent of increase?

22. A man weighed 160 lb. in September and 10 lb. more the next April. What per cent had he gained on his weight in September?

23. A farm of 300 acres has 48 acres of woodland. What per cent of the farm is woodland?

24. In a certain school 22 of the pupils are in the sixth grade. There are 275 pupils in the school. What per cent are in the sixth grade?

25. A contractor has agreed to erect a building for $48,000. He has been paid $18,000 on account. What per cent of the contract price has he been paid?

26. In an arithmetic class it is planned to solve 550 problems in a certain number of days. All but 44 have been solved. What per cent of the problems remain to be solved? What per cent of them have been solved?

27. A baseball team has won 32 games and lost 28 games. What per cent of the total number of games has it won? What per cent has it lost?

28. A cubic foot of ice weighs 57½ lb. A block of ice containing 4 cu. ft. loses 28¾ lb. by melting. What per cent does it lose?

29. A cubic foot of water weighs 62½ lb. From a tank containing 16 cu. ft. there is drawn off an amount of water that would weigh 125 lb. What per cent of the water is drawn off?

30. To a class of 32 pupils 8 more are admitted. What per cent of the original number are admitted? What per cent of the present number are admitted? If 10 pupils are now taken out, what per cent are taken out?

228. One Number is a Given Per Cent of what Number? Because the percentage is the product of the base and the rate (§ 225), therefore

The base equals the percentage divided by the rate.

For example, if 15% of a number is 390, what is the number?

$$390 \div 15\% = 39{,}000 \div 15 = 2600. \ \textit{Ans.}$$

The simple equation may be used in cases of this kind, it being explained that $2x$ means 2 times some number that is to be found.

For example, $0.15x = 390.$

Therefore $x = 2600,$

by dividing these equals by 0.15

EXERCISE 132

Of what numbers are the following the given per cents?

1. 720, 12%.
2. 300, 15%.
3. 690, 23%.
4. 175, 25%.
5. 360, 75%.
6. 480, 12½%.
7. $84, 6%.
8. $75, 5%.
9. $88, 4%.

10. What is the number of which $75 is 125%? That is, what number increased by 25% equals $75?

11. What number increased by 33⅓% equals $480?

12. What number increased by 50% equals $600?

13. $260 is 4% more than (or 104% of) what number?

14. $420 is 5% more than what number?

15. $375 is 75% of what number? That is, what number decreased by 25% equals $375?

16. What number decreased by 25% equals $900?

17. $435 is 6% of what sum of money?

18. What number increased by 33⅓% equals $500?

19. A city increased 14% in population in ten years. The increase was 3332. What was the population before this increase?

20. A city increased 16% in population in ten years. The increase was 5200. What was the population both before and after this increase?

21. The weekly pay roll in a certain shop was increased 8%. The increase amounted to $192. What was the pay roll before the increase?

22. The attendance at a certain school has increased 14, which is 5% of the attendance last year. What was the attendance last year?

23. A lady paid 95¢ less for groceries this week than last week. This was a decrease of 10%. How much did she pay last week? this week?

24. A man purchased this year 2½ tons of coal less than he did last year, a reduction of 12½%. How many tons did he purchase last year? this year?

25. In a certain month there were seven stormy days, or 25% of the total number of days. What month was it?

26. A man sold a farm at a profit of 16⅔% on the cost, thereby making $540. What did the farm cost him?

27. A man has a meadow to mow. After he has mowed 24 acres, 37½% of the work is done. How many acres has he then to mow?

28. A man buying a farm is told that if he will pay cash he can get it for 6¼% less, and can thereby save $500. What would the farm then cost him?

29. A state is building a certain number of miles of road this year. It has already finished 33⅓% of it, or 280 miles. How many miles ~~is~~ it to build this year?

EXERCISE 133

PROBLEMS WITHOUT NUMBERS

1. How do you find some required per cent of a number?

2. How do you express per cent as a decimal fraction?

3. How do you express a decimal fraction as per cent?

4. How do you express per cent as a common fraction?

5. How do you express a common fraction as per cent?

6. Name eight of the most important per cents, and the corresponding common fractions.

7. If you know the base and rate, how do you find the percentage?

8. How do you find what per cent one number is of another?

9. How do you find the number of which a given number is a certain per cent?

10. If you know your weight a year ago and also to-day, how do you find the gain? the per cent of gain?

11. If you know how many pounds you have gained, and the per cent, how do you find your weight a year ago?

12. Knowing how much a man paid for his farm, and the amount for which he is selling it, how can you find the per cent of gain or loss?

13. Knowing that a farmer paid a certain amount for a team of horses, and sold them at a certain per cent of profit, how do you find the selling price?

14. If a boy had a kite string of a certain length, and lost a certain per cent of it, how do you find the amount left?

15. If a boy had a fishing line of a certain length, and you know how many feet of it he lost, how do you find the per cent of loss?

CHAPTER XII

PROFIT AND LOSS

229. Profit and Loss. In business transactions profit and loss are often computed as certain per cents of the cost of the property. The following may be taken as typical problems in profit and loss, including the marking of goods:

(1) At what price must a merchant mark goods that cost $275, so as to gain 20%?

If he gains 20% of $275 he gains $\frac{1}{5}$ of $275, or $55. Therefore he must mark the goods $275 + $55, or $330.

(2) If goods marked $75 are sold at a bargain sale for 15% off the marked price, at what price are they sold?

If the goods are sold at 15% off the marked price, they are sold for $75 less 15% of $75, or $75 – $11.25, or $63.75.

(3) If some goods are damaged so that they are marked down to $120, or 25% below cost, how much did they cost?

100% of the cost = the cost.
25% of the cost = the loss.
75% of the cost = the selling price = $120.
1% of the cost = $\frac{1}{75}$ of $120.
100% of the cost = $\frac{100}{75}$ (or $\frac{4}{3}$) of $120, or $160.

(4) A dealer sold a hat that cost $2.40 so as to gain 25%. The selling price was 20% less than the marked price. What was the marked price?

He gained 25% (or $\frac{1}{4}$) of $2.40, or $0.60. Therefore the selling price was $2.40 + $0.60, or $3. Now show that

80% of the marked price = the selling price = $3.
1% of the marked price = $\frac{1}{80}$ of $3.
100% of the marked price = $\frac{100}{80}$ (or $\frac{5}{4}$) of $3, or $3.75.

EXERCISE 134

For how much must a merchant sell goods that he bought at the price here given, so as to gain the per cent specified?

1. $30, 10%.
2. $45, 12%.
3. $75, 15%.
4. $25, 12%.
5. $36, 14%.
6. $65, 15%.
7. $72, 18%.
8. $85, 25%.
9. $240, 12½%.
10. $375, 15%.
11. $675, 16%.
12. $175, 8½%.
13. $190, 9¼%.
14. $250, 12%.
15. $375, 15%.
16. $735, 18%.
17. $480, 16⅔%.
18. $540, 33⅓%.
19. $720, 12½%.
20. $172.80, 12½%.
21. $373.20, 16⅔%.
22. $45,550, 17½%.
23. $27,475, 22%.
24. $75,250, 25%.

If goods marked as here shown are sold at a bargain sale for the given per cent off, find the selling price:

25. $60, 15%.
26. $85, 18%.
27. $72, 22%.
28. $75, 10%.
29. $65, 12%.
30. $96, 15%.
31. $176, 6¼%.
32. $712, 12½%.
33. $972, 16⅔%.
34. $720, 6¼%.
35. $840, 12½%.
36. $636, 16⅔%.
37. $975, 20%.
38. $960, 25%.
39. $981, 33⅓%.
40. $225.40, 25%.
41. $27,250, 20%.
42. $22,400, 17½%.

If goods are sold for the sum specified, which is the given per cent below cost, find the cost:

43. $71.25, 5%.
44. $72.16, 12%.
45. $58.48, 14%.
46. $66.75, 11%.
47. $72.98, 11%.
48. $140, 20%.
49. $726, 12%.
50. $735, 12½%.
51. $506, 12%.
52. $382.50, 15%.
53. $848.40, 12½%.
54. $736.89, 12½%.
55. $207.20, 30%.
56. $265.60, 17%.
57. $358.80, 22%.
58. $568.80, 21%.
59. $226.20, 13%.
60. $11,780, 20%.
61. $12,300, 18%.
62. $14,025, 15%.
63. $14,700, 12½%.

Given the cost and rate of gain as follows, find the amount of gain:

64. $275, 15%.
65. $365, 12½%.
66. $4723, 14%.
67. $3965, 16%.
68. $7287, 22%.
69. $275.40, 15%.
70. $346.50, 16%.
71. $492.75, 18%.
72. $575.50, 24%.
73. $812.50, 16%.
74. $4275.50, 8%.
75. $2936.40, 12%
76. $3275.75, 16%.
77. $4206.03, 33⅓%.
78. $3742.20, 16⅔%.

Given the cost and rate of loss as follows, find the amount of loss:

79. $275, 6%.
80. $345, 4%.
81. $463, 8%.
82. $362.50, 4%.
83. $427.50, 8%.
84. $732.25, 12%.
85. $4725.25, 8%.
86. $2632.50, 12%.
87. $4170.20, 15%.

88. Some goods were sold for $1015.30 at a loss of 31¼%. Find the loss.

89. Beans bought @ $1.60 a bushel are sold @ 7¢ a quart. Find the per cent of gain or of loss.

90. A dealer sold some goods that cost $875 so as to gain 15%. What was the selling price?

91. A man bought a farm for $4800 and sold it at a gain of 25%. What was the selling price?

92. A merchant's stock of goods cost him $8450. It was damaged 45% by fire. What was it then worth?

93. Papers bought at the rate of 60 for 50¢ are sold for 1¢ each. Find the per cent of gain and the cost of 90 papers.

94. A box of 180 oranges is bought for $5.40. The oranges are sold at 50¢ per dozen. Find the gain per cent.

95. A dealer buys 320 yd. of cloth at 87½¢ a yard. At what price per yard must he sell the cloth to make a profit of $40?

96. A merchant marked his goods by private signs so as to conceal the exact cost. He used this scheme:

B L A C K	H O R S E
1 2 3 4 5	6 7 8 9 0

An article worth between \$1 and \$2 was marked $\frac{\text{BLE}}{\text{BKE}}$, indicating the cost and the selling price. What was the cost? the selling price? the gain? the per cent of gain?

97. Using the key words of Ex. 96, what is the per cent of gain on a hat that cost LEE and sells for LKE?

98. Using EDUCATIONX for 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, what is the per cent of profit on an arithmetic that costs AA and sells for OX?

99. Using the key of Ex. 98, how must a dealer mark a pair of shoes that cost UUX so as to sell them at a profit of 33⅓%?

100. Using ARITHMEtiC as the key, and X and Y as duplicate symbols for 0, how may a dealer mark goods that cost tTX and IiY so as to make a profit of 25% on the former and 33⅓% on the latter?

The class may be encouraged to devise similar key words and make examples to correspond.

101. A merchant sold some goods that cost $\$1^{7}5$ so as to gain 20%. The selling price was 20% less than the marked price. What was the marked price?

102. A dealer sold some furniture for which he had paid \$240 so as to gain 12½%. The selling price was 25% less than the marked price. What was the marked price? What per cent was the marked price above cost?

103. A grocer buys 50 bu. of potatoes at 60¢ a bushel and retails them at 20¢ a peck. Find his whole gain and his gain per cent.

104. A merchant sold some goods at 10% below cost, this being 20% below his marked price. The goods cost $300. What was the marked price?

105. A wholesale dealer paid $2.50 per barrel for apples. He sold them to a retailer at a gain of 8%, and the retailer sold them at a gain of 16⅔%. What was the retail price?

106. A clothier marked a suit of clothes that cost him $13.20 so as to gain 40%. He then deducted from the marked price 16⅔%. What was the actual selling price?

107. A man bought a horse for $150 and a carriage for $120. He sold them, gaining 8% on the horse and losing 12% on the carriage. Find his net gain or loss.

108. A farmer received $160 for some produce. It passed through the hands of a shipper, a commission merchant, and a retailer, who gained respectively 12½%, 3⅓%, and 20% on what it cost them. What was the retail price?

109. Some goods cost a merchant $100. At what price must he mark them so that he can take off 20% and still make a profit of 20%?

110. Some goods cost a merchant $240. At what price must he mark them so that he can take off 25% and still make a profit of 25%?

111. At what per cent above cost must a merchant mark his goods so that he can take off 20% and still make a profit of 20%?

112. A man sold two houses for $9600 each. On one he gained 20% and on the other he lost 20%. Did he gain or lose on the whole transaction, and how much?

113. A grocer bought 625 doz. pound packages of a certain food for $900, less 16⅔% discount. He sold each package for 16¢. How much did he gain in all?

114. A merchant marks goods 12½% above cost and sells them 20% below the marked price. Find his per cent of loss. Write the analysis.

115. A dealer bought some goods for $2400 and sold them for $3000. What was the per cent of gain on the purchase price of the goods?

116. A merchant's stock of goods cost him $12,500. It was damaged by fire so that he sold out the entire stock for $7500. What was the per cent of loss?

117. A merchant paid $2250 for a carload of furniture. He sold $1250 worth at a profit of 40%, but the rest having been damaged by fire was sold at a loss of 50%. Did he gain or lose on the entire carload, and how much?

118. A cattle dealer bought a lot of cows for $1250. He sold them at a profit of 30% and invested the proceeds in cattle which he was obliged to sell at a loss of 30%. Did he gain or lose on the whole transaction, and how much?

119. A dealer in horses bought a carload of ponies for $1250. He sold them at a loss of 20%, and with the proceeds he bought some horses that he sold at a profit of 20%. Did he gain or lose on the whole transaction, and how much?

120. A merchant invested $4500 in goods and sold them at a profit of 33⅓%. He invested the proceeds again, selling this stock at a profit of 28%. How much did he make on the two transactions?

121. A man bought some land and sold it off in building lots. On two thirds of it he gained 33⅓%, and on one third of it he gained 66⅔%. On which did he gain the more?

122. A man bought some property for $2400 and sold it at a profit of 12½%. He invested the proceeds in some more property and sold this at a profit of 12½%. What per cent did he gain on his original investment of $2400?

CHAPTER XIII

THE EQUATION

230. The Equation Method. There is a better method for solving many of the problems of arithmetic than any that have thus far been given. It is known as the *equation method.*

231. Equation. An indicated equality between two expressions is called an *equation.*

For example, $2 + x = 5$ is an equation.

232. Unknown Quantity. In the equation $2 + x = 5$, x is called the *unknown quantity.*

We may indicate an unknown quantity by x, or by some initial, as d for dollars, or n for number. We may also use $x\%$, or any other convenient symbol.

233. Indicated Multiplication. We indicate the multiplication of $5 \times x$ by writing simply $5x$.

234. Solving an Equation. To find the value of the unknown quantity in an equation is to *solve* the equation.

235. How to Solve. In the equation $2 + x = 5$, we may subtract 2 from these two equal quantities, and we have $x = 5 - 2$, or $x = 3$.

To solve an equation, do to one side what is necessary to leave x by itself, and then perform the same operation on the other side.

Thus, if $3x = 15$, divide both sides by 3.
Then $x = 5$.
If $\frac{1}{3}x = 7$, multiply both sides by 3.
Then $x = 21$.

236. Applications of the Equation. The following are illustrations of the special application of the equation to percentage:

1. 25 is what per cent of 225?

Let $x\% =$ the required rate per cent.

Then $x\% \times 225 = 25.$

Dividing by 225, $x\% = \frac{25}{225}$

$= \frac{1}{9}$, or $11\frac{1}{9}\%$.

2. $8.40 is 12% of what number?

Let $x =$ the required number.

Then $0.12\,x = \$8.40.$

Dividing by 0.12, $x = \$8.40 \div 0.12$

$= \$70.$

EXERCISE 135

Find what per cent the first number is of the second:

1. 25, 750.
2. 37, 148.
3. 5 ft., $37\frac{1}{2}$ ft.
4. 528 ft., 2 mi.
5. $8.60, $77.40.
6. $1\frac{1}{2}$, $7\frac{1}{2}$.
7. $1.80, $3.
8. 7 ft., 7 yd.
9. $4.90, $29.40.
10. 73 ft., 194 yd. 2 ft.

11. A man's income is $1650 a year, and he spends $693. What per cent of his income does he save?

12. In a certain village 576 out of the 1200 pupils in school are boys. What per cent are boys? girls?

13. The purity of gold is measured in carats, or 24ths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in an 18-carat ring?

14. What is the per cent of pure gold in a watch case that is 16 carats fine? in a chain that is 12 carats fine? in a ring that is 14 carats fine?

15. A horse and carriage together cost $375, and the carriage cost twice what the horse did. What did the horse cost?

We may explain the work as follows:

Let $x =$ the *number of dollars* the horse cost.
Then $2x =$ the *number of dollars* the carriage cost,
and $x + 2x =$ the *number of dollars* both cost, which is 375.
Therefore $3x = 375$, because x (that is, $1x$) and $2x$ are $3x$.
Therefore $x = 125$, by dividing both sides by 3.
Therefore the horse cost $125.

The actual work of this problem is as follows:

$$3x = 375.$$
$$x = 125.$$

16. After increasing 6%, a certain sum amounts to $318. What is the sum?

We may explain the work as follows:

Let $x =$ the *number of dollars* in the sum.
Then $0.06x =$ the *number of dollars* increase.
Adding, $1.06x =$ the *number of dollars* after the increase.
Therefore $1.06x = 318$.
Therefore $x = 318 \div 1.06 = 300$.
Therefore the sum is $300, as is easily proved.

The actual work of this problem is as follows:

$$1.06x = 318.$$
$$x = 300.$$

17. A man sold a carriage for $120, which was 20% less than it cost. How much did it cost?

We may explain the work as follows:

Let $x =$ the *number of dollars* of cost.
Then $0.20x =$ the *number of dollars* lost,
and $x - 0.20x =$ the *number of dollars* received.
Therefore $0.80x = 120$, because $1x - 0.20x = 0.80x$.
Therefore $x = 120 \div 0.80 = 150$.
Therefore the cost was $150, as is easily proved.

The actual work of this problem is as follows:

$$0.80x = 120.$$
$$x = 150.$$

18. Of what length is 112 ft. 6 in. exactly 45%?

19. After gaining 12%, a certain sum amounts to $392. What is the sum?

20. If to $\frac{1}{5}$ of a certain number 730 is added, the result is 855. What is the number?

21. A boy lost $\frac{2}{5}$ of his marbles and had 18 left. How many marbles had he at first?

22. A man sold $\frac{3}{7}$ of his cattle and had 68 left. How many cattle had he at first?

23. A certain sum was increased 50% and then amounted to $505.50. What was the sum?

24. A man sold a house for $2250, which was 20% less than it cost. How much did it cost?

25. A man saves $675.20 a year, which is 32% of his income. How much is his income?

26. A school has 20% of its pupils in the sixth grade, which numbers 29. How many are there in the school?

27. If a school is in session 50% of the days of a certain year, and is in session 183 days, is that a leap year or not?

28. With $2250 a man bought 20 acres of land from Mr. A, who thereby gained 25% on what it cost him. How much did Mr. A pay per acre for the land?

29. A man sold a farm of 100 acres at the rate of $56.25 an acre. He lost 10% on the cost of the farm. How much did the farm cost him?

30. A man buys some lumber and sells it for $60, thereby gaining 20% on the cost. The $60 is what per cent of the cost? What is the cost?

31. A certain school has 119 boys, which is 85% of the number of girls. How many girls are there in the school? What is the total number of pupils?

32. A man sells 41 sheep, which is 33⅓% of all he owned. He received $5.50 a head. At this rate, what is the value of the sheep remaining in his flock?

33. A man's salary has been increased 22% this year. It is now $1525. What was it last year?

$1.22x$ = what number?

34. If I have added 60% to the size of my farm, and now have 240 acres, how many did I have before the increase?

35. If you weigh 78.1 lb., and have increased 42% in weight in the last 5 years, how much did you weigh five years ago?

36. If our regular army has been increased 21%, and now numbers 73,810, how many did it number before the increase?

37. If a library has increased in size 16⅔% in the past five years, and now has 8323 volumes, how many volumes had it before the increase?

38. A dealer bought two horses at the same price. He sold one, at a profit of 20%, for $102. The other he sold at a loss of 10%. How much did he receive for the latter?

39. A certain village, after gaining 10% on its population in 1910, had 10% less population than another village of 4180 inhabitants. What was the population of the first village in 1910?

40. A furniture dealer bought 5 sets of furniture at $65 a set, 3 sets at $90, and 2 sets at $40. The $40 sets he sold at a profit of 30%, and the $65 sets at a profit of 20%, but on the $90 sets he lost 10%. What per cent did he gain on the lot?

41. If you have read all but 40% of a book, and have read 234 pages, how many pages has the book?

$$x - 0.40x = 0.60x. \quad 0.60x = \text{what number?}$$

42. If the width of a schoolhouse is 10% less than the length, and the width is 46.8 ft., what is the length?

43. If you lose 10% of a kite string, and have 585 ft. left, how long was the string? How many feet did you lose?

44. How much is a man's income if he pays 12% of it for rent at the rate of $15 a month?

45. If an automobile going 14 mi. an hour increases its speed 37%, at what rate will it then be traveling?

46. If the boys in a certain school are 45% of the total, and there are 66 girls, how many boys are there?

47. What sum of money increased by 33⅓% of itself becomes $600? What sum of money decreased by 33⅓% of itself becomes $600?

48. A stenographer can write 105 words a minute, which is 25% more than she could write three months ago. What was her rate then?

49. A stenographer can write 95 words a minute. Three months ago she wrote 20% less than now. What was her rate then?

50. A cubic foot of platinum weighs 1220 lb., and a cubic foot of pressed gold weighs 10 lb. less. Platinum is what per cent heavier than pressed gold?

51. A bank averaged $1200 a day of new deposits in November, and 26% more a day in December. What were the average daily deposits in December?

52. A clerk's salary is $12 a week. If he spends 20% of it for rent, 10% for clothes, and 25% for other expenses, and saves the rest, how much does he save in 52 wk.?

CHAPTER XIV

TRADE DISCOUNT

237. Trade Discount. A reduction from the marked price of an article, from the amount of a bill of goods, or from the amount of a debt is called *trade discount.*

Trade discount is also called *commercial discount.*

Discounts are usually reckoned at some common fraction of, or at some rate per cent of, the amount from which the discount is made.

238. Cash Discount. A discount that is taken off because a purchaser is paying cash at once instead of waiting to make the payment later, is called a *cash* discount.

The amount of the bill after the discount has been made is called the *net amount,* or the *net price.*

The price given in a printed list is called the *list price.*

What is the net amount of a bill of $125 after a discount of 20% is made?

Amount of bill	$125
Less 20%, or ⅕, of $125	25
Net amount	$100

Instead of speaking of "20% off," merchants often speak of "⅕ off." It is not so convenient to use common fractions for 12%, 7%, etc.

EXERCISE 136

Find the discount on the following at the rates stated:

1. $250, 10%.
2. $375, 20%.
3. $450, 15%.
4. $575, 4%.
5. $675, 6%.
6. $825, 3%.
7. $1250, 2%.
8. $1400, 8%.
9. $2250, 12%.

Find the net prices of goods billed as follows, with discounts as stated:

10. $250, 20%.

11. $175, 10%.

12. $360, 10%.

13. $475, 8%.

14. $560, 12%.

15. $775, 15%.

16. $480, $16\frac{2}{3}$%.

17. $720, $12\frac{1}{2}$%.

18. $810, $33\frac{1}{3}$%.

19. A sewing machine was marked $50 but was sold at $12\frac{1}{2}$% discount. What was the net price?

20. A piano was marked $800 but was sold at 8% discount. What was the net price?

21. A dealer bought some goods for $975.25, less 20%. What was the net price?

22. A lady bought a $50 cloak at a bargain sale at 25% discount. What did it cost her?

23. A dealer offers some goods for $75, or 4% less for cash. How much does a purchaser save by paying cash?

24. A dealer offers some goods for $80, or for $72 if the purchaser pays cash. What per cent discount does he offer?

25. A dealer offers some goods for $10.80 less than the regular price, a discount of 12%. What is the regular price?

26. The regular price of some goods is $496, but they cost a dealer 85% of this sum. How much is the discount?

27. A dealer buys a bill of goods amounting to $1875. He is allowed a discount of 8% on this bill. How much is the discount?

28. A dealer buys a bill of goods amounting to $2250. He is allowed a discount of 6% on this bill. What is the net amount of the bill?

29. A dealer buys a bill of goods amounting to $2800, but by paying cash he settles it for $2576. What is the rate of discount allowed?

30. A dealer buys a bill of goods, and by paying cash he saves $94.50. The rate of discount being 7%, what is the original bill? How much does he pay?

31. A dealer buys a bill of goods amounting to $2575, but by paying cash he settles it for $231.75 less. What is the rate of discount allowed?

32. The discount on some goods listed at $460 is $115. What is the rate of discount?

33. The discount on some goods is $41.60 at 8%. What is the list price?

34. The discount on some goods is $2.05 at 12½%. What is the list price?

35. A bicycle was marked $45, but was sold at 12% discount. What was the net price?

36. A piano was listed at $550, but was sold at 30% discount. What was the selling price?

37. A jobber bought some goods listed at $2450. He was allowed a discount of 22%. What was the net price?

38. A merchant bought 850 yd. of muslin at 6¢ a yard, less 18%. How much did it all cost?

39. A merchant bought 975 yd. of lawn for $146.25, less 20%. How much did he pay per yard?

40. A grocer bought 36 doz. cans of soup listed at $1.75 a dozen. He was allowed a discount of 25%. What was the net price?

41. A dealer sold some boxes of soap, 60 cakes to the box, listed at $2.25 a box. He allowed a discount of 20%. How much did he receive per cake?

42. What per cent of profit will a man make by paying $17.60 for an article, with a discount of 20% from the list price, and selling it at the list price?

43. Find the net amount of a bill for $470, the discount being 18%.

44. On a bill for $1760 a discount of $105.60 is allowed. What is the rate of discount?

45. A grocer bought 24 doz. pound packages of macaroni for $31.20, less 15%. He sold it for 15¢ per package. How much did he gain in all?

46. A merchant sold at 12½% discount some goods that were marked $272.80. What was the selling price?

47. A grocer bought 160 small bags of flour, 12¼ lb. to the bag, listed at 4¢ a pound. He was allowed a discount of 15%. What did the flour cost him?

48. Find the net amount of a bill for $880 subject to a discount of 12½%; of 6¼%; of 25%; of 2½%.

49. On a bill of $1650 a discount of 8½% was allowed. What was the net price?

50. Find the net cost of a piano marked $675, a discount of 8½% being allowed.

51. What is the marked price of an automobile the discount on which is $140 at 8%?

52. If the marked price of an automobile is $1850 and the selling price is $1702, what is the rate of discount?

53. Which is the better for the purchaser, and how much better: a discount of 16⅔% on a bill for $842.50, or a flat reduction of $140?

54. Which is the better for the seller, and how much better: a discount of 12½% on a bill for $1275, or a flat reduction of $159?

55. Which is the better for the purchaser, and how much better: a discount of 6¼% on a bill for $625.75, or a flat reduction of $39.25?

239. Bills Discounted. A bill for goods purchased by a dealer and subject to discount is usually made out as follows:

New York, Aug. 1, 1914

Mr. Thos. K. Ryder, Syracuse, N.Y.

Bought of **Hibbard and Company**

Terms: *4%* **Dealers in Stationery**

July	*5*	*60 gro. Pens @ 79¢*	*$47*	*40*		
		12 doz. Penholders @ 8¢		*96*		
		40 gro. Lead pencils @ \$3$^{\underline{40}}$	*136*			
			$184	*36*		
		Less 4%	*7*	*37*	*$176*	*99*

Received payment
Hibbard & Co.

For bills not subject to trade discount, see page 171.

EXERCISE 137

Write bills similar to the above, inserting places, dates, and names, making the extensions and footing, and then deducting the discount:

1. 25 M (or 25,000) envelopes @ $2.30 per M, 480 lb. paper @ 14¢, 16 doz. bottles ink @ 36¢ per dozen, 12 doz. bottles mucilage @ 35¢ per dozen. Discount 5%.

2. 60 doz. cans soup @ $2.96 per dozen, 20 doz. jars meat extract @ $4.16 per dozen, 20 doz. cartons raisins @ $2.39 per dozen, 40 doz. baskets washed figs @ $2.42 per dozen. Discount 8%.

3. Supply the proper numbers in the spaces marked with an asterisk (*) in the following bill:

Ithaca, N.Y., March 7, 1915

Mr. Robert J. Richards, Elmira, N.Y.

Bought of **Thompson Bros.**

Terms: *6% 30 da., net 60 da.*

March	7	50 gro. Pens @ 78¢	$39			
		15 doz. Notebooks @ $1.70	*			
		4 doz. Rulers @ 1.32	*			
		30 gro. Lead pencils @ 3.35	*			
			*			
		Less 6%	*		*	

Make out bills for the following:

4. Bought 40 arithmetics @ 45¢, 65 geographies @ 90¢, 25 readers @ 35¢. Discount 12%.

5. Bought 3 doz. locks @ $4.25, 4 doz. bolts @ $1.10, 2½ doz. hinges @ $2.35. Discount 30%.

6. Bought 4 doz. spoons @ $15, 8 doz. plated tablespoons @ $5, 6½ doz. teaspoons @ $10.50. Discount 8%.

7. Bought 3 doz. pencils @ 25¢, ½ doz. rulers @ $1.00, 1 pt. ink @ 25¢, 75 notebooks @ 15¢, 3 doz. bottles of ink @ 38¢, 4 lb. paper @ 30¢. Discount 10%.

8. Bought 575 yd. cotton flannel @ 38¼¢, 684 yd. cotton @ 7⅓¢, 364 yd. buckram @ 9½¢, 648 yd. sheeting @ 26¾¢, 327 yd. Java canvas @ 62½¢, 340 yd. taffeta @ 96¢, 480 yd. lawn @ 23¢. Discount 12%.

9. 8 doz. fountain pens @ $21.75 per dozen, 20 doz. boxes fine note paper @ $4.80 per dozen, 8 gross erasers @ $3.60 per gross. Terms: 6%.

10. 25 doz. cartons prunes @ $1.38 per dozen, 8 doz. packages citron @ $2.72 per dozen, 2 gro. cartons dates @ $8.80 per gross, 2 gro. gelatin @ $9.60 per gross, 60 bbl. flour @ $5.48. Terms: 6%.

11. 8 gro. macaroni @ $13.78 per gross, 10 doz. bottles olive oil @ $2.38 per dozen, 3 gro. packages beans @ $17.80 per gross, 8 gro. packages oatmeal @ $27.70 per gross, 70 gal. sirup @ 42¢ per gallon. Terms: 8%.

12. 5 gro. baking powder @ $23.80 per gross, 8 doz. jars mustard @ $1.91 per dozen, 8 doz. boxes table salt @ $1.22 per dozen, 16 doz. boxes cinnamon @ $3.61 a dozen, 18 doz. cans molasses @ $1.96 per dozen. Terms: 6%.

13. 40 M envelopes @ $2.70 per M, 3 doz. fountain pens @ $18.50 per dozen, 8 doz. bottles ink @ 32¢ per dozen, 5 gross erasers @ $3.30 per gross. Terms: 8%.

14. 80 bbl. flour @ $5.23, 60 gal. molasses @ 38¢, 75 gal. fancy sirup @ 50¢, 3 doz. cartons rye flour @ $2.64, 260 lb. coffee @ 9.65¢. Terms: 6%.

15. 10 doz. hatchets @ $10.75 per dozen, 6 doz. pr. hinges @ $4.35 per dozen, 5 doz. carpenter's squares @ $34.50 per dozen, ½ gross locks @ $42.50 per gross, 8 doz. files @ $6.25 per dozen. Terms: 40%.

16. 500 yd. cotton flannel @ 38¼¢, 600 yd. cotton @ 7⅓¢, 360 yd. buckram @ 9½¢, 640 yd. sheeting @ 26¾¢, 320 yd. Java canvas @ 62½¢. Terms: 12%.

17. D. P. Howard purchased from Brown Bros. 84 gross bone buttons @ 18½¢, 72 doz. pearl buttons @ 9¾¢, 690 yd. cambric @ 17½¢, 360 yd. cashmere @ 82¼¢, 400 yd. chambray @ 21¾¢. Terms: 12%.

240. Discount Series. The changes in cost of production from time to time, and the variation due to the credit of the buyer, and the quantity of goods purchased, give rise frequently to several discounts known as a discount series.

Goods that are selling at a discount of 20% may be produced at a lower cost owing, for example, to an unusually large cotton crop. An additional discount of 10% may then be allowed. Likewise a person whose credit is exceptional, who buys an especially large quantity, or who pays cash, may be entitled to a still further discount of 5%.

In a discount series the first denotes the discount from the list price, the second denotes the discount from the remainder, and so on.

Thus $100 less 20%, 10%, and 5%, means that 20% is taken from $100, leaving $80; then 10% from $80, leaving $72; then 5% from $72, leaving $68.40. The order is, however, immaterial.

When there is a fraction of a cent in a remainder, the seller usually counts it a whole cent in his favor.

The regular discount is quoted first, and then the special discounts.

Advantages of a discount series are that it saves reprinting elaborate catalogues, allows of easy change, enables the dealer easily to offer different discounts to different customers, and allows for several options to purchasers.

If a bill is due at once it usually bears the words "Terms cash." If due in 60 days without discount, it may bear the marks "N/60" or "Net 60 da." If due in 60 days, but with a special discount of 3% if paid in 10 days, it may read "3/10, N/60," or "3% 10 da., net 60 da."

EXERCISE 138

Given these list prices and discounts, find the net prices:

1. $225, 20% and 6%
2. $368, 25% and 5%.
3. $480, 12½% and 3%.
4. $350, 15% and 4%.
5. $560, 25% and 10%.
6. $270, 33⅓% and 10%.
7. $480, 25%, 10%, 5%.
8. $560, 25%, 20%, 3%.
9. $729, 33⅓%, 10%, 10%.
10. $280, 25%, 10%, 10%.
11. $2760, 33⅓%, 20%, 10%.
12. $5550, 33⅓%, 25%, 15%.

13. How much will 240 ft. of iron pipe cost if listed at $1.50 a foot, a discount of 40% and 10% being allowed?

14. How much will a gross of jars of mixed pickles cost at $6 a dozen, a discount of 20% and 10% being allowed?

15. The list price of 2-pound cartons of prunes is $24 per 100. A discount of 20% and 10% is allowed. What is the net price of 300 cartons?

16. A book dealer can buy an encyclopedia for $60 less 20% and 10%, or for the same price less 15% and 15%. Which is the better for him, and how much better?

17. A grocer buys 100 dozen jars of table salt listed at $1.50 a dozen, a discount of 25% and 8% being allowed. If he sells the salt at 12¢ a jar, how much does he gain?

18. A dry goods dealer buys 960 yd. of silk listed at $1.50 a yard, a discount of 33⅓% and 5% being allowed. If he sells it at $1.50 a yard, how much does he gain?

19. A dealer buys 36 doz. pocket knives listed at $8.40 a dozen, a discount of 25% and 10% being allowed. If he sells them at 75¢ apiece, how much does he gain?

20. A furniture dealer buys 16 bedroom sets listed at $36 each, a discount of 16⅔% and 10% being allowed. If he sells the furniture at $36 a set, how much does he gain?

21. A hardware dealer bought a shipment of stoves. After deducting a discount of 33⅓% and 25%, and paying $9.75 freight and $2.50 cartage, the stoves cost him $252.25. What was the list price of the stoves?

22. A dry goods dealer bought a shipment of cloth. After deducting a discount of 25% and 10%, and paying $8.50 for freight and cartage, the cloth cost him $410. What was the list price?

241. Bills with Several Discounts. The following is a common form of a bill of goods purchased from a wholesale dealer and subject to a discount series.

Binghamton, N.Y., March 15, 1917

Mr. R. D. Scanlan, Cortland, N.Y.

Bought of **Lane & Company, Jewelers**

Terms: *20%, 10%*

Feb.	*10*	*8 doz. Spoons @ $14.75*	*$118*	*00*		
		7 doz. Forks @ $4.20	*29*	*40*		
			$147	*40*		
		Less 20%, 10%	*41*	*27*	*$106*	*13*

EXERCISE 139

Make out bills for the following and discount them:

1. 36 doz. files at $6.25. Discounts 30%, 20%.

2. 960 yd. silk at $1.40. Discounts 25%, 10%.

3. 4 doz. pr. hinges at $4.60; 25 doz. table knives at $8.40. Discounts 20%, 10%.

4. 12 doz. locks at $4.20; 6 doz. mortise locks at $4.60. Discounts 25%, 8%.

5. 360 yd. taffeta at 98¢; 4 gro. pompons at $144; 3 doz. pieces braid at $20.40. Discounts 10%, 5%.

6. 60 lb. brads at 6¢; 200 bolts at $4.80 per C; 6 doz. knives at $7.20. Discounts 40%, 20%, 10%.

7. 480 yd. silk at $1.60; 640 yd. lawn at 22¢; 960 yd. taffeta at 96¢. Discounts 10%, 5%, 5%.

8. 2 doz. tennis rackets @ $21, 8 doz. balls @ $3, $\frac{3}{4}$ doz. tennis nets @ $19.20. Discounted @ 12%, 5%, 2%.

9. $\frac{1}{2}$ doz. rugs at $84, 750 yd. carpet @ 82¢, 3 doz. hassocks @ $6. Discounted @ $\frac{1}{5}$, $\frac{1}{4}$.

10. 5 doz. bottles ink @ $3, 300 lb. paper @ 30¢, 2 doz. bottles mucilage at $4, 6 gro. pencils @ $2.75. Discounted @ 3%, 2%, 1%.

11. 18 brass bedsteads @ $17.25; 15 woven wire mattresses @ $3.45; 3 oak sideboards @ $17.25. Discounted @ $\frac{1}{6}$, $\frac{1}{10}$.

12. 50 M envelopes @ $2.10, 3 doz. fountain pens @ $19.50, 8 doz. bottles ink @ 34¢, 5 gro. erasers @ $3.38. Terms: 8% 30 da., net 60 da.

13. 75 bbl. flour at $5.23, 63 gal. molasses @ 39¢, 75 gal. fancy sirup at 58¢, 3 doz. cartons flour @ $2.64, 260 lb. coffee @ 9.65¢. Discounted @ 6%.

14. 8 doz. hatchets @ $10.75, 6 doz. pr. hinges @ $4.35, 5 doz. carpenter's squares @ $34.50, $\frac{1}{2}$ gro. locks @ $42.50, 3 doz. pr. shears @ $7.50, 8 doz. files @ $6.25. Discounted @ 40%, 10%, 5%.

15. 21 Ohio steers, 1226 lb. (average), @ $5.10 (per 100 lb.); 20 do. (ditto = the same), 1247 lb., @ $5.10; 20 do., 1112 lb., @ $4.95. Discounted @ $\frac{1}{10}$.

16. 20 Illinois steers, 1157 lb. (see Ex. 15), @ $4.60; 28 do., 996 lb., @ 4.12\frac{1}{2}$; 20 Kentucky steers, 1230 lb., @ $4.80; 7 Indiana steers, 1078 lb., @ $4.70. Discounted @ $\frac{1}{10}$, $\frac{1}{10}$.

17. 75 180-pound bags Western potatoes @ $1.48, 50 150-pound do. @ $1.28, 125 168-pound do. @ $1.42, 1620 lb. Jersey potatoes in bulk @ $1.47 per 180 lb. Discounted @ $\frac{1}{6}$, $\frac{1}{10}$.

242. Single Discount equal to a Series. It is often convenient to find a single discount equal to a series.

For example, find the single discount equal to the discount series 20%, 10%, and 10%.

Let the list price be represented by 100 % of the list price.
Then deducting 20% of this, or 20 % of the list price,
we have 80 % of the list price.
Deducting 10% of this, or 8 % of the list price,
we have 72 % of the list price.
Deducting 10% of this, or 7.2% of the list price,
we have 64.8% of the list price.

Then 100% − 64.8% = 35.2%, the single discount.

EXERCISE 140

Find the single discount equal to the discount series:

1. 10%, 8%.
2. 15%, 6%.
3. 18%, 3%.
4. 14%, 9%.
5. 20%, 10%.
6. 30%, 10%.
7. 25%, 12½%.
8. 10%, 10%, 10%.
9. 25%, 15%, 10%.
10. 25%, 20%, 10%.
11. 30%, 20%, 10%.
12. 33⅓%, 10%, 5%.
13. 33⅓%, 10%, 10%.
14. 16⅔%, 10%, 10%.
15. 20%, 12½%, 10%.
16. 25%, 12½%, 10%.

17. Compare the single discount equal to the series 10% and 5%, with the sum of 10% and 5% minus the product of 10% and 5%.

18. By the method suggested in Ex. 17, find the single discount equal to the series 20%, 5%.

19. How does the single discount equal to the discount series 20%, 10%, and 5%, compare with that equal to the discount series 10%, 5%, and 20%? Prove it.

EXERCISE 141

REVIEW

1. Find the net amount of a bill for $880 subject to discounts of 2½%, 5%, and 10%.

2. On a bill of $620 trade discounts of 5% and 5% are allowed. What is the net price?

3. Find the net cost of a piano marked $450, with trade discounts of 3%, 8%, and 10%.

4. What is the list price of an article whose net cost is $6.15 after discounts of 20% and 25% are deducted?

5. What per cent of profit will a man make by paying $17.10 for an article, with discounts of 20%, 10%, and 5% from the list price, if he sells it at the list price?

6. Furniture is bought at trade discounts of 5% and 16% from the list price, and the net cost is $518.70. Find the list price of the furniture.

7. Find the net amount of a bill of $470, the trade discounts being 20% and 5%. What single rate of discount is equivalent to these discounts?

8. What single discount is equivalent to a discount series of 40%, 20%, 10%, and 5%?

9. Which is the better for the purchaser, and how much better: two successive trade discounts of 25% and 10% on a bill of $500, or a single discount of 33⅓%?

10. Which is the better for the seller, and how much better: trade discounts of 20%, 12%, and 3%, on a bill of $800, or a single discount of 31¾%?

11. Which is the better for the purchaser, and how much better: trade discounts of 10%, 10%, and 10%, on a bill of $625.75, or a single discount of 27$\frac{1}{10}$%?

EXERCISE 142

PROBLEMS WITHOUT NUMBERS

1. If you know the list price of some goods and the single rate of discount, how do you find the net price?

2. If you know the list price and net price, how do you find the rate of discount?

3. If you know the net price and the rate of discount, how do you find the list price?

4. If you know the list price and the several rates in a discount series, how do you find the net price?

5. If you have a discount series, is it right to add the several rates to find the single rate of discount? State the reason.

6. If you know the several rates in a discount series, how do you find the single rate equivalent to them?

7. If you know the several rates in a discount series, and the net price, how do you find the list price?

8. Does it make a difference in the net price if you vary the order in a discount series? State the reason.

9. If the rate of discount is given in per cent, how can you express it as a common fraction?

10. If the rate of discount is given as a common fraction, how can you express it as per cent?

11. If a discount series is given in per cents, how can you express it as a single common fraction?

12. If a discount series is given in common fractions, how can you express it as a single rate per cent?

13. If a dealer marks some goods a certain rate above cost, and then takes off that same rate as a discount from the marked price, does he gain or lose? Why?

CHAPTER XV

COMMISSION

243. Commission or Brokerage. A sum paid by one person to another for transacting business is called *commission* or *brokerage*.

For example, if a man sells a house for another man, at 2% commission, and the price received is $5000, the amount of the commission is 2% of $5000, or $100.

Similarly, if a man collects a bill of $250 for another man, at 5% commission, he receives 5% of $250, or $12.50.

244. Principal and Agent. The person for whom another transacts business is called the *principal.* The person who transacts the business is called the *agent.*

245. Commission Merchant and Broker. An agent who has actual possession of the goods of his principal is called a *commission merchant.*

An agent who makes sales or purchases for his principal without having actual possession of the goods is called a *broker.*

Commission and brokerage are usually reckoned at a certain per cent of the money involved in the transaction. If goods are bought, the commission is a certain per cent of the cost; if goods are sold, the commission is a certain per cent of the amount received; if a collection is made, the commission is a certain per cent of the amount collected.

Brokerage is often a fixed sum per bushel (as of wheat purchased), a fixed sum per 100 bales (as of cotton), a fixed sum per tierce (as of lard), and so on.

In all problems, unless otherwise directed, discard any fraction of a cent less than $\frac{1}{2}$ in a final result; when a fraction of a cent is $\frac{1}{2}$ or more, count it as a full cent.

EXERCISE 143

Find the commissions on sales or purchases of the following amounts, at the rates specified:

1. $275, 2%.
2. $345, 3%.
3. $675, 4%.
4. $725, 5%.
5. $875, 6%.
6. $4125, 2%.
7. $7250, 3%.
8. $9325, 4%.
9. $2750, 1½%.
10. $3275, 2½%.
11. $2760, 3½%.
12. $4175, 4½%.
13. $6375, 3⅓%.
14. $6540, 5%.
15. $8650, 6%.
16. $7775, 8%.
17. $8000, ⅛%.
18. $7500, ¼%.
19. $6200, ½%.
20. $3520, ⅜%.
21. $4800, 1⅛%.
22. $8000, 1¼%.
23. $2400, 1¼%.
24. $4800, 2½%.

Find the net amounts remaining after deducting commissions on sales as follows, at the rates specified:

25. $4800, ⅛%.
26. $6400, ¼%.
27. $3500, ⅕%.
28. $400, 2%.
29. $600, 4%.
30. $850, 5%.
31. $1250, 4½%.
32. $2800, 2¼%.
33. $5600, 2⅛%.
34. $6400, 3½%.
35. $1500, 1½%.
36. $2200, 1¼%.
37. $3600, 1¾%.
38. $4800, 3¾%.
39. $275.50, 4%.
40. $612.50, 8%.
41. $721.50, 6%.
42. $5600, ⅛%.
43. $4200, ¼%.
44. $3200, ⅜%.
45. $6400, ⅝%.

46. After paying 5% for collecting a debt of $1200, how much remains?

47. After paying 3% for selling some property for $3200, how much remains?

48. An agent received $84, or 2%, for selling some property. What was the selling price?

49. An agent received $75, or 2½%, for selling some property. What was the selling price?

50. An agent sold 325 bu. of wheat @ 80¢ a bushel, charging a commission of 5%. Find the net proceeds.

51. An agent charged $6.63 for selling 52 bu. of tomatoes @ 85¢ a bushel. Find the rate of his commission.

52. What is the commission at 3½% on a sale of potatoes to the amount of $1270?

53. A real-estate agent charged $246.25 for selling a piece of property for $9850. What was the rate of his commission?

54. A real-estate agent charged $312.50 for selling a piece of property for $12,500. What was the rate of commission?

55. An agent sold 44 wagons @ $135 each and paid his principal $5197.50 as the proceeds of the sale. Find the rate of the agent's commission.

56. An agent bought for a dealer 240 baskets of peaches @ 75¢ a basket, charging 3½% commission. Find the total cost to the dealer.

57. A commission merchant sold 45 bbl. of apples @ $2.80 a barrel, deducting 6¢ a barrel for storage and a commission of 5%. Find the net proceeds of the sale.

58. An agent sold some produce for $935, charging 3% commission. How much did he remit to his principal?

59. An agent sold some produce, charging 3% for his services. His commission amounted to $28.05. How much did he remit to his principal, after deducting his commission?

60. An agent sold some produce, retaining his commission of $3.69 and remitting the proceeds, $119.31. What was the rate of commission?

61. A commission merchant sold some produce, and, after deducting his commission, remitted to his principal $906.95. His commission amounted to $28.05. What was the selling price? What was the rate of his commission?

62. What is an architect's commission on a house costing $9472, at 5%?

63. An architect's commission on a house costing $8750 is $437.50. What is the rate?

64. What is an auctioneer's commission for selling $358 worth of furniture, at 6%?

65. A real-estate dealer sells 168 acres of land at $40 an acre. How much is his commission at 2½%?

66. A lawyer charged $40 for collecting a debt of $750. What is the rate?

67. An agent sells our school $65 worth of maps, and his firm pays him 7½% commission. How much is this?

68. A farmer sends some butter and eggs to the city, and they are sold for $72.50. He is charged 8% by his agent. What is the net amount received by the farmer?

69. What is the commission on 87 tubs of butter, each containing 50 lb., if sold at 27¢ a pound, the rate of commission being 3½%?

70. An agent sold a house for a man, charging 5% commission. He sent to his employer $4512.50 as the proceeds. At what price did he sell the house?

71. A commission merchant sold for a farmer some produce for $260, and sent to the farmer $239.20 as the proceeds. What was the rate of commission?

72. An agent sold some land for a man and received $9500. He sent to his employer $9215 as the proceeds. What was the rate of commission?

73. A salesman's sales in a department store in a city amounted, during the six days of one week, to $257.70, $103.10, $175.02, $132.30, $302.25, $50.63. He receives 2½% on his sales. How much did he receive that week?

74. What is the commission at 7½% on a sale of an automobile for $1250?

75. What is the commission at 2½% for collecting a debt of $750?

76. What is the net amount remaining after deducting a commission of 3% from a sale amounting to $850?

77. A broker bought 850 bales of cotton for a customer, charging $5 per hundred bales as brokerage. What was the brokerage?

78. A broker bought 750 tierces of lard for a customer, charging 2½¢ a tierce. A tierce being 340 lb., how many pounds did he buy, and what was the brokerage?

79. Lard selling at $7.03 per tierce, how much will 1250 tierces cost? How many pounds are there? What is the brokerage on the purchase at 2½¢ per tierce?

80. A commission merchant sells for a factory 6900 lb. of cheese at 12¢ a pound, charging 5% commission. What sum should he remit to his principal?

81. A commission merchant sold 1700 doz. eggs for a dealer, charging him 5% on the sale, and remitting $339.15. At what price per dozen did he sell the eggs?

82. A commission merchant sold 160 tubs of butter, 50 lb. to the tub, for $1920, charging 5% commission. What did the commission amount to per pound?

83. A commission merchant sold 240 tubs of butter, 50 lb. to the tub, for $3360. His commission being 1⅖¢ a pound, what was his rate of commission?

84. A broker buys 1750 bags of coffee for a customer at 10.43¢ per pound. There being 130 lb. to the bag, and the broker charging $10 per 250 bags, how many pounds are bought and what is the total cost including the brokerage?

85. Given the net proceeds $438.44, and the commission $13.56, find the rate of commission.

86. A real-estate agent charged $446.25 for selling a piece of property for $12,750. What was the rate of his commission?

87. The net proceeds of the sale of 244 bbl. of flour is $1124.23, the commission being 3%. Find the selling price of the flour per barrel.

88. An agent bought for a dealer 360 baskets of peaches @ 82½¢ a basket, charging 3½% commission. Find the total cost to the dealer.

89. An agent sold 82 horses @ $112.50 each and paid his principal $8763.75 as the proceeds of the sale. Find the rate of the agent's commission.

90. A real-estate agent charged $562.50 for selling a piece of property. The rate of commission being 4½%, at what price did he sell the property?

91. A commission merchant sold 225 bbl. of apples @ $2.87 a barrel, charging 7¢ a barrel for storage and a commission of 5%. Find the net proceeds of the sale.

92. A commission merchant sold some produce, charging 3½% for his services. He remitted to his principal $1621.20. At what price did he sell the produce?

93. A manufacturer sent his agent $3177.81 to pay for some leather, and the commission of ½%. How much did the agent invest and how much was his commission?

94. A broker bought 700 bales of cotton for a customer @ 12.1¢ per pound. Taking the bales as weighing 500 lb. each, and the brokerage at $5 per hundred bales, find the number of pounds purchased and the total cost including the brokerage.

95. On which will the net amount after deducting the commission be greater, a sale of $980 less 3% commission, or one of $970 less 2% commission? How much greater?

96. A clerk receives $4 a week and 2% commission on goods sold. If he sells $372 worth of goods in a week, what is his income for the week?

97. On which will the net amount after deducting the commission be greater, a sale of $1500 less 4% commission, or one of $1600 less 10% commission? If there is any difference, what is it?

98. A clerk receives $10 a week and 1½% commission on goods sold. If he sells $492 worth of goods in a week, how much is his income for the week?

99. A city grocer sends a man out to his customers' residences every morning to take orders. If the man receives 8% commission on all orders, and if he takes orders to the amount of $9800 a year, how much does he earn?

100. An agent sells some property for one man for $7500 on a commission of 2%, and some for another man for $6000 on a commission of 2½%. On which does he receive the greater commission, and how much commission does he receive in all?

101. A saleswoman is offered her choice of $8.50 a week, $5 a week with 1% on all sales, or 4% on all sales. She takes the last plan, and her sales for 52 weeks average $265 a week. How much does she gain over each of the other two plans?

102. In a large dry-goods store in New York a woman is employed to "shop" for out-of-town customers. She receives from the dealers a commission of 5% on all purchases. Her purchases amount to $422 a month, on the average. How much is her commission during the year?

EXERCISE 144

Problems without Numbers

1. If you know the amount paid by an agent for some property, and his rate of commission, how do you find the amount of his commission?

2. If you know the amount received by a broker for goods sold, and his rate of brokerage, how do you find the net amount remitted to his principal?

3. If you know the net amount of a sale remitted by an agent to his principal, and the rate of his commission, how do you find the price at which the property was sold?

4. If you know the net amount left after the commission has been deducted, and the price at which the property was sold, how do you find the rate of commission?

5. If you know the amount collected by an agent, and the rate of his commission, how do you find the net amount to be remitted by him to his principal?

6. If you know an agent's rate of commission for collecting some money, and the net amount remitted by him to his principal, how do you find the amount collected?

7. If you know the price per acre at which an agent sells some land, the number of acres, and the rate, how do you find his commission?

8. If you know the price per acre at which an agent sells some land, the number of acres, and the commission, how do you find the rate of commission?

9. If a farmer sends a certain number of barrels of apples to a commission merchant, and these are sold at a certain price per barrel, and the freight per barrel and the rate of commission are known, how do you find the amount due the farmer?

CHAPTER XVI

INTEREST

246. Interest. Money paid for the use of money is called *interest.*

Interest is computed as a certain per cent of the money used. Thus, if a man borrows \$500 for a year, at 6%, the interest is 6% of \$500, or \$30.

247. Principal. Money loaned on which interest is paid is called the *principal.*

In the example just given, \$500 is the principal.

The sum of the principal and interest is called the *amount.*

248. Rate of Interest. The rate per cent of the principal paid in interest for one year is called the *rate of interest.*

In the example under § 246, the rate of interest is 6%, and the amount is \$500 + \$30, or \$530.

For example, consider the following:

(1) Find the interest on \$840 at 5% for 2 yr.

Since the rate is 5% a year,

$$\text{Interest for 1 yr.} = 5\% \text{ of } \$840 = \$42.$$
$$\text{Interest for 2 yr.} = 2 \times \$42 = \$84.$$

(2) Find the interest on \$750 at $5\frac{1}{2}\%$ for $3\frac{1}{2}$ yr.

Since the rate is $5\frac{1}{2}\%$ a year,

$$\text{Interest for 1 yr.} = 5\tfrac{1}{2}\% \text{ of } \$750 = \$41.25.$$
$$\text{Interest for } 3\tfrac{1}{2} \text{ yr.} = 3\tfrac{1}{2} \times \$41.25 = \$144.38.$$

Therefore, *to find the interest, given the principal, rate, and time, multiply the principal by the rate, and the product by the number that expresses the time in years.*

EXERCISE 145

Given the principal, rate, and time, find the interest:

1. $100, 6%, 2 yr.
2. $100, 4%, 3 yr.
3. $100, 3%, 4 yr.
4. $200, 6%, 3 yr.
5. $300, 6%, 2 yr.
6. $600, 3%, 2 yr.
7. $250, 6%, 2 yr.
8. $375, 4%, 3 yr.
9. $650, 6%, 5 yr.
10. $275, 5%, 2 yr.
11. $350, 4%, 3 yr.
12. $460, 6%, 2 yr.
13. $525, 5%, 3 yr.
14. $265, 4%, 2 yr.
15. $425, 5%, 3 yr.
16. $635, 6%, 4 yr.
17. $875, 6%, 2 yr.
18. $925, 5%, 5 yr.
19. $100, 6%, 2½ yr.
20. $100, 5%, 2½ yr.
21. $200, 6%, 3¼ yr.
22. $200, 5%, 2¼ yr.
23. $375, 6%, 2½ yr.
24. $675, 4%, 3¼ yr.
25. $480, 6%, 2¾ yr.
26. $540, 6%, 3¾ yr.
27. $640, 5%, 2¾ yr.
28. $975, 4%, 2 yr.
29. $975, 5½%, 2 yr.
30. $725, 4½%, 3 yr.
31. $835, 3½%, 2 yr.
32. $425.50, 6%, 2½ yr.
33. $375.75, 6%, 3½ yr.
34. $420.50, 5%, 2¼ yr.
35. $2450, 4%, 3¾ yr.
36. $3575, 5%, 2¾ yr.

37. A man borrowed $250 and agreed to pay 6% interest. He kept the money a year. How much interest did he pay?

38. A man borrowed $250 and agreed to pay 6% interest. He kept the money two years. How much interest did he pay?

39. How much is the interest on $400 at 6% for 1 yr.? for 2 yr.? for ½ yr.? for 1½ yr.? for 2 yr. 6 mo.?

40. How much is the interest on $600 at 4% for 1 yr.? for 2 yr.? for 6 mo.? for 1 yr. 6 mo.? for 2 yr. 6 mo.?

41. What is the interest on $200 at 6% for 1 yr.? for 1 mo.? for 2 mo.? for 7 mo.?

42. What is the interest on $600 at 6% for 1 yr.? for 1 mo.? for 3 mo.? for 10 mo.? for 8 mo.?

43. What is the interest on $500 at 6% for 2 yr.? for $2\frac{5}{12}$ yr.? for 2 yr. 5 mo.? for 2 yr. 7 mo.?

Given the principal, rate, and time, find the interest:

44. $300, 2 yr. 5 mo., 4%.
45. $400, 2 yr. 5 mo., 3%.
46. $600, 1 yr. 6 mo., 6%.
47. $600, 1 yr. 8 mo., 6%.
48. $600, 1 yr. 8 mo., 5%.
49. $500, 1 yr. 9 mo., 6%.
50. $600, 1 yr. 9 mo., 5%.
51. $500, 2 yr. 10 mo., 6%.
52. $800, 3 yr. 5 mo., 6%.
53. $900, 2 yr. 3 mo., 5%.
54. $1200, 1 yr. 7 mo., 6%.
55. $1200, 1 yr. 7 mo., 5%.
56. $1200, 2 yr. 2 mo., 6%.
57. $1500, 2 yr. 2 mo., 6%.
58. $1500, 2 yr. 9 mo., 6%.
59. $1500, 2 yr. 9 mo., 5%.
60. $1800, 3 yr. 8 mo., 6%.
61. $1800, 3 yr. 2 mo., 6%.
62. $1800, 3 yr. 2 mo., 5%.
63. $2500, 4 yr. 3 mo., 6%.

64. What is the interest on $436, at 6%, for 1 yr.? for 6 mo.? for 4 mo.? for 8 mo.? for 3 yr. 10 mo.?

65. What is the interest on $360, at 5%, for 1 yr.? for 6 mo.? for 10 mo.? for 1 mo.? for 2 yr. 9 mo.?

66. If the interest on a certain sum is $93.84 for 1 yr., how much is it for 3 mo.? for 6 mo.? for 1 yr. 9 mo.?

67. If the interest on a certain sum is $36.48 for 1 yr., how much is it for 1 mo.? for 5 mo.? for 3 yr. 10 mo.?

68. What is the interest on $540, at 4%, for 1 yr.? for 1 mo.? for $\frac{1}{2}$ mo.? for $2\frac{1}{2}$ mo.? for $5\frac{1}{3}$ mo.?

69. What is the interest on $175.50, at 4%, for 1 yr.? for 6 mo.? for 4 mo.? for 3 mo.? for 1 mo.? for $2\frac{2}{3}$ mo.?

249. Interest when the Time involves Days. If the time involves days, we may reduce it to a fraction of a year and proceed as in § 248.

For example, find the interest on $480 for 3 yr. 3 mo. 10 da. at 5%.

$$10 \text{ da.} = \tfrac{10}{30} \text{ mo.} = \tfrac{1}{3} \text{ mo.}$$

$$3\tfrac{1}{3} \text{ mo.} = \tfrac{10}{3} \text{ of } \tfrac{1}{12} \text{ yr.} = \tfrac{5}{18} \text{ yr.}$$

$$3\tfrac{5}{18} \times 5\% \text{ of } \$480 = \frac{59 \times 5 \times \$480}{18 \times 100} = \$78.67.$$

It is the custom in computing interest in this way to take 30 da. to the month, or 360 da. to the year, and to find the interest to the nearest cent.

EXERCISE 146

Find the interest on:

1. $100, at 6%, for 2 yr. 6 mo.
2. $100, at 6%, for 2 yr. 6 mo. 10 da.
3. $100, at 6%, for 3 yr. 4 mo. 15 da.
4. $200, at 5%, for 3 yr. 4 mo. 20 da.
5. $300, at 5%, for 3 yr. 8 mo. 20 da.
6. $275, at 4%, for 1 yr. 9 mo. 15 da.
7. $375, at 5%, for 2 yr. 9 mo. 15 da.
8. $450, at 6%, for 3 yr. 10 mo. 6 da.
9. $525, at 6%, for 4 yr. 10 mo. 6 da.
10. $675, at 5%, for 2 yr. 3 mo. 10 da.
11. $575, at 5%, for 3 yr. 3 mo. 3 da.
12. $825, at 4%, for 2 yr. 3 mo. 3 da.
13. $1250, at 5½%, for 2 yr. 6 mo. 15 da.
14. $1500, at 4½%, for 3 yr. 6 mo. 20 da.
15. $2500, at 3½%, for 2 yr. 4 mo. 15 da.
16. $3500, at 3½%, for 3 yr. 7 mo. 10 da.

250. Difference between Dates. The time between two dates may be found as here shown. For example, required to find the time from April 16, 1914, to February 5, 1919.

yr.	mo.	da.
1919	2	5
1914	4	16
4	9	19

Since February is the second month of the year, and April the fourth, the work is arranged as here shown. We then subtract as with other compound numbers; 5 – 16 is impossible, but 35 – 16 = 19. Proceeding in the usual manner, the result is 4 yr. 9 mo. 19 da. In such cases allow 30 da. to the month.

EXERCISE 147

Find the difference in time between these dates:

1. Sept. 2, 1914, Oct. 15, 1919.
2. Oct. 5, 1914, Jan. 25, 1920.
3. Aug. 7, 1915, June 4, 1921.
4. Nov. 10, 1916, Jan. 5, 1920.
5. Aug. 23, 1915, Jan. 9, 1920.
6. What is the interest on $375 from July 4, 1914, to July 4, 1917, at 6%?
7. What is the interest on $650 from July 20, 1914, to May 20, 1917, at 5%?
8. What is the interest on $825 from May 12, 1915, to Feb. 12, 1919, at 4½%?
9. What is the interest on $360 from June 8, 1916, to April 6, 1918, at 6%?
10. What is the interest on $480 from March 7, 1916, to April 9, 1918, at 5%?
11. What is the interest on $750 from Nov. 8, 1916, to Feb. 22, 1918, at 5%?
12. What is the amount of $650 at interest at 6% from Sept. 27, 1915, to March 12, 1918?

251. Promissory Notes. A written promise to pay a specified sum of money on demand or at a specified time is called a *promissory note*, or simply a *note*.

252. Parties to a Note. The person who signs a note is called the *maker*. The person to whom a note is payable is called the *payee*. The lawful owner of a note is called the *holder*.

A note may be made payable to *bearer*, in which case it is lawful to pay it to any one who presents it to the maker.

It may be made payable to some person named in the note, in which case he or his legal representatives must present it. Such a note is said to be *non-negotiable*.

It may be made payable to the payee, *or order*, in which case the payee may transfer it to another person by writing his own name on the back, thus *indorsing* the note. Such a note is said to be, like one payable to bearer, *negotiable*.

An indorser of a note is responsible for its payment if the maker does not pay it, unless he writes the words "without recourse" above his signature.

253. Form of a Note. A common form of note is as follows:

$375.25. *Albany, N.Y., June 1, 1913*

Six months after date, I promise to pay Frank Mills, or order, Three Hundred Seventy-five and $\frac{25}{100}$ Dollars, with interest at 5%, for value received.

James Lang

Here the maker is James Lang, and the payee is Frank Mills.

If the payee writes across the back,

Payable to the order of John B. Brown,
Frank Mills

he becomes an indorser, and John B. Brown becomes the holder.

If Frank Mills writes simply his name across the back, the note becomes payable to bearer, that is, to any one who may have it.

This note is due on Dec. 1, 1913, which is called the *day of maturity*.

The words "with interest," without specifying the rate, carry the legal rate of the state in which the note is made.

EXERCISE 148

Find the day of maturity and amount due, having given:

	Face of Note	*Date of Note*	*Time*	*Rate of Interest*
1.	$300	July 7, 1913	60 da.	6%
2.	$500	Jan. 4, 1913	6 mo.	5%
3.	$400	Feb. 8, 1913	1 yr.	4½%
4.	$751.50	Mar. 7, 1913	3 mo.	6%
5.	$525.25	Apr. 15, 1914	90 da.	6%
6.	$255.50	May 16, 1914	30 da.	5%

Write notes for the following, and compute the interest:

7.	$400	June 6, 1914	6 mo.	6%
8.	$600	Nov. 9, 1914	4 mo.	5%
9.	$750	Jan. 7, 1913	3 mo.	6%

Find the amount due at maturity on the following notes:

10. $200. ROCHESTER, N.Y., Oct. 20, 1914

Sixty days after date, I promise to pay George A. Wood, or order, Two Hundred Dollars, with interest at 5%. Value received. JAMES SINCLAIR

11. $1200. UTICA, N.Y., Mar. 7, 1914

Ninety days after date, I promise to pay James Goodwin, or order, Twelve Hundred Dollars, with interest at 6%. Value received. JAMES HILTON

12. $375.50. ELMIRA, N.Y., Apr. 6, 1914

One year after date, I promise to pay Francis Warren, or order, Three Hundred Seventy-five and $\frac{50}{100}$ Dollars, with interest at 5%. Value received. ALFRED GIBSON

13. F. H. Holmes borrows $350, at 6%, for 1 yr., from A. B. Seel. He gives a note payable to Mr. Seel or order. Mr. Seel sells the note to B. C. Dean. Make out the note, indorse it, and find the amount due at the end of the year.

Make out and indorse, payable to the order of the buyer, the following notes, and find the amount due at the end of the time stated:

	Maker	*Payee*	*Buyer*	*Face*	*Rate*	*Time*
14.	J. R. Roberts	A. N. Cole	P. R. Jones	$375	6%	6 mo.
15.	M. L. King	A. R. Owen	E. F. Pease	$750	5%	2 yr.
16.	A. K. Jewett	S. M. Wood	E. L. Brown	$75	6%	10 mo.
17.	B. S. Stevens	O. N. House	B. R. Lodge	$250	5%	3 yr. 6 mo.
18.	O. M. Richards	D. E. Dodge	C. King	$125	6%	1 yr. 2 mo.
19.	A. R. Castle	A. J. Jackson	G. F. Ray	$250	5½%	2 yr. 4 mo.
20.	F. E. Taylor	A. B. Baker	G. F. Coe	$350	4½%	1 yr. 3 mo.
21.	F. E. Glover	M. T. Thorn	F. Taylor	$250	6%	4 yr. 1 mo.

22. A man borrowed $250 on Jan. 15, for 9 mo., at 6%. Write the note and compute the amount of the principal and interest due on Oct. 15.

23. A man borrowed $350 on Jan. 27, for 1 yr. 1 mo., at 6%. Write the note and compute the amount of the principal and interest due on Feb. 27 of the following year.

24. A man bought 12 head of cattle on Feb. 1, at $45 a head, giving his promissory note at 6%. He paid it on the 16th of the following April. What was the amount of principal and interest?

25. A man bought 75 acres of land on March 3, at $85 an acre, giving his promissory note at 5½%. He paid it on the 18th of the following September. What was the amount of principal and interest?

254. Problems in Interest. Of the four elements used in computing interest,—principal, rate, time, and interest (or amount, which is only the sum of the principal and interest),—we can find any one if the other three are known.

For, since the interest is the product of three factors (principal, rate, and time), if it is divided by the product of any two of these factors the quotient is the third.

The following examples will illustrate:

(1) What principal will, in 3 yr. 6 mo., yield $52.50 interest at 6%?

Since the interest for $3\frac{1}{2}$ yr. is $52.50,
therefore the interest for 1 yr. is $52.50 ÷ $3\frac{1}{2}$, or $15.

Since 6% of the principal is $15,
therefore 1% of the principal is $2.50,
and 100% of the principal is $250.

(2) What principal will amount to $570 in 2 yr. 4 mo. at 6%?

The amount of $1 for 2 yr. 4 mo. at 6% is $1.14.

Since $1.14 is the amount of $1,
$570 is the amount of $\frac{\$570}{1.14}$, or $500.

(3) In what time will $540 yield $109.80 interest at 6%?

The interest on $540 at 6% for 1 yr. is $32.40.

Since $32.40 is the interest for 1 yr.,
$109.80 is the interest for $\frac{109.80}{32.40}$ yr., or $3\frac{7}{18}$ yr.

$3\frac{7}{18}$ yr. = 3 yr. $4\frac{2}{3}$ mo. = 3 yr. 4 mo. 20 da.

(4) At what rate will $600 yield $81 interest in 3 yr.?

The interest on $600 at 1% for 3 yr. is $18.

Since $18 is the interest at 1%,
$81 is the interest at $\frac{81}{18}$%, or $4\frac{1}{2}$%.

EXERCISE 149

Find the principal that will yield the following interest:

1. $20, in 2 yr. 8 mo., at 6%.
2. $93⅓, in 3 yr. 4 mo., at 5%.
3. $9.59, in 3 yr. 6 mo., at 4%.
4. $40.47, in 1 yr. 7 mo., at 6%.
5. $3.64, in 5 mo. 18 da., at 6%.
6. $45.90, in 4 yr. 3 mo., at 4½%.

Find the principal that will amount to the following:

7. $482.40, in 3 yr. 4 mo., at 6%.
8. $1151.41, in 2 yr. 4 mo., at 4½%.
9. $1210.50, in 2 yr. 5 mo., at 5%.

Find the time, given the principal, rate, and interest:

10. $840, 5%, $147.
11. $625, 4%, $56.25.
12. $350, 6%, $49.
13. $1250, 6%, $187.50.

Find the rate, given the principal, time, and interest:

14. $330, 1 yr. 2 mo., $19.25.
15. $68.50, 3 yr. 6 mo., $9.59.
16. $87.50, 2 yr. 8 mo. 24 da., $14.35.
17. How long will it take $1 to yield $1 interest at 6%?
18. How long will it take $1 to double itself at 5%?
19. What sum must be put at interest at 6% so that it shall amount to $1000 in 16⅔ yr.?
20. What sum must be put at interest at 5% so that it shall amount to $1000 in 20 yr.?

255. Formulas in Interest. We may express the principles involved in the problems on page 285 by using initial letters to represent the elements. In this way we have convenient *formulas.*

Using the initial letters, and letting the time be expressed in years, we have

Interest = principal × rate × time (§ 248), or $I = P \times R \times T$; therefore

$$\text{principal} = \frac{\text{interest}}{\text{rate} \times \text{time}}, \qquad \text{or } P = \frac{I}{R \times T};$$

$$\text{rate} = \frac{\text{interest}}{\text{principal} \times \text{time}}, \text{ or } R = \frac{I}{P \times T};$$

$$\text{time} = \frac{\text{interest}}{\text{principal} \times \text{rate}}, \text{ or } T = \frac{I}{P \times R}.$$

Thus, if the interest on $200 for 3 yr. at 6% is $36, we have

$$I = 3 \times 6\% \text{ of } \$200 = \$36,$$

$$P = \frac{\$36}{3 \times 6\%} = \$200,$$

$$R = \frac{\$36}{3 \times \$200} = 6\%,$$

$$T = \frac{\$36}{6\% \text{ of } \$200} = 3.$$

Instead of using capital letters for the initials, we may use small letters if we wish. We shall then have:

$$i = p \times r \times t, \quad p = \frac{i}{r \times t}, \quad r = \frac{i}{p \times t}, \quad t = \frac{i}{p \times r}.$$

The simple equation and the use of the unknown quantity x may profitably be used in the solution of some problems, thus: $36 is the interest on $200 for how many years at 6%?

Since $\$36 = x \times 6\%$ of $200, we may divide these equals by 6% of $200, and we have

$$x = \frac{\$36}{6\% \text{ of } \$200} = 3.$$

The same principle is involved as in the preceding solution in which T is found.

EXERCISE 150

1. Given $r = 6\%$, $i = \$90$, and $t = 3$, find p.
2. Given $i = \$64$, $p = \$800$, and $t = 2$, find r.
3. Given $i = \$16$, $p = \$200$, and $r = 4\%$, find t.

Find the missing element, marked x:

	p	r	t	i		p	r	t	i
4.	\$300	x	4 yr.	\$60	**16.**	\$375	4%	4 yr.	x
5.	x	4%	4 yr.	\$64	**17.**	x	5%	2 yr.	\$70
6.	\$700	3%	x	\$42	**18.**	\$600	x	3 yr.	\$90
7.	\$650	5%	2 yr.	x	**19.**	\$700	6%	2 yr.	x
8.	x	5%	3 yr.	\$30	**20.**	\$600	3%	x	\$54
9.	\$300	x	3 yr.	\$36	**21.**	x	5%	3 yr.	\$30
10.	\$300	3%	x	\$36	**22.**	\$700	x	2 yr.	\$70
11.	\$700	5%	3 yr.	x	**23.**	\$600	5%	4 yr.	x
12.	x	6%	5 yr.	\$60	**24.**	\$500	4%	x	\$80
13.	\$400	x	2 yr.	\$48	**25.**	x	3%	3 yr.	\$36
14.	\$500	5%	x	\$75	**26.**	\$250	x	2 yr.	\$20
15.	\$800	4%	2 yr.	x	**27.**	\$900	6%	x	\$162

28. A note for \$275 at 6% runs for 3 yr. without any payments being made. What is the amount then due?

29. A note for \$300 at 6% amounts to \$336. How long has the note run?

30. A note for \$400 amounts in 3 yr. to \$460. What is the rate of interest?

31. A note amounts to \$224 in 2 yr. The face of the note is \$200. What is the rate of interest?

32. Write a note for \$275 due 2 yr. 6 mo. after this date, at 6%, and find the amount due at maturity.

EXERCISE 151

REVIEW OF INTEREST

Given the principal, rate, and time, find the interest:

1. $250, 5%, 4 mo. 10 da.; 8 mo. 20 da.

2. $520, 6%, 2 yr. 9 mo.; 2 yr. 9 mo. 25 da.

3. $760, 5%, 4 yr. 6 mo.; 4 yr. 6 mo. 15 da.

4. $875, 6%, 3 yr. 7 mo.; 3 yr. 7 mo. 16 da.

5. $1250, 5%, 1 yr. 10 mo.; 1 yr. 10 mo. 5 da.

6. $2500, 5½%, 2 yr. 10 mo.; 2 yr. 10 mo. 7 da.

7. $2275, 5½%, 3 yr. 11 mo.; 3 yr. 11 mo. 2 da.

8. $3525, 4½%, 1 yr. 11 mo.; 1 yr. 11 mo. 3 da.

Given the principal, rate, and time, find the amount:

9. $175, 6%, 6 mo.; 8 mo.; 1 yr. 2 mo.

10. $225, 6%, 8 mo. 3 da.; 1 yr. 8 mo. 3 da.

11. $625, 5%, 4 mo. 3 da.; 2 yr. 4 mo. 3 da.

12. $1250, 5%, 3 mo. 8 da.; 2 yr. 3 mo. 8 da.

13. $2275, 5½%, 10 mo. 14 da.; 3 yr. 10 mo. 14 da.

14. Find the interest on $275 for 3 yr. 6 mo. at 5%.

15. Find the interest on $1250 for 7 yr. 11 mo. at 6%.

16. Find the interest on $630 for 2 yr. 4 mo. 5 da. at 6%.

17. Find the interest on $345 for 2 yr. 3 mo. 3 da. at 5%.

18. Find the interest on $290 for 1 yr. 6 mo. 5 da. at 5%.

19. Find the interest on $350 for 1 yr. 9 mo. 15 da. at 4%.

20. Find the interest on $650 for 2 yr. 8 mo. 10 da. at 6%.

21. Find the interest on $1575 for 3 yr. 1 mo. 6 da. at 5½%; at 4½%; at 6%.

22. What is the interest on $375 from July 4, 1915, to May 1, 1918, at 6%?

Given the principal, interest, and time, find the rate:

23. $275, $27.50, 2 yr.
24. $420, $50.40, 3 yr.
25. $121.25, $9.70, 2 yr.
26. $320.50, $57.69, 3 yr.
27. $275.25, $55.05, 5 yr.
28. $3500, $87.50, 10 mo.
29. $220, $9.35, 1 yr. 5 mo.
30. $250.50, $30.06, 2 yr.
31. $480, $36, 1 yr. 6 mo.
32. $240, $24, 2 yr. 6 mo.
33. $325, $45.50, 3 yr. 6 mo.
34. $240, $10.50, 1 yr. 3 mo.
35. $160, $19.80, 2 yr. 3 mo.
36. $480, $26.60, 1 yr. 7 mo.

Given the principal, interest, and rate, find the time:

37. $240, $36, 6%.
38. $3000, $155, 2%.
39. $175, $12.25, 4%.
40. $600, $42.75, 5%.
41. $360, $19.91, 5½%.
42. $280, $28, 4%.
43. $350, $17.50, 3%.
44. $525, $47.25, 6%.
45. $230, $20.70, 4½%.
46. $2500, $37.50, 4½%.

47. How long will it take a sum to double itself at 4%?

48. How long will it take $220, together with the interest, to amount to $253, at 6%?

49. How long will it take $1750, together with the interest, to amount to $1968.75, at 5%?

50. For how long must $1250 be invested at 5% so that it shall amount, together with the interest, to $1281.25? to $1385? to $1416.25?

51. A girl had $250 invested for her, at 6%, on her birthday. When she became 21 it amounted, with interest, to $400. How old was she when it was invested?

52. A father gave his son his promissory note for $225, due when the son became 21 years old. The rate of interest was 5%, and when the note became due the principal and interest together amounted to $303.75. How old was the son when the note was given?

53. Which yields the better income, $1675 at 5% or $1375 at 6%? What is the difference for 2 yr. 6 mo.?

54. A man receives $2187.50 interest in 2 yr. 6 mo. on an investment of $17,500. What is the rate of interest?

55. A man has $250 in a savings bank at 3%. He leaves the principal, but draws out the interest as it is due. How much does he draw out in four years?

56. A man having $13,500 invested at 5% reinvests $4000 of it at 5½%, $5000 of it at 3½%, and leaves the rest as before. What is the annual difference in income?

57. A boy has $300 given to him the day he is 14 years old. His father invests it for him in a 5% promissory note, due on the day he is 21 years old. What is the amount on the day the note is due?

58. A real estate dealer buys 360 acres of farm land at $30 an acre, and after keeping it 7 months sells it at an advance of $3.75 an acre. The money being worth to him 5% a year, how much does he gain by the transaction?

59. A man having $26,750 invested in business has found that his annual profits average 18% a year. He is offered $35,000 for the business, which he can invest at 4¼%. If he sells out and retires, what will be his loss in income?

60. On April 10 a coal dealer borrowed $33,250 at 5%, with which he purchased his summer's supply of coal at $4.75 a ton. He sold the coal at $5.65 a ton, the buyers paying for unloading and delivery, and paid his debt on November 16. How much did he gain?

61. On October 15 a dealer purchased $1750 worth of goods for the holiday trade on 30 days' credit. At the end of that time he gave his note for 2 months at 6% a year. He sold the goods for $2105, and paid the note when due. What per cent did he make on the original purchase price?

EXERCISE 152

PROBLEMS WITHOUT NUMBERS

1. Given the principal, rate, and time, how do you find the interest?

2. Given the principal, rate, and interest, how do you find the time?

3. Given the principal, time, and interest, how do you find the rate?

4. Given the rate, time, and interest, how do you find the principal?

5. How do you find the amount of principal and interest?

6. Given the principal, rate, and time, how do you find the amount of principal and interest?

7. Given the amount of principal and interest, the time, and the principal, how do you find the rate?

8. Given the amount of principal and interest, the rate, and the principal, how do you find the time?

9. How do you find the day of maturity of a promissory note?

10. Given the date of a note, the day of maturity, the principal, and rate, how do you find the interest?

11. How do you find how long it will take any sum of money to double itself at a given rate of interest?

12. How do you find the rate of interest at which a given sum of money will double itself in a given time?

13. Given the face of a note, and its amount in a given number of years, how do you find the rate of interest?

14. Given the amount of a note for a given number of years, and the rate of interest, how do you find the face of the note?

TABLES OF DENOMINATE NUMBERS

Table of Length

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
5½ yards or 16½ feet = 1 rod (rd.)
320 rods or 5280 feet = 1 mile (mi.)

mi.	rd.	yd.	ft.	in.
			1	12
		1	3	36
	1	5½	16½	198
1	320	1760	5280	63,360

A *hand* (4 in.) is used in measuring the height of horses; a *fathom* (6 ft.) and *cable length* (120 fathoms) in measuring depths of water; a *knot* (nautical mile, 1.152 common or statute miles, or 6080.27 ft.) in measuring distances at sea. Carpenters, mechanics, and others usually write 2′ 6″ for 2 ft. 6 in.

Table of Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
				1	144
			1	9	1296
		1	30¼	272¼	39,204
	1	160	4840	43,560	6,272,640
1	640	102,400	3,097,600	27,878,400	4,014,489,600

Table of Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

cu. yd.	cu. ft.	cu. in.
	1 =	1728
1 =	27 =	46,656

A perch of stone or masonry is usually 1 rd. long, 1 ft. high, and 1½ ft. thick, and contains 24¾ cu. ft., but this varies in different parts of the country. A cubic yard of earth is considered a load.

Table of Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

T.	lb.	oz.
	1 =	16
1 =	2000 =	32,000

100 lb. is sometimes called a *hundredweight* (cwt.). The ton of 2000 lb. is sometimes called the *short ton*, there being an old ton of 2240 lb., known as the *long ton*, which is used in some wholesale transactions in mining products.

Table of Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
31½ gallons = 1 barrel (bbl.)
63 gallons = 1 hogshead (hhd.)

	bbl.	gal.	qt.	pt.	gi.
				1 =	4
			1 =	2 =	8
	1 =	4 =	8 =	32	
1 =	31½ =	126 =	252 =	1008	

A gallon contains 231 cu. in.

Casks holding from 28 gal. to 43 gal. are called barrels, and casks holding from 54 gal. to 63 gal. are called hogsheads. Whenever barrels or hogsheads are used as *measures*, a barrel means 31½ gal., and a hogshead means 63 gal. A barrel contains about 4¼ cu. ft.

Table of Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

bu.	pk.	qt.	pt.
		1	2
	1	8	16
1	4	32	64

A bushel contains 2150.42 cu. in., or about 1¼ cu. ft. A dry quart contains 67.2 cu. in., while the liquid quart contains only 57.75 cu. in. In measuring grain, seeds, and small fruits the measure must be *even* full. In measuring apples, potatoes, and other large articles the measure must be *heaping* full.

Table of Surveyors' Linear Measure

7.92 inches (in.) = 1 link (l.)
25 links = 1 rod (rd.)
100 links = 1 chain (ch.)
80 chains = 1 mile (mi.)

mi.	ch.	rd.	l.	in.
			1	7.92
		1	25	198
	1	4	100	792
1	80	320	8000	63,360

Table of Surveyors' Square Measure

16 square rods (sq. rd.) = 1 square chain (sq. ch.)
10 square chains = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
1 square mile = 1 section (sec.)
36 sections = 1 township (T.)

sq. mi.	A.	sq. ch.	sq. rd.
		1	16
	1	10	160
1	640	6400	102,400

†

Table of Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days = 1 common year (yr.)
366 days = 1 leap year

yr.	mo.	wk.	da.	hr.	min.	sec.
					1 =	60
				1 =	60 =	3600
			1 =	24 =	1440 =	86,400
		1 =	7 =	168 =	10,080 =	604,800
1 =	12 =		365 =	8760 =	525,600 =	31,536,000
			366 =	8784 =	527,040 =	31,622,400

Angle Measure

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 4 right angles

Arc Measure

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circumference

United States Money

10 mills = 1 cent (ct. or ¢)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
10 dollars = 1 eagle (E.)

The term *eagle* is not used. The mill is not coined, although the name is frequently used in speaking of fractional parts of a cent.

English Money

4 farthings = 1 penny (d.)
12 pence = 1 shilling (s. or /)
20 shillings = 1 pound (£) or sovereign

†

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ANSWERS

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12. 26.
13. 22.
14. 17.
15. 32.
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17. 24.
18. 35.
19. 28.
20. 35.
21. 37.
22. 33.
23. 30.
24. 33.
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26. 31.
27. 36.
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31. 40.
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40. 55.
41. 52.
42. 64.
43. 50.
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61. 62.
62. 85.
63. 79.
64. 82.
65. 79.

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1. 99.
2. 99.
3. 124.
4. 111.
5. 228.
6. $102.
7. 269.
8. 184.
9. 137.
10. 193.
11. 283
12. $130.
13. 147.
14. 159.
15. 172.
16. 271.
17. 438.
18. 420.
19. 358.
20. 470.
21. 438.
22. 394.
23. 265.
24. 293.
25. 276.
26. 439.
27. 461.
28. 500.
29. 489.
30. 544.
31. 390.
32. 400.
33. 410.
34. 220.
35. 195.
36. 226.
37. 238.
38. 218.
39. 268.
40. 253.
41. 359.
42. 499.
43. 507.
44. 400.
45. 487.
46. 410.
47. 435.
48. 512.
49. 609.
50. 726.
51. 810.
52. 780.
53. 788.

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1. 1388.
2. 2210.
3. 1652.
4. 1662.
5. 2210.
6. 20,000.
7. 6024.
8. 14,030.
9. 23,058.
10. 16,886.
11. 99,936.
12. 116,404.
13. 156,827.
14. 142,877.
15. 198,887.
16. 210,264.
17. 265,241.
18. 249,853.
19. 92,097.
20. 134,487.
21. 22,027.
22. 1157.
23. $34,593,348.
24. 3,026,789.
25. 9,436,416.
26. $102,354,579.
27. $752,856,806.
28. $2,055,150,998.
29. 76,303,387.

Exercise 8. Page 16

1. $13.67.
2. $6.46.
3. $23.30.
4. $14.
5. $22.
6. $21.10.
7. $24.41.
8. $11.49.
9. $16.27.
10. $91.
11. $29.50.
12. $40.39.
13. $30.41.
14. $40.04.
15. $247.91.

Exercise 9. Page 18

1. 412.
2. 322.
3. 292.
4. 313.
5. 330.
6. 206.
7. 531.
8. 333.
9. 111.
10. 246.
11. $6.23.
12. $3.21.
13. $8.23.
14. $5.50.
15. $6.51.
16. $13.55.
17. $21.10.
18. $22.35.
19. $31.15.
20. $62.36.
21. $11.54.
22. $80.61.
23. $72.50.
24. $60.
25. $24.42.
26. $154.10.
27. $121.63.
28. $134.55.
29. $361.90.
30. $415.23.
31. 51,113.
32. 11,341.
33. 34,225.
34. 13,463.
35. 11,221.
36. 12,363.
37. 7505.
38. 24,152.
39. 12,920.
40. 52,371.
41. 320.
42. 325.
43. $345.
44. 1411.
45. 240.
46. 1112.
47. 115.
48. 832 miles.
49. 35,100 square miles.
50. 106,110 square miles.
51. 140 square miles.
52. 31,602,024 bushels.

Exercise 10. Page 21

1. 212.
2. 309.
3. 408.
4. 538.
5. 608.
6. 717.
7. 829.
8. 892.
9. 292.
10. 183.
11. 262.
12. 342.
13. 672.
14. 782.
15. 289.
16. 578.
17. 685.
18. 436.
19. 287.
20. 377.
21. 478.
22. 670.
23. 557.
24. 244.
25. 310.
26. 401.
27. 512.
28. 300.
29. 318.
30. 117.
31. 217.
32. 309.
33. 135.
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35. 205.
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42. 89.
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50. 161.
51. 268.
52. 213.
53. 2662.
54. 3209.
55. 5108.
56. 8267.
57. 2093.
58. 4492.
59. 6172.
60. 4091.
61. 7093.
62. 2487.
63. 5078.
64. 3057.
65. 4087.
66. 6082.
67. 6185.
68. 2087.
69. 1287.
70. 4895.
71. 1869.
72. 14,859.
73. 9339.
74. 21,833.
75. 13,389.
76. 17,887.
77. 16,789.
78. 10,083.

Exercise 11. Page 22

1. 1943.
2. 14,529.
3. 2768.
4. 28,571.
5. 19,148.
6. 24,110.
7. $2413.75.
8. $61.75.
9. $370.95.
10. $43.50.
11. $555.61.
12. $782.50.
13. $165.
14. $240.90.

Exercise 12. Page 26

1. 414.
2. 1227.
3. 1224.
4. 3520.
5. 3048.
6. 5614.
7. 4840.
8. 8181.
9. 874.
10. 1578.
11. 3292.
12. 3575.
13. 2472.
14. 5691.
15. 4896.
16. 6570.
17. 1498.
18. 2052.
19. 6216.
20. 4536.
21. 12,174.
22. 18,222.
23. 32,384.
24. 35,240.
25. 20,454.
26. 32,942.
27. 54,464.
28. 38,763.
29. 5526.
30. 13,488.
31. 33,108.
32. 39,395.
33. 55,416.
34. 62,601.
35. 75,584.
36. 66,303.
37. 59,892.
38. 51,429.
39. 71,104.
40. 89,991.
41. 82,406.
42. 78,621.
43. 142,820.
44. 213,045.
45. 185,556.
46. 282,023.
47. 563,544.
48. 725,625.
49. 154,468.
50. 247,887.
51. 190,904.
52. 419,605.
53. 433,662.
54. 444,003.
55. 386,368.
56. 327,924.
57. 473,274.
58. 629,902.
59. 454,312.
60. 899,991.
61. $336.
62. $2400.
63. 3240.
64. 3934.

Exercise 13. Page 27

1. $3.75.
2. $8.10.
3. $10.60.
4. $13.
5. $13.15.
6. $31.55.
7. $14.88.
8. $22.32.
9. $29.96.
10. $40.25.
11. $28.96.
12. $51.28.
13. $29.25.
14. $65.52.
15. $3164.
16. $1956.
17. $1405.
18. $3432.
19. $1596.
20. $7191.
21. $3468.
22. $45.75.
23. $65.10.
24. $50.48.
25. $128.60.
26. $86.
27. $115.55.
28. $113.40.
29. $225.
30. $288.12.
31. $264.25.
32. $173.04.
33. $662.08.
34. $319.50.
35. $428.67.
36. $12,852.
37. $15,710.
38. $12,378.
39. $4214.
40. $30,380.
41. $26,901.
42. $46,736.
43. $364.50.
44. $813.60.
45. $501.
46. $1226.80.
47. $1607.55.
48. $1243.75.
49. $1973.40.
50. $3256.50.
51. $901.25.
52. $2136.40.
53. $1694.
54. $3402.
55. $3354.48.
56. $8889.12.
57. $84,682.
58. $64,263.
59. $105,604.
60. $158,100.
61. $540,519.
62. $544,581.
63. $508,488.
64. $111.
65. $22.75.
66. $13.50.
67. $25.90.
68. $31.50.
69. $142.
70. $171.
71. $6915.
72. $71.
73. $6.84.
74. $77.46.
75. $2300.
76. $339.75.
77. $112.50.
78. $29.75.
79. $65.95.

Exercise 14. Page 29

1. 1770.
2. 19,080.
3. 14,280.
4. 23,500.
5. 55,800.
6. 15,300.
7. 32,100.
8. $30,100.
9. $284.
10. $439.20.
11. $186.20.
12. $345.60.
13. $546.
14. $504.
15. 128,100.
16. 320,000.
17. 2,940,000.
18. 3,150,000.
19. $3000.
20. $2400.
21. 89,100,000.

Exercise 15. Page 30

1. 55,608.
2. 151,032
3. 110,628.
4. 192,696.
5. 95,788.
6. 195,642.
7. 184,522.
8. 436,408.
9. 200,129.
10. 273,208.
11. 644,982.
12. 433,519.
13. 338,706.
14. 287,616.
15. 656,197.
16. 366,816.
17. 452,898.
18. 720,802.
19. 268,128.
20. 653,315.
21. 508,776.
22. 651,817.
23. 410,720.
24. 397,467.
25. 627,480.
26. 383,808.
27. 639,878.
28. 859,068.
29. 989,901.
30. 1,553,171.
31. 1,515,192.
32. 1,469,264.
33. 1,389,789.
34. 3,071,880.
35. 4,015,920.
36. 2,980,670.
37. 3,376,360.
38. 2,398,484.
39. 3,044,672.
40. 1,644,032.
41. 5,019,258.
42. 1,542,940.
43. 6,778,982.
44. 3,517,701.
45. 4,480,852.
46. 2,811,468.
47. 5,915,182.
48. 7,411,656.
49. 4,005,980.
50. 7,728,993.
51. 5,485,172.
52. 3,403,944.
53. 3,471,794.
54. 6,195,441.
55. 39,930,966.
56. 23,887,808.
57. 26,611,944.
58. 50,024,883.
59. $22,605.
60. $32,900.
61. $30,660.
62. $26,010.
63. $19,411.
64. $41,088.
65. $75,078.
66. $3,360,000.
67. $5,250,000.
68. $1,920,000.
69. $4,800,000.
70. $13,654.75.
71. $10,174.50.
72. $13,639.92.
73. $25,502.40.
74. $32,181.32.
75. $43,421.84.
76. $13,048.82.
77. $32,484.06.
78. $52,706.70.
79. $734,447.04.
80. $350,513.52.
81. $337,085.60.
82. $411.25.
83. $1940.40.
84. $529.20.
85. $1482.
86. $2386.30.

Exercise 16. Page 32

1. $1050.
2. 42,240.
3. 88,000 pounds.
4. 264 feet.
5. 22,500 feet.
6. $23.80.
7. $195.
8. $1200.
9. $1020.
10. $25.50.
11. $3000.
12. 1960.
13. $4875.
14. $13,875.
15. 15,504.
16. 2100.
17. 126,720.
18. 13,392.
19. 19,456.
20. 4900.

21. 2448.
22. $4200.
23. 86,400.
24. 620,400 pounds.
25. 20,664 pounds.
26. 62,060 pounds.
27. 8640 ounces.
28. 52,095.
29. 28,224 pounds.
30. $1575.
31. $4668.75.
32. $5378.75.
33. $2047.
34. $257.50.
35. $1424.44.
36. 135,000 feet.
37. 16,092 pounds.
38. 16,524 pounds.
39. 5568 pounds.
40. 6,065,400.
41. 62.
42. $1008.
43. $369.60.
44. $31.36.
45. The horses; $136.
46. 17,280,000 miles.

Exercise 17. Page 37

1. 116.
2. 115.
3. 104.
4. 65.
5. 123.
6. 132.
7. 74.
8. 93.
9. 1621.
10. 1374.
11. 1501.
12. 603.
13. 672.
14. 439.
15. 921.
16. 891.
17. $2777\frac{1}{2}$.
18. $2004\frac{2}{3}$.
19. $1053\frac{1}{4}$.
20. $578\frac{3}{5}$.
21. $1045\frac{1}{6}$.
22. $1050\frac{6}{7}$.
23. $1140\frac{3}{8}$.
24. $897\frac{4}{9}$.
25. $23.62.
26. $11.05.
27. $7.18.
28. $6.55.
29. $3.55.
30. $4.21.
31. $8.21.
32. $9.29.
33. 20,251.
34. 5801.
35. 5211.
36. 5720.
37. 8791.
38. 3557.
39. $15{,}600\frac{1}{2}$.
40. $11{,}359\frac{1}{3}$.
41. $6557\frac{3}{4}$.
42. $14{,}004\frac{3}{5}$.
43. $13{,}389\frac{1}{6}$.
44. $4188\frac{5}{7}$.
45. $5293\frac{1}{8}$.
46. $8467\frac{6}{9}$.
47. $\$17.60\frac{1}{2}$.
48. $\$24.14\frac{1}{3}$.
49. $\$15.30\frac{3}{4}$.
50. $\$14.47\frac{1}{5}$.
51. $\$13.66\frac{5}{6}$.
52. $\$6.00\frac{3}{7}$.
53. $\$8.12\frac{4}{8}$.
54. $\$10.40\frac{5}{9}$.

Exercise 18. Page 38

1. $39.
2. 27 feet.
3. 52.
4. 144.
5. 59.
6. 44 feet.
7. 124.
8. 288.
9. $136.
10. 576.
11. 720.
12. $1.65.
13. $1.35.
14. $2.48.

Exercise 19. Page 39

1. 34.
2. 36.
3. 18.
4. 16.
5. 15.
6. 14.
7. 12.
8. 12.
9. 4.
10. 8.
11. $27\frac{3}{10}$.
12. $21\frac{3}{20}$.
13. $10\frac{25}{30}$.
14. $6\frac{35}{40}$.
15. $4\frac{36}{50}$.
16. $4\frac{47}{70}$.
17. $7\frac{52}{70}$.
18. $4\frac{71}{80}$.
19. $3\frac{8}{90}$.
20. $8\frac{57}{90}$.
21. 203.
22. 137.
23. 214.
24. 181.
25. $63\frac{1}{500}$.
26. $45\frac{403}{600}$.
27. $85\frac{221}{700}$.
28. $52\frac{748}{800}$.
29. $109\frac{107}{900}$.
30. 42.
31. 82.
32. 93.
33. $94\frac{700}{6000}$.
34. $61\frac{2820}{7000}$.
35. $92\frac{6623}{8000}$.
36. $95\frac{2542}{9000}$.
37. 24.
38. 17.
39. 27.
40. 37.
41. 165.
42. 17.
43. 175; $250.
44. 17; $1100.
45. 291; 1260.

Exercise 20. Page 41

1. 13 quarts.
2. 15 quarts.
3. 14.
4. 65.
5. 385 pounds.
6. 8120 pounds.
7. 63 pounds.
8. 12 pounds; 3 pounds.
9. Red, 14 pounds; crimson, 15 pounds.
10. 10.

Exercise 21. Page 43

1. 32.
2. 41.
3. 62.
4. 88.
5. 78.
6. 87.
7. 67.
8. 83.
9. 12.
10. 14.
11. 21.
12. 32.
13. 37.
14. 39.
15. 47.
16. 28.
17. 19.
18. 15.
19. 17.
20. 32.
21. 22.
22. 33.
23. 37.
24. 111.
25. 123.
26. 112.
27. 101.
28. 219.
29. 407.
30. 107.
31. 511.
32. 462.
33. 531.
34. 617.
35. 609.
36. 701.
37. 803.
38. 809.
39. 456.
40. 862.
41. 877.
42. 888.
43. 880.
44. 990.
45. 999.
46. 987.
47. 998.
48. 777.
49. 888.
50. 999.
51. 567.
52. 909.
53. 808.
54. 809.

Exercise 22. Page 44

1. 73.
2. 92.
3. 105.
4. 63.
5. 204.
6. 65.
7. 48.
8. 77.
9. 129.
10. 129.
11. 333.
12. 426.
13. 916.
14. 862.
15. 999.
16. 709.
17. 809.
18. 707.
19. 2111.
20. 4706.
21. 9009.
22. 352 miles.
23. 720.
24. 3075.
25. 2009.
26. 640.
27. 225.
28. 346.
29. 696.
30. 504.
31. 625.
32. 4025.
33. 329 miles.
34. 3168; 8800.

Exercise 23. Page 47

1. 84.
2. 82.
3. 76.
4. 726.
5. 950.
6. 8760.
7. 503.
8. 1060.
9. 626.
10. 1050.
11. 365.
12. 467.
13. 675.
14. 4268.
15. 468.
16. 207.
17. 783.
18. 858.
19. 367.
20. 780.
21. 1332.
22. 1875.
23. 2397.
24. 2086.
25. 98.
26. 79.
27. $111\frac{25}{69}$.
28. $112\frac{10}{76}$.
29. $372\frac{6}{23}$.
30. $365\frac{12}{56}$.
31. $283\frac{1}{64}$.
32. $259\frac{34}{57}$.
33. $416\frac{4}{46}$.
34. $564\frac{15}{47}$.
35. $467\frac{6}{59}$.
36. $682\frac{20}{69}$.
37. $617\frac{8}{82}$.
38. $436\frac{8}{232}$.
39. $890\frac{10}{124}$.
40. $612\frac{6}{412}$.
41. $185\frac{25}{653}$.
42. $617\frac{6}{422}$.
43. $831\frac{100}{232}$.
44. 36.
45. 54.
46. 37.
47. 75.
48. 77.
49. 98.
50. $564\frac{56}{2836}$.
51. $384\frac{40}{6257}$.
52. $873\frac{45}{5947}$.
53. $144.
54. $21.
55. $41.
56. $81.
57. 1411.
58. 1350.
59. 6480.
60. $25.
61. 5746.
62. $4938.
63. $18,246.
64. 476.
65. $780.
66. $90.

Exercise 24. Page 49

1. $8.76.
2. $1.88.
3. $29.35.
4. $625.72.
5. $572.30.
6. $4213.01.
7. $682.02.
8. $617.20.
9. $527.30.
10. $309.87.
11. $481.20.
12. $681.39.
13. $502.75.
14. $491.76.
15. $123.13.
16. $33.87.
17. $879.17.
18. $932.60.
19. $49.50.
20. $79.
21. $77.
22. $43.
23. $70.40.
24. $873.
25. $574.
26. $390.
27. $1198.34.
28. $2.17.
29. $1.57.
30. $9.45.
31. $2.07.
32. $3.10.
33. $3.15.
34. $2.84.
35. $2.67.
36. $4.93.
37. $2.75.
38. $3.20.
39. $3.10.
40. $10.08.
41. $2.50.

Exercise 25. Page 52

1. $9.45.
2. $8.75 ; $1.25.
3. $2.10.
4. $3.50.
5. $1.65.
6. $58.50 ; $120.25.
7. $79.60.
8. $0.21.
9. 12 ; 36.
10. 31.
11. 7425.
12. $71.55.
13. $168.75.
14. $201.60.
15. $12.70.
16. 8538 feet.
17. 36,605 miles.
18. $30.15.
19. 33,645 feet.
20. $5.49.
21. 475 tons.
22. 60 miles.
23. 348 miles.
24. 576 miles.
25. 884 ; 272.
26. $0.93.
27. 2 hours.

Exercise 29. Page 60

1. 0.20 ; 0.17.
2. 0.80 ; 0.08.
3. 0.700 ; 0.002.
4. 0.90 ; 0.13.
5. 0.600 ; 0.123.
6. 0.5 ; 0.7.
7. 0.40 ; 0.05.
8. 0.100 ; 0.357.
9. 0.120 ; 0.103.
10. 0.720 ; 0.008.
11. 0.69 ; 0.07.
12. 0.3 ; 0.7.
13. 0.990 ; 0.999.
14. 0.75 ; 0.63.
15. 0.5 ; 0.4.
16. 0.850 ; 0.392.
17. 0.001 ; 0.110.
18. 0.2070 ; 0.2376.
19. 0.412 ; 0.932.
20. 0.6810 ; 0.6881.
21. 0.9 ; 0.7.
22. 0.62 ; 0.57.
23. 0.090 ; 0.655.
24. 0.5840 ; 0.5834.

Exercise 30. Page 61

1. $\frac{1}{5}$.
2. $\frac{2}{5}$.
3. $\frac{3}{5}$.
4. $\frac{4}{5}$.
5. $\frac{1}{10}$.
6. $\frac{3}{20}$.
7. $\frac{1}{4}$.
8. $\frac{1}{8}$.
9. $\frac{1}{6}$.
10. $\frac{2}{3}$.
11. $\frac{1}{8}$.
12. $\frac{5}{8}$.
13. $\frac{1}{4}$.
14. $\frac{1}{2}$.
15. $\frac{3}{8}$.
16. $\frac{147}{200}$.
17. $\frac{7}{8}$.
18. $\frac{1}{3}$.
19. $1\frac{1}{6}$.
20. $1\frac{2}{3}$.
21. $1\frac{1}{8}$.
22. $1\frac{1}{40}$.
23. $1\frac{3}{8}$.
24. $1\frac{1}{20}$.
25. $1\frac{5}{8}$.
26. $1\frac{3}{4}$.
27. $1\frac{9}{50}$.
28. $1\frac{1}{3}$.
29. $2\frac{2}{3}$.
30. $2\frac{1}{6}$.
31. $2\frac{1}{4}$.
32. $2\frac{1}{2}$.
33. $3\frac{3}{4}$.
34. $6\frac{1}{4}$.
35. $1\frac{7}{8}$.
36. $18\frac{3}{4}$.
37. $187\frac{1}{2}$.
38. $3\frac{3}{8}$.
39. $3\frac{3}{8}$.
40. $3\frac{3}{8}$.
41. $\frac{12}{16}$.
42. $\frac{12}{16}$.
43. $\frac{4}{16}$.
44. $\frac{8}{16}$.
45. $1\frac{1}{2}$.
46. $2\frac{1}{4}$.
47. $3\frac{1}{5}$.
48. $5\frac{3}{4}$.
49. $2\frac{2}{5}$.
50. $6\frac{4}{5}$.
51. $4\frac{13}{20}$.
52. $5\frac{7}{20}$.

Exercise 31. Page 62

1. 0.5.
2. 0.25.
3. 0.125.
4. 0.75.
5. 0.375.
6. 0.625.
7. 0.875.
8. 0.2.
9. 0.4.
10. 0.6.
11. 0.8.
12. 0.0625.
13. 0.1875.
14. 0.3125.
15. 0.4375.
16. 0.9375.
17. 0.03125.
18. 0.09375.
19. 0.333$\frac{1}{3}$.
20. 0.666$\frac{2}{3}$.
21. 0.166$\frac{2}{3}$.
22. 0.833$\frac{1}{3}$.
23. 0.12.
24. 0.28.
25. 1.36.
26. 3.053$\frac{1}{3}$.
27. 0.1375.
28. 6.2875.
29. 1.38.
30. 6.125.

Exercise 32. Page 64

1. 700¢.
2. 2900¢.
3. 3500¢.
4. 17,500¢.
5. 240¢.
6. 360¢.
7. 970¢.
8. 1280¢.
9. 345¢.
10. 965¢.
11. 805¢.
12. 2963¢.
13. 14,175¢.
14. 24,385¢.
15. 69,205¢.
16. 90,009¢.
17. $3.
18. $4.50.
19. $5.75.
20. $6.06.
21. $7.32.
22. 5.22\frac{1}{2}$.
23. 6.33\frac{1}{3}$.
24. 1.66\frac{2}{3}$.
25. $26.42.
26. $49.
27. $72.75.
28. $96.54.
29. $0.02.
30. $0.05.
31. $0.55.
32. $0.75.
33. 27,630 m.
34. $5.24.
35. $276; 27,600¢.

Exercise 33. Page 65

1. 59.5.
2. 80.73.
3. 85.46.
4. 30.82$\frac{1}{4}$.
5. 28.12.
6. 214.47.
7. 271.28.
8. 637.59.
9. 167.88$\frac{3}{4}$.
10. 151.75$\frac{1}{5}$.
11. 40.
12. 40.
13. 40.
14. 40.08.
15. 35.55.
16. 51.166.
17. 40.61.
18. 60.1.
19. 50.2.
20. 46.96.
21. 50.
22. 50.
23. 34.86.
24. 30.
25. 35.45.
26. 20.606.
27. 2.041.
28. 83.087.
29. 334.87.
30. 918.4546.
31. 194.04.
32. 99.986.
33. 52.9.
34. 368.33$\frac{1}{3}$.
35. 365.08.
36. 206.
37. 201.
38. 88.1 miles.
39. 148.1 miles.
40. 236.2 miles.
41. 498.64 miles.
42. 141.3 miles; 84.6 miles; 88.1 miles.
43. 207.57$\frac{2}{3}$.
44. Yes.

Exercise 34. Page 68

1. 22.79.
2. 35.92.
3. 53.06.
4. 58.69.
5. 67.13.
6. 20.48.
7. 28.47.
8. 44.62.
9. 14.59.
10. 34.05.
11. 18.67.
12. 24.518.
13. 18.975.
14. 65.727.
15. 24.757.
16. 67.115.
17. 11.933.
18. 39.2235.
19. 7.175.
20. 0.53074.
21. 6.7302.
22. 2.7291.
23. 4.9727.
24. 4.86$\frac{2}{3}$.
25. 6.73$\frac{1}{3}$.
26. 5.37$\frac{1}{2}$.
27. 162.75 acres.
28. $0.1335.

29. $318.91\frac{2}{3}$ barrels.
30. 3.0997 grains.
31. 46.25 feet.
32. $267.96\frac{2}{3}$ acres.
33. 9.16 feet.
34. 11.1 miles.
35. 510.8 miles.
36. 67.5 miles.
37. 177.8 miles.

Exercise 35. Page 70

1. 13.4.
2. 39.2.
3. 6.3.
4. 4.8.
5. 16.25.
6. 19.25.
7. 20.
8. 60.
9. 17.
10. 25.5.
11. 35.175.
12. 56.072.
13. 31.425.
14. 10.
15. 10.
16. 40.
17. 33.
18. 32.643.
19. 42.624.
20. 42.924.
21. 18.496.
22. 0.3.
23. 1.94.
24. 0.222.
25. 60.
26. 9.98.
27. 3.125.
28. 4.
29. 210.
30. 100.198.
31. 1002.001.
32. 127.75.
33. 702.675.
34. 49.9125.
35. 83.754.
36. $130.725\frac{1}{3}$.
37. 115.635.
38. 318.3231.
39. 111.11.
40. 57.75 inches; 49 inches.
41. 24.5 inches; 21.245 inches.
42. \$5; \$9.
43. 472.44; 984.25; 1456.69.
44. 165.3; 279.908.
45. \$24.125.
46. 3.2515.
47. \$131.3955.
48. 331.6 miles.
49. 158 miles.
50. 27.6.
51. 468.8 feet.
52. 12.24 acres.
53. \$8171.625.

Exercise 36. Page 72

1. 684.
2. 1884.
3. 4680.
4. 496.
5. 4960.
6. 2843.4.
7. 253.6.
8. 3128.4.
9. 31,284.
10. 312,840.
11. 50.
12. 100.
13. 10.
14. 5584.25.
15. 5,394,400.
16. 157,480.
17. \$11,325.
18. 156,250 pounds.
19. 70 inches.
20. 125.664 feet.

Exercise 37. Page 73

1. 0.49.
2. 0.81.
3. 1.36.
4. 2.34.
5. 3.75.
6. 5.36.
7. 9.43.
8. 18.24.
9. 83.52.
10. 0.186.
11. 0.576.
12. 0.264.
13. 0.774.
14. 1.284.
15. 13.3774.
16. 42.9359.
17. 67.47.
18. 37.7798.
19. 0.0889.
20. 0.1.
21. 0.1.
22. 0.4.
23. 2.4615.
24. 21.7761.
25. 81.68325.
26. 0.53801.
27. 0.007416.
28. 57.5 pounds.
29. 15 pounds.
30. 173.75 pounds.
31. 559.375 pounds.
32. 131.9472 inches.
33. 25,132.8 miles.
34. 2,722,196.4 miles.
35. \$9.8904.
36. \$12.0897.
37. \$10.3385.

Exercise 38. Page 75

1. 1.46.
2. 5.
3. 36.
4. 23.94.
5. 30.
6. 105.
7. 148.
8. 220.
9. 356.25.
10. 478.5.
11. 6.02.
12. 1100.
13. 3200.
14. 14.625.
15. 4.3967.
16. 315.99.
17. 5.0532.
18. 8.2362.
19. 3366.
20. 3366.
21. 67.552.
22. 67.552.
23. 6.7552.
24. 37.2375.
25. 1.1025.
26. 0.00665.
27. 26.162.
28. 0.63.
29. 6.3.
30. 63.
31. 300.
32. 30.
33. 3.
34. 2841.3.
35. 28.413.
36. 16.4836.
37. 1648.36.
38. 13.104.
39. 13.104.
40. 10.6436.
41. 54.912.
42. 11.718.
43. 0.7098.
44. 0.9604.
45. 334.645.
46. 3.66.
47. 1.9588.
48. 9.604.
49. 1124.12.
50. 5.9049.
51. 10.3518.
52. 553.311.
53. 16.7265.
54. 10.4247.
55. 6.39.
56. 0.639.
57. 0.000035.
58. 0.000369.
59. 4.720472.
60. 11.125497.
61. 3201.64.
62. 32.0164.
63. 1,540,000.
64. 154,000.
65. 15.4.
66. 5.577153.
67. 12.561256.
68. 6.060505.
69. 121.60608.
70. 10.271611.
71. 49.972.
72. 17.2788.
73. 2.59809.
74. 13.4442.
75. 35.552.
76. 8.888.
77. 39.715.
78. 42.42.

Exercise 39. Page 76

1. 2.26.
2. 0.62.
3. 0.45.
4. 4.5.
5. 0.045.
6. 0.2937.
7. 1.089.
8. 3.684.
9. 5.873.
10. 6.324.
11. 0.7059.
12. 465.2.
13. 0.07564.
14. 0.09852.
15. 0.00479.
16. $0.43.
17. 45.15 miles.
18. $27\frac{9}{25}$ miles.
19. 62.5.

Exercise 40. Page 78

1. 2.8.
2. 12.5.
3. 7.2.
4. 7.3.
5. 7.8.
6. 12.1.
7. 0.0422.
8. 0.241.
9. 0.234.
10. 0.0017.
11. 136.
12. 178.
13. 91.
14. 143.
15. 460.
16. 520.
17. 0.17.
18. 0.24.
19. 0.03.
20. 280.
21. 11.
22. 1.85.
23. 76.
24. 340.
25. 0.29.
26. 0.207.
27. 0.333.
28. 0.66.
29. 0.44.
30. 0.29.
31. 20.
32. 0.03125.
33. 2.03.
34. 146.61 feet.
35. 20.
36. 24,500.
37. 80,000.
38. 174.
39. 2200 bushels.
40. 32.
41. 164.
42. $7.20.
43. 720 +.
44. 2.78.
45. 1.92.
46. 1.20 ounces.
47. 275.

Exercise 41. Page 80

1. 2.5.
2. 25.
3. 2.5.
4. 1.7.
5. 0.17.
6. 17.
7. 1.7.
8. 2.3.
9. 23.
10. 2.3.
11. 2.1.
12. 21.
13. 2.1.
14. 1000.
15. 10,000.
16. 10,000.
17. 2090.
18. 20.9.
19. 2090.
20. 50.
21. 0.5.
22. 7.6.
23. 76.
24. 0.76.
25. 340.
26. 3.4.
27. 0.34.
28. 1.7.
29. 17.
30. 17.
31. 8.2.
32. 11.792.
33. 9100.
34. 51.
35. 6.
36. 980.
37. 78.
38. 7.1.
39. 81.
40. 65.
41. 91.
42. 0.7.
43. 21.
44. 1.2.
45. 25.
46. 2.5.
47. 9.1.
48. 0.39.
49. 5.
50. 182.
51. 290.
52. 28.5.
53. 2.95.
54. 3.1.
55. 493.
56. 493.
57. 0.493.
58. 0.502.
59. 5.02.
60. 1.1.
61. 11.
62. 1.85.
63. 1076.
64. 10.76.
65. 2000.
66. 2.03.
67. 2.03.
68. 24,500.
69. 800.
70. 174.
71. 2205.
72. 2.16.
73. 216.
74. 7.059.
75. 4652.
76. 14.118.
77. 35.295.
78. 111.

Exercise 42. Page 81

1. $0.60.
2. $0.70.
3. $14.
4. 10¢.
5. 22¢.
6. $1.14.
7. 500; 450.
8. 90¢.
9. 0.23.
10. $360.
11. 0.25.
12. 0.28.
13. $728.
14. 0.26.
15. $552; $624; $240; $1416.
16. $528.
17. $83.09.
18. 586.4.
19. $15,000.
20. 4.43.
21. 3.28.
22. 360.9375.
23. $45.
24. $48.75.
25. $3.60.
26. $3.60.
27. $35.52.
28. $46.61.
29. $680; $746.67.
30. $26; $29.60.
31. $121.60.
32. 1446.3 feet.
33. 31.3 feet.
34. 12.4 feet.
35. $1497.
36. 3.
37. 27.
38. 51.
39. 1.095.
40. $93.33\frac{1}{3}$.
41. 0.913.
42. 13.11.
43. 637.5625.
44. 72.45.
45. 0.05.
46. $50.74.
47. 212.25.

Exercise 44. Page 89

1. $2.50.
2. $82,510.
3. $62.50.
4. $55.
5. $37.
6. $287.50.
7. $250.
8. $6375.
9. $3,211,200.
10. $37,610.
11. $43,077.50.
12. $3600.
13. $16,000.
14. $8125.
15. $16,875.
16. $637.50.
17. $1881.25.
18. $80,000.
19. $61,000.
20. $80,000.
21. $800,000.
22. $350,000.
23. 1200.
24. 1600.
25. 4800.
26. $10,410.
27. 210,500.
28. $10,605.
29. 238,324.
30. $1,053,250.
31. $10,560,000.
32. $105,600.
33. 80,000,000.
34. $30.
35. $2900.
36. $2700.

37. $62.50.
38. $310.
39. $4.50.
40. $12.
41. $1200.
42. $71,320.
43. $900; $1200.
44. 300 pounds; 600 pounds.
45. $7687.50; $10,250.

Exercise 45. Page 90

1. 513.
2. 612.
3. $1.11.
4. $2.40.
5. $186.
6. $324.
7. 5163.
8. $276.
9. $460.
10. $721.
11. $244.
12. $29.
13. 651.
14. 2244.
15. 192.4.
16. 19.24.
17. 366,000.
18. 0.02.
19. $3600.
20. $4125.
21. $5400.
22. $11,000.
23. $72.
24. $171⅞.
25. $363.
26. $273.

Exercise 46. Page 92

1. $746.30.
2. $28.96.
3. $4.96.
4. $443.10.
5. $624.47.
6. 34,446.4.
7. 365.
8. 493.
9. 250.6.
10. 502.
11. $45.75.
12. $98.25.
13. 228.
14. 261.
15. 144.
16. 102.
17. 128.
18. 92.
19. 496.
20. 640.
21. 720.
22. 405.
23. 495.
24. 39.
25. 3450.
26. 5030.
27. 144.
28. 14; 11.
29. 675.
30. $54.
31. $1375.
32. $1400.
33. $46,875.
34. $742.50.
35. $108.
36. $156.25.
37. 1760.
38. $2250.
39. 2700.
40. 4000.
41. 63.
42. 87.
43. $800.
44. 1448.
45. 1111.
46. $41,500.

Exercise 47. Page 94

1. 122.
2. 1220.
3. 12,200.
4. 1.22.
5. 483.
6. 4830.
7. 48,300.
8. 4.83.
9. 1827.
10. 178.7.
11. 134.
12. 1.34.
13. 0.134.
14. $176.
15. $43.
16. $97.
17. 159.
18. 1590.
19. 15,900.
20. 159.
21. 365.
22. 3650.
23. 36.5.
24. 0.365.
25. $518.
26. $46.
27. $112.
28. $99.
29. $248.
30. $328.
31. $93.
32. $29.
33. 637.
34. 6.37.
35. 6370.
36. 63.7.
37. 637,000.
38. 364.
39. 3640.
40. 3640.
41. 147.6.
42. 12.4.
43. 12.4.
44. 1.24.
45. 180.
46. 18.
47. 1.8.
48. 0.02.
49. 1940.
50. 4208.
51. 3443.2.
52. 17.44.
53. 211.2.
54. 1.856.
55. 18.56.
56. 262.4.
57. 4512.
58. 8.25.
59. 97.8.
60. 870.
61. 642.
62. 78.6.
63. 63.6.
64. 832[illegible].
65. 780.
66. 90.
67. 384.
68. 0.384.
69. 888.

Exercise 48. Page 95

1. $\frac{21}{100}$.
2. $1\frac{1}{3}$.
3. $\frac{6}{35}$.
4. 3.
5. $3\frac{1}{2}$.
6. $3\frac{1}{3}$.
7. 56.
8. 1.
9. 10.
10. 7.
11. 5.
12. 9.
13. $\frac{1}{10}$.
14. $\frac{3}{10}$.
15. $\frac{1}{10}$.
16. 1.
17. 5.

Exercise 49. Page 96

10. 19.809.
11. 1.3263.
12. 15.151.
13. 1.0087.
14. 0.159.
15. 0.122.
16. 79.572.
17. 27.006.
18. 57.529.
19. 30.09.
20. 10.06.
21. 0.35.
22. $1566.96.
23. $3524.04.
24. $1114.72.
25. $2287.22.
26. $341.71.
27. $775.14.
28. $2447.13.
29. $572.88.
30. $701.25.
31. $397.38.
32. 1876.2345.
33. $136.69.
34. $12,144.50.
35. 0.3.
36. $1829.23.
37. $100.
38. $496.40.
39. 15.836 gallons.
40. 17.444 inches.
41. 42.432 feet.
42. 3494.54 square feet; $5241.81.
43. 132.795432 feet.
44. $29.75.
45. 138.8 feet.
46. $1925.25.
47. $24.15.
48. $9037.50.
49. 9.296.
50. 70.992.
51. 4259.52.
52. 2424.558.
53. 1563.25.
54. 2165.44.
55. 5517.54.
56. 32.7096.
57. 15.76204.
58. 282.9.
59. 53.9352.
60. 53.63205.
61. 4392.63.
62. 6348.861.
63. 1244.35.
64. 408.04.
65. 5431.69.
66. 227.151.
67. 830.025.
68. 2546.2116.
69. 1744.6.
70. 52.5525.
71. 2782.35.
72. 154.8504.
73. 3238.17.
74. 225.024.
75. 329.4308.
76. 453.22.
77. 14.63775.
78. 0.170625.
79. 0.0345.
80. 1.168.
81. 0.0001052.
82. 0.01624.
83. 0.01031.
84. $4.06.
85. $1.05.
86. $1.19.
87. $7.11.
88. $6.85.
89. 74.
90. 7.198.
91. 196.54.
92. 317.2.
93. 235.68.
94. 11.853.
95. 1568.889.
96. 7810.72.
97. 45.351.
98. 270.565.
99. 11.326.
100. 6.213.
101. 3.115.
102. 66.104.

Exercise 50. Page 100

1. 655 in.
2. 873 in.
3. 84 ft.; 1008 in.
4. $16\frac{1}{2}$ ft.; 198 in.
5. 143 yd.; 429 ft.
6. 1760 yd.; 5280 ft.
7. 41 mi. 4224 ft.
8. 25 mi. 3960 ft.
9. 126 mi. 3696 ft.
10. 880 yd.
11. 1600 rd.
12. 12,320 yd.
13. 7200 rd.
14. 17,160 ft.
15. 2640 ft.
16. 3168 ft.
17. 3432 ft.
18. 17.6 yd.
19. 552.96 mi.

Exercise 51. Page 101

1. 2 yd. 2 ft.
2. 2 yd. 2 ft. 4 in.
3. 3 yd. 7 in.
4. 1 yd. 2 ft.
5. 3 yd.
6. 1 rd. 2 yd. 2 ft. 6 in.
7. 2 rd.
8. 2 rd. 4 yd.
9. 6 rd. 2 yd.
10. 2 mi. 60 rd.
11. 3 mi.
12. 4 mi.
13. 12 ft. 6 in.
14. 13 ft. 9 in.
15. 30 ft.
16. 33 ft. 4 in.
17. 50 ft.
18. 60 ft.
19. 80 ft.
20. 81 ft.
21. 20 yd.
22. 21 yd. 2 ft.
23. 166 yd. 2 ft.
24. 250 yd.
25. 2 yd.
26. 2 yd. 1 ft.
27. 1 yd. 14 in.
28. 2 yd. 8 in.

Exercise 52. Page 102

1. 114 in.
2. 436 in.
3. 513 in.
4. 347 in.
5. 30 in.
6. 66 in.
7. 44 in.
8. 56 in.
9. 110 in.
10. 159 in.
11. 238 in.
12. 318 in.
13. 14 ft.
14. 67 ft.
15. 101 ft.
16. 124 ft.
17. 100 ft.
18. 200 ft.
19. 200 ft.
20. 227 ft.
21. $82\frac{1}{2}$ ft.
22. $848\frac{1}{2}$ ft.
23. 16,560 ft.
24. 21,252 ft.
25. 121 yd.
26. 154 yd.
27. 5280 yd.
28. 1815 yd.
29. $3833\frac{1}{2}$ yd.
30. 5720 yd.
31. 160 rd.
32. 384 rd.
33. 1200 rd.
34. 4 ft.
35. 40 ft.
36. 30 ft.
37. 22 ft. 11 in.
38. 27 ft. 1 in.
39. 37 ft. 6 in.
40. 52 ft. 11 in.
41. 49 ft. 2 in.
42. 104 ft. 2 in.
43. 426 yd. 2 ft.
44. 2550 yd.
45. 1760 yd.
46. 598 yd. 1 ft.
47. 11 yd. 29 in.
48. 24 yd. 27 in.
49. 4 rd.
50. 20 rd.
51. 6 rd.
52. 4 rd. 9 ft.
53. 1 rd.
54. 63 rd. $10\frac{1}{2}$ ft.

Exercise 53. Page 103

1. 1872 sq. in.
2. 4032 sq. in.
3. 8640 sq. in.
4. 387 sq. ft.
5. 558 sq. ft.
6. 3751 sq. yd.
7. 960 sq. rd.
8. 2400 sq. rd.
9. 5600 sq. rd.
10. 3200 A.
11. 16,000 A.
12. 96,000 A.
13. 10,836 sq. in.
14. 2650 sq. in.
15. 476 sq. ft.
16. 1 sq. rd. 21 sq. yd. 8 sq. ft. 20 sq. in.
17. 1 A. 16 sq. rd.
18. 4 A. 102 sq. rd. 25 sq. yd. 4 sq. ft. 72 sq. in.

Exercise 54. Page 104

1. 172.8 sq. in.
2. 244.8 sq. in.
3. 252 sq. in.
4. 648 sq. in.
5. 3240 sq. in.
6. 907.2 sq. in.
7. 507 sq. in.
8. 368 sq. in.
9. 648 sq. in.
10. 2 sq. ft.
11. 5 sq. ft.
12. 6 sq. ft.
13. 7 sq. ft.
14. 531 sq. ft.
15. 78.3 sq. ft.
16. 382.5 sq. ft.
17. 3 sq. ft.
18. $1361\frac{1}{4}$ sq. ft.
19. 1769.625 sq. ft.
20. 1973.8125 sq. ft.
21. $35,392\frac{1}{2}$ sq. ft.
22. 1 sq. yd.
23. 5 sq. yd.
24. 12 sq. yd.
25. 89 sq. yd.
26. 1830.125 sq. yd.
27. $3660\frac{1}{4}$ sq. yd.
28. $18,301\frac{1}{4}$ sq. yd.
29. $7320\frac{1}{2}$ sq. yd.
30. 4840 sq. yd.
31. 2420 sq. yd.
32. 3630 sq. yd.
33. $1613\frac{1}{3}$ sq. yd.
34. 1 sq. rd.
35. 4 sq. rd.
36. 20 sq. rd.
37. 40 sq. rd.
38. 4 sq. rd.
39. 8 sq. rd.
40. 20 sq. rd.
41. 10 sq. rd.
42. 1120 sq. rd.
43. 240 sq. rd.
44. 280 sq. rd.
45. 260 sq. rd.
46. 3 A.
47. 5 A.
48. 7 A.
49. 1 A.
50. 5 A.
51. $5\frac{1}{2}$ A.
52. 4480 A.
53. 1600 A.
54. 2400 A.
55. 2 sq. mi.
56. 3 sq. mi.
57. $\frac{1}{2}$ sq. mi.
58. $3\frac{1}{2}$ sq. mi.
59. 7 sq. mi.
60. 9 sq. mi.

Exercise 55. Page 105

1. 5842 sq. ft.
2. 5320 sq. in.
3. 18,450 sq. yd.
4. 220 sq. ft.
5. 19 sq. ft.
6. 430 sq. in.
7. $4\frac{2}{3}$ sq. ft.
8. $101\frac{1}{2}$ sq. ft.
9. 512.
10. $29\frac{37}{40}$.
11. $3334\frac{5}{8}$.
12. 18 rd.

Exercise 56. Page 106

1. 15,072 sq. ft.
2. $503\frac{3}{4}$ sq. ft.
3. $92\frac{1}{2}$ A.
4. $91\frac{2}{3}$ sq. ft.
5. 27 ft.
6. 26 ft.
7. 32 ft.
8. 64 rd.

Exercise 57. Page 107

1. 658 sq. ft.
2. $937\frac{1}{2}$ sq. ft.
3. 1443.75 sq. ft.
4. 2000 sq. rd.
5. $1093\frac{1}{2}$ sq. in.
6. $10\frac{1}{2}$ sq. ft.
7. 1591 sq. in.
8. $19\frac{5}{6}$ sq. ft.
9. $47\frac{13}{16}$ sq. ft.
10. $16\frac{5}{8}$ sq. ft.
11. 8 ft.
12. 20 ft.
13. 22 ft.
14. 54 in.
15. $10,260.25.
16. 15.57 sq. in.
17. $157\frac{1}{2}$ sq. ft.
18. $10\frac{1}{3}$ in.
19. $4\frac{1}{2}$ in.
20. $3\frac{1}{3}$ ft.
21. $154\frac{3}{8}$ sq. ft.
22. 52.25 sq. ft.
23. 636 ; 2544.
24. 60 rd.
25. $127\frac{1}{2}$ sq. ft.
26. 41,925 sq. in.
27. $6254\frac{3}{4}$ sq. ft.
28. $18\frac{1}{16}$ sq. in.; $9\frac{1}{32}$ sq. in.; $18\frac{1}{16}$ sq. in.; $18\frac{1}{16}$ sq. in.

Exercise 58. Page 110

1. 28.
2. 32.
3. 22.
4. 25.
5. 30.
6. 42.
7. 56.
8. 89.
9. 109.
10. 103.
11. 98.
12. 103.
13. $5.94.
14. $7.26.
15. $7.04.
16. $8.80.
17. $6.60.
18. $6.16.
19. $6.16.
20. $5.06.
21. $5.50.
22. $5.72.
23. $10.78.
24. $13.42.
25. $0.34.
26. $0.38.
27. $0.46.
28. $0.42.
29. $0.68.
30. $0.80.
31. $0.76.
32. $1.16.
33. $1.66.
34. $2.34.
35. $3.12.
36. $4.44.
37. $37.67.
38. $24.30.
39. $26.50.

Exercise 59. Page 111

1. 56.
2 46$\frac{2}{3}$.
3. 58$\frac{2}{3}$.
4. 30.
5. 34.
6. 44$\frac{1}{3}$.
7. 66$\frac{2}{3}$.
8. 69$\frac{1}{3}$.
9. 106$\frac{2}{3}$.
10. $34.42.
11. 8; 24 in.
12. $26.63.

Exercise 60. Page 113

1. 3456 cu. in.
2. 43,200 cu. in.
3. 129,600 cu. in.
4. 216,000 cu. in.
5. 57,600 cu. in.
6. 115,200 cu. in.
7. 675 cu. ft.
8. 972 cu. ft.
9. 3375 cu. ft.
10. 3200 cu. ft.
11. 16,000 cu. ft.
12. 64,000 cu. ft.
13. 2 cu. ft.
14. 5 cu. ft.
15. 91 cu. yd.
16. 181 cu. yd. 13 cu. ft.
17. 1 cu. ft. 272 cu. in.
18. 2 cu. ft. 804 cu. in.
19. 43,265 cu. in.
20. 415$\frac{1}{2}$ cu. ft.
21. 10 cd. 10 cu. ft.;
47 cu. yd. 21 cu. ft.
22. 10,336 cu. in.
23. 9504 cu. in.
24. 8160 cu. in.
25. 9367 cu. in.
26. 5220 cu. ft.
27. 1143 cu. ft.
28. 1302 cu. ft.
29. 1462$\frac{1}{2}$ cu. ft.
30. 2016$\frac{2}{3}$ cu. ft.
31. 35 cu. ft.
32. 46$\frac{1}{4}$ cu. ft.
33. 183$\frac{11}{24}$ cu. ft.
34. 1292.
35. $83.20.
36. $68.95.
37. 1830.
38. 4096 cu. in.
39. 7840.
40. 262 sq. in.
41. 36$\frac{3}{4}$ sq. ft.
42. 218$\frac{29}{72}$.
43. 62$\frac{1}{2}$ lb.
44. 2025 lb.
45. 6 ft.
46. 12 ft.
47. 8 sq. in.

Exercise 61. Page 115

1. 288.
2. 403$\frac{1}{5}$.
3. 540.
4. 768.
5. 1152.
6. 1209$\frac{3}{5}$.
7. 240.
8. 527.
9. 681$\frac{29}{40}$.
10. 750.
11. 1062$\frac{1}{2}$.
12. 1218$\frac{3}{4}$.
13. 1875.
14. 2031$\frac{1}{4}$.
15. 2325.
16. 4000.
17. 7000.
18. 9375.
19. 20,000.
20. 27,900.
21. 35,000.
22. 46,875.
23. 83,700.
24. 74,000.

25. $41,218\frac{3}{4}$.
26. $273.60.
27. $4896.
28. 12.
29. 24.
30. 13.
31. $8\frac{1}{7}$.
32. 10.
33. $10\frac{4}{5}$.
34. 15.
35. $13\frac{5}{7}$.
36. $11\frac{2}{3}$.
37. $6\frac{17}{18}$.
38. $10\frac{10}{21}$.
39. $6\frac{3}{14}$.
40. $71\frac{3}{7}$ lb.
41. $59\frac{11}{21}$ lb.
42. $1666\frac{2}{3}$.
43. $156\frac{4}{27}$.
44. $5555\frac{5}{9}$.
45. $3280.50.
46. 59.84.

Exercise 62. Page 117

1. 330 ft.
2. 59.4 ft.
3. 81.4 ft.
4. 9.24 ft.
5. 1.144 in.
6. 13,420 in.
7. 15,620 in.
8. 160.6 in.
9. 396 in.
10. 484 in.
11. 616 in.
12. 1320 in.
13. 387.2 in.
14. 431.2 in.
15. 536.8 ft.
16. 55.44 ft.
17. 84.8232 ft.
18. 135.0888 ft.
19. 100.5312 yd.
20. 14.76552 yd.
21. 1.947792 ft.
22. 2.3562 ft.
23. 7.0686 ft.
24. 3.989832 in.
25. 169.6464 in.
26. 238.7616 in.
27. 383.2752 in.
28. 30.15936 ft.
29. 11.93808 yd.
30. 16.96464 yd.
31. 3.015936 ft.
32. 3.895584 ft.
33. 20 ft.
34. 110 in.
35. 160 in.

Exercise 63. Page 118

1. 3.1416 sq. in.
2. 28.2744 sq. in.
3. 19.635 sq. in.
4. 50.2656 sq. in.
5. 201.0624 sq. in.
6. 132.7326 sq. in.
7. 78.54 sq. in.
8. 63.6174 sq. in.
9. 50.2656 sq. in.
10. 38.4846 sq. in.
11. 95.0334 sq. in.
12. 314.16 sq. in.
13. 1256.64 sq. in.
14. 2827.44 sq. in.
15. 3.1416 sq. in.
16. 50.2656 sq. in.
17. 28.2744 sq. in.
18. 78.54 sq. in.
19. 314.16 sq. in.
20. 1256.64 sq. in.
21. 0.7854 sq. in.
22. $0.3490\frac{2}{3}$ sq. in.
23. 0.031416 sq. in.
24. 0.19635 sq. in.
25. 1.76715 sq. in.
26. $1.3962\frac{2}{3}$ sq. in.
27. 19.635 sq. in.
28. 33.18315 sq. in.
29. 16.619064 sq. in.
30. 12.5664 in.
31. 7.0686 sq. in.
32. 10.9956 in.
33. 47.124 in.
34. 10 in.
35. 31.416 ft.
36. 21.46 sq. ft.
37. 220 ft.; 38,013.36 sq. ft.
38. 8.2957875 sq. in.
39. 11.0446875 sq. in.

Exercise 64. Page 121

1. 351 cu. in.
2. 612 cu. in.
3. 4750 cu. in.
4. $365\frac{1}{2}$ cu. in.
5. 405 cu. in.
6. 35 cu. ft. 1320 cu. in.
7. $2\frac{3}{8}$ ft.
8. 6 ft. 3 in.
9. 18 sq. ft.
10. 19 sq. ft.
11. 13 sq. ft.
12. 15 sq. ft. 36 sq. in.
13. 1357.1712 cu. in.
14. 2827.44 cu. in.
15. 6433.9968 cu. in.
16. 9160.9056 cu. in.
17. 376.992 cu. in.
18. 10.472 cu. ft.

Exercise 65. Page 122

1. 60.
2. 75.
3. $50\frac{5}{8}$.
4. $101\frac{1}{4}$.
5. $7031\frac{1}{4}$.
6. $9445\frac{5}{16}$.
7. $282\frac{6}{7}$.
8. 400.
9. $265\frac{85}{336}$.
10. $2121\frac{3}{7}$.
11. $15{,}910\frac{5}{7}$.
12. 21,120.
13. 288,750.
14. $776{,}242\frac{1}{2}$.
15. $637{,}215\frac{105}{128}$.
16. $424\frac{2}{7}$.
17. $318\frac{3}{14}$.
18. $754\frac{2}{7}$.
19. $433\frac{1}{8}$.
20. $435\frac{15}{256}$.
21. $725\frac{25}{256}$.

Exercise 66. Page 123

1. 248.
2. 585.
3. $408\frac{1}{3}$.
4. 900.
5. 1066.
6. 1248.
7. $18\frac{1}{3}$.
8. $26\frac{2}{3}$.
9. $18\frac{1}{3}$.
10. 50.
11. $105.60.
12. 36,740; $303.11.
13. 1152; 46.08; $414.72.
14. 61,424; $509.82.
15. 210,364.

Exercise 67. Page 125

1. $9\frac{1}{2}$ T.
2. 510,000 lb.
3. 7000 lb.
4. 12,000 lb.
5. 45,000 lb.
6. 40,000 lb.
7. 100,500 lb.
8. 4800 lb.; 13,200 lb.
9. 4800 lb.; 1200 lb.; 120,000 lb.
10. 5600 lb.; 1400 lb.; 140,000 lb.
11. 800 lb.; 8000 lb.; 80,000 lb.
12. 30; 16.
13. $33\frac{1}{3}$.
14. $62\frac{1}{2}$.
15. $\frac{14}{15}$; $\frac{14}{15}$.
16. $\frac{1}{2}$; $\frac{1}{3}$.
17. $\frac{1}{6}$; $\frac{1}{6}$.
18. $\frac{1}{2}$; $\frac{1}{3}$.
19. $\frac{1}{14}$; $\frac{1}{7}$.
20. $\frac{1}{5}$; $\frac{7}{15}$.
21. $362.50.
22. $984.38.
23. $770.25.
24. $14.70.
25. 375 lb.; 4375 lb.
26. $1\frac{3}{40}$ T.
27. 0.925.
28. 256 oz.
29. 40 oz.
30. 32,000 lb.
31. 57,000 lb.
32. 11 lb.
33. 12 lb. 8 oz.
34. 1 T.
35. 5 T.
36. 476 oz.
37. 906 oz.
38. 867 oz.
39. 437 oz.
40. 54,000 lb.
41. 5400 lb.
42. 9 lb.
43. 6700 lb.
44. $98\frac{23}{24}$ lb.
45. $2023\frac{7}{16}$ lb.
46. 31.25 lb.; 6.25 lb.; 50 lb.
47. $\frac{1}{5}$; $\frac{2}{5}$; $\frac{4}{5}$; $\frac{1}{2}$.
48. 625 lb.; 1875 lb.
49. $31\frac{1}{4}$ lb.; $15\frac{5}{8}$ lb.; $7\frac{13}{16}$ lb.
50. 125 lb.
51. 15 lb.
52. $1206\frac{1}{4}$ lb.
53. $5250.
54. 70 lb.
55. 15.
56. 4975 lb.
57. $768\frac{9}{32}$.

Exercise 68. Page 129

1. 74 pt.
2. 98 pt.
3. 198 pt.
4. 512 pt.
5. 33 pt.
6. 75 pt.
7. 175 pt.
8. 497 pt.
9. 384 qt.
10. 472 qt.
11. 538 qt.
12. 2150 qt.
13. 188 pk.
14. 226 pk.
15. 371 pk.
16. 1709 pk.
17. 1 bu. 4 qt.
18. 1 bu. 6 qt.
19. 1 bu. 2 pk. 1 qt.
20. 2 bu.
21. 1 bu. 3 pk. 4 qt.
22. 3 bu. 2 qt.
23. 24 bu.
24. 22 bu. 1 pk.
25. 18 bu. 3 pk.
26. 2 bu. 1 pk. 6 qt.
27. 4 bu. 3 pk. 4 qt.
28. 39 bu.
29. 144 pt.
30. 171 pt.
31. 137 pt.
32. 784 pt.
33. 856 pt.
34. 1212 pt.
35. 128 pt.
36. 2240 pt.
37. 1824 pt.
38. 38 pt.
39. 94 pt.
40. 144 pt.
41. 304 qt.
42. 388 qt.
43. 478 qt.
44. 625 qt.
45. 2400 qt.
46. 2800 qt.
47. 88 qt.
48. 296 qt.
49. 288 pk.
50. 358 pk.
51. 18 pk.
52. 36 pk.
53. 84 bu.
54. 129 bu.
55. 193 bu.
56. 239 bu.
57. 11 bu.
58. 61 bu.
59. 31 bu.
60. 99 bu.
61. 300.
62. 720.
63. 43,200 lb.
64. 5.
65. 22,400 lb.

Exercise 69. Page 131

1. 108 gi.
2. 184 gi.
3. 138 pt.
4. 174 pt.
5. 356 qt.
6. 272 qt.
7. 257 qt.
8. 125 pt.
9. 3 gal.
10. 12 gal.
11. 1 gal. 3 gi.
12. 10 gal. 2 qt. 1 pt.
13. 75 gal.
14. 59 gal. 1 qt. 1 pt.
15. 23 gal. 1 pt.
16. 9 gal. 1 qt. 1 pt. 2 gi.
17. 99 pt.
18. 223 pt.
19. 253 pt.
20. 636 pt.
21. 99 qt.
22. 149 qt.
23. 3707 qt.
24. 291 qt.
25. 1024 oz.
26. 9504 cu. in.
27. 156 oz.
28. 1080 sq. in.
29. 192,000 lb.
30. $196\frac{5}{8}$ sq. yd.
31. 97,500 lb.
32. 400 sq. rd.
33. 33 lb.
34. 39 lb.
35. 23 lb. 7 oz.
36. 28 lb. 2 oz.
37. 1 T. 187 lb. 8 oz.
38. 1 T. 11 lb.
39. 6 T. 500 lb.
40. 37 T. 1550 lb.
41. 12 oz.
42. $3\frac{1}{2}$ oz.
43. 1158 oz.
44. $466\frac{1}{2}$ oz.
45. 1550 oz.
46. 476 oz.
47. 35,200 oz.
48. 157 oz.
49. 85,500 lb.
50. 131,600 lb.
51. 7200 lb.
52. $6666\frac{2}{3}$ lb.
53. 27 lb.
54. 302 lb.
55. 4800 lb.
56. 10,650 lb.
57. 59,360.
58. 175,000.
59. 1032.
60. 625,000 lb.
61. One of 50 lb., and one of 25 lb.
62. $207\frac{1}{2}$ lb.; 398.4 lb.
63. 4.15 lb.; 1.0375 lb.; 6.225 lb.; 20.75 lb.
64. 27.9 lb.; 15.45 lb.; 21.675 lb.
65. 1087.5 lb.; 2946.5 lb.; 380.14 lb.; 12,450 lb.
66. 261.45 lb.

67. 8.549 lb.; 2.13725 lb.; 1.068625 lb.
68. 6.557 lb.; 206.5455 lb.
69. 5.81 lb.; 183.015 lb.
70. 559.375 lb.; 74.285 lb.
71. 8.217 lb.; 258.8355 lb.
72. 192 cu. ft.; 331,776 cu. in.; 1436.26 gal.
73. 132 cu. ft.; 228,096 cu. in.; 987.428 gal.
74. 864.
75. 77½ lb.

Exercise 70. Page 134

9. 4; 8. **10.** 3; 4; 6; 5. **11.** 12; 24; 20; 28. **12.** 8; 48; 16; 32; 2.

Exercise 71. Page 135

1. 240d.
2. 480d.
3. 60d.
4. 264d.
5. 348d.
6. 672d.
7. 270d.
8. 833d.
9. 1108½d.
10. 40s.
11. 100s.
12. 140s.
13. 30s.
14. 75s.
15. 139s.
16. 4s.
17. 8s.
18. 20s.
19. £3.
20. £7.
21. £24.
22. £1.
23. £2.
24. £5.
25. £35.
26. £3.
27. £75.
28. $233.76.
29. $365.25.
30. $2313.25.
31. £5.
32. £7.
33. £110.

Exercise 72. Page 136

1. 8.50 fr.
2. 3.75 fr.
3. 4.80 fr.
4. 12.50 fr.
5. 4 M.
6. 1.75 M.
7. 16 M.
8. 92.75 M.
9. $\frac{3}{4}$.
10. $\frac{3}{4}$.
11. 0.772.
12. 0.965.
13. 0.952.
14. 0.952.
15. 18.85 fr.
16. 1.53 M.

Exercise 73. Page 138

1. 21,600 sec.
2. 1080 min.
3. 2160 min.
4. 384 hr.
5. 864 hr.
6. 1164 hr.
7. 364 da.
8. 182 da.
9. 325 da.
10. 96 mo.
11. 192 mo.
12. 384 mo.
13. 6 min.
14. 12 hr.
15. 3 wk.
16. 1 wk.
17. 11 yr.
18. ½ yr.
19. 0.25 of a century.
20. 52 wk. 1 da.
21. 92.
22. 153.
23. 18,262½.
24. 500.
25. 1928; 1940; 1960; 1972; 2000.
26. Saturday.
27. 55,937 sec.
28. 3329 min.
29. 87,963 min.
30. 29; 696.
31. 4 wk. 1 da. 12 hr. 44 min. 3 sec.
32. $71.50.
33. 30.
34. 1 hr. 5¼ min.
35. 1 hr. 21 min.
36. 2 hr. 55 min.
37. 13 hr.
38. $\frac{5}{11}$.
39. 1.55 P.M.
40. 4.30 P.M.
41. 5.30 P.M.
42. 1 hr. 26¼ min.
43. 1.50 P.M.
44. 100; 135; 210.

Exercise 74. Page 141

1. 1020″.
2. 1920″.
3. 2880″.
4. 3600″.
5. 10,800″.
6. 97,200″.
7. 11,100″.
8. 14,880″.
9. 29,108″.
10. 32,762″.
11. 54,630″.
12. 66,035″.
13. 324,000″.
14. 162,000″.
15. 109,830″.
16. 8′.
17. $9\frac{1}{3}$′.
18. 7′.
19. 15′.
20. 6′.
21. 4′.
22. 1620′.
23. 2100′.
24. 2700′.
25. 549′.
26. 610′.
27. 2130′.
28. 1676′.
29. 2930′.
30. 3560′.
31. 1° 20′.
32. 1°.
33. $\frac{1}{2}$°.
34. 2° 40′.
35. 45°.
36. 90°.
37. 14°.
38. 30°.
39. 60°.
40. 180°; 90°.
41. 21,600′.
42. 5400′; 324,000″.
43. $11\frac{1}{4}$°.
44. 45°; 6°.
45. 1410′.
46. 43°.

Exercise 75. Page 142

1. 576.
2. 216; 1296; 5040.
3. 6; 32; 69.
4. $63.
5. 80; 60; 90.
6. 48; 840; 1800.
7. $2.10; 336.
8. $2.40.
9. $27.
10. 60¢.
11. 1680.
12. $1.19; $0.81; $1.50.
13. 32¢.

Exercise 76. Page 143

1. $1\frac{1}{4}$.
2. $2\frac{1}{4}$.
3. $2\frac{11}{32}$.
4. $2\frac{1}{2}$.
5. $2\frac{1}{4}$.
6. $3\frac{3}{4}$.
7. 3.
8. $4\frac{15}{32}$.
9. $5\frac{2}{3}$.
10. $18.75.
11. $37.69.
12. $17\frac{3}{4}$.
13. $12.03.
14. $25.31.
15. $4.
16. 24 ft.
17. 4 ft.
18. 480.
19. $12\frac{3}{8}$.
20. $16\frac{13}{32}$.
21. $42.19.
22. 65.63.
23. $5\frac{1}{4}$.
24. $7\frac{13}{16}$.
25. $16.41.

Exercise 77. Page 145

1. 2 yd.
2. 37 ft. 10 in.
3. 51 lb. 12 oz.
4. 35 rd. 15 ft.
5. 36 rd.
6. 143 lb. 8 oz.
7. 4 lb. 7 oz.
8. 50 lb.
9. 100 rd.
10. 50 mi.
11. 80 mi. 10 rd.
12. 83 yd.
13. 71 ft. 1 in.
14. 51 lb. 2 oz.
15. 58 lb.
16. 71 rd.
17. 151 mi. 5 rd.
18. 51 yd. 2 in.
19. 64 yd. 18 in.
20. 24 da. 12 hr.
21. 9 wk. 1 da.
22. 78 bu. 3 pk.
23. 94 bu. 1 pk.
24. 24 ft. 3 in.
25. 250 sq. ft. 71 sq. in.
26. 258 cu. ft. 626 cu. in.
27. 1431 lb. 11 oz.
28. 100 ft. 5 in.
29. 211 sq. ft. 65 sq. in.
30. 251 cu. ft. 920 cu. in.

31. 1857 lb. 11 oz.
32. 139 gal. 3 qt.
33. 241 gal. 5 pt.
34. 139 ft.
35. 21 hr. 38 sec.
36. 209 gal. 3 qt.
37. 237 bu. 3 pk.
38. 284 ft. 6 in.
39. 25° 16″.
40. 166 qt.
41. 94 pk. 1 qt.
42. 86 wk. 2 da.
43. 104 hr. 12 min. 57 sec.
44. 130° 58′ 47″.
45. 10 A.
46. 19 sq. rd.
47. 18 cu. yd. 1 cu. ft.
48. 30 lb. 13 oz.

Exercise 78. Page 148

1. 9 in.
2. 1 gal. 3 qt.
3. 3 in.
4. 49 ft. 4 in.
5. 7 lb. 14 oz.
6. 8 rd. $12\frac{1}{2}$ ft.
7. 10 mi. 160 rd.
8. 6 yd. 27 in.
9. 17 rd. $12\frac{1}{2}$ ft.
10. 2 mi. 280 rd.
11. 6 rd. $2\frac{1}{2}$ ft.
12. 5 rd. 10 ft.
13. 7 rd. $15\frac{1}{4}$ ft.
14. 15 bu. 3 pk.
15. 7 mi. 40 rd.
16. 9 mi. 300 rd.
17. 46 mi. $319\frac{3}{4}$ rd.
18. 21 gal. 2 qt.
19. 9 lb. $3\frac{1}{3}$ oz.
20. 8 lb. $14\frac{2}{3}$ oz.
21. 5 yd. $31\frac{3}{4}$ in.
22. 26 qt. 1 pt.
23. 12 ft. 9 in.
24. 1 yd. 1 ft.
25. 103 lb. 6 oz.
26. 12 gal. 3 qt.
27. 17 ft. $10\frac{1}{4}$ in.
28. 9 bu. 7 qt.
29. 74 lb. 8 oz.
30. 34 qt. 1 pt.
31. 48 sq. ft. 130 sq. in.
32. 7 hr. 15 min.
33. 74 cu. ft. 797 cu. in.
34. 17 mi. 268 rd.
35. 20 sq. ft. $48\frac{3}{4}$ sq. in.
36. 15 A. 122 sq. rd.
37. 47 cu. ft. 1453 cu. in.
38. 56 min. 38 sec.
39. 31 ft. 11 in.
40. 20 lb. 15 oz.
41. 5 rd. $15\frac{1}{2}$ ft.
42. 7 yd. 24 in.
43. 15 mi. 229 rd.
44. 17 yd. 32 in.
45. 16 ft. 9 in.
46. 116 mi. 275 rd.
47. 48 lb. 14 oz.
48. 28 rd.
49. 19 mi. 240 rd.
50. 27 rd. 7 ft.
51. 1 yr. 10 mo. 29 da.
52. 4 yr. 4 mo. 18 da.
53. 2 yr. 5 mo. 14 da.
54. 2° 20′ 47″.
55. 24′ 4″.
56. 1° 53′ 50″.
57. 48° 36′ 38″.
58. 13′ 37.5″.

Exercise 79. Page 151

1. 6 ft. 8 in.
2. 30 ft.
3. 73 ft. 6 in.
4. 27 yd. 18 in.
5. 65 yd. 4 in.
6. 43 yd. 6 in.
7. 1187 ft. 6 in.
8. 4709 yd. 12 in.
9. 481 ft. 9 in.
10. 1127 lb.
11. 2725 yd. 24 in.
12. 262 ft. 6 in.
13. 748 ft. 9 in.
14. 1069 ft. 9 in.
15. 603 qt. 1 pt.
16. 1568 gal. 3 qt.
17. 3007 bu.
18. 92 hr. 42 min.
19. 60,137 mi. 1940 ft.
20. 393 min. 36 sec.
21. 724 min.
22. 450 min.
23. 110 lb. 4 oz.
24. 60 rd. 6 ft.
25. 45 lb. 12 oz.
26. 29 gal. 1 qt.
27. 25 gal. 2 qt.

28. 51 mi. 720 ft.
29. 2283 lb. 12 oz.
30. 3916 mi. 267 ft.
31. 9560 ft. 10 in.
32. 20,723 lb. 15 oz.
33. 30,712 yd. 11 in.
34. 191 sq. ft. 132 sq. in.
35. 57 cu. ft. 1116 cu. in.
36. 216 cu. ft. 1600 cu. in.
37. 988 lb. 12 oz.
38. 774 min. 40 sec.
39. 1110 min.
40. 150 bu.
41. 39 T. 600 lb.
42. 29,492 mi. 5110 ft.
43. 10,500 mi. 500 yd.
44. 10,501 mi. 280 rd.
45. 2003 lb. 12 oz.
46. 920 ft. 10 in.
47. 1145 ft. 10 in.
48. 35 ft.
49. 31 ft.
50. 472 gal. 2 qt.
51. 21 lb.
52. 4 hr. 30 min.
53. 2191 da. 10 hr. 52 min. 36 sec.
54. 3000 lb.
55. 24,375 lb.
56. 24,375 lb.
57. 14,625 lb.
58. 109 ft. 8 in.

Exercise 80. Page 153

1. 2 ft. 5 in.
2. 21 ft. 11 in.
3. 3 lb. 2 oz.
4. 4 yd. 33 in.
5. 3 mi. 4864 ft.
6. 24 min. 21 sec.
7. 7.
8. 9.
9. 4.
10. 11.
11. 9.
12. 20.
13. 8.
14. 32.
15. 1 min. 13 sec.
16. 8; 242.
17. $1\frac{4}{5}$.
18. 72.
19. 8.
20. 35.
21. 6.
22. 7.
23. $7\frac{1}{2}$.
24. 144.
25. 112.
26. $17\frac{1}{2}$.

Exercise 83. Page 159

1. 392 in.
2. 23 ft. 6 in.; $23\frac{1}{2}$ ft.
3. $2.75.
4. 1200.
5. 7.
6. 3.
7. 447.
8. $34\frac{18}{23}$.
9. 3963 mi. 1562 ft.
10. $21\frac{9}{16}$.
11. 30 in.; $2\frac{1}{2}$ ft.; 2.5 ft.; $\frac{5}{6}$ yd.; $\frac{5}{33}$ rd.
12. 56 in.; 4 ft. 8 in.; $4\frac{2}{3}$ ft.; $4.66\frac{2}{3}$ ft.
13. 62.5 lb.; 62 lb. 8 oz.; $62\frac{1}{2}$ lb.; 62.5 lb.
14. $16,008\frac{1}{3}$.
15. $129\frac{1}{31}$.
16. 84 ft.; $27\frac{1}{2}$ fathoms.
17. 5 hr. 52 min.
18. 1 mi. 4764 ft.
19. 25,875 T.
20. 1.85 min.
21. 1.42 min.
22. 6 min. 47 sec.
23. 6 min. 48 sec.
24. 3200; 20.
25. 40; 160.
26. 1200.
27. 12,243.75 lb.
28. 12.5.
29. $11\frac{1}{5}$ T.; 18 T.
30. $195\frac{5}{12}$ lb.; $53,202\frac{3}{16}$ lb.; 4256 T. 350 lb.
31. $62.08.
32. $31.62.
33. 12 da.; $73.50.
34. $232.75.
35. 12,375 lb.
36. 1056 lb.
37. $1518.
38. 62,475 lb.
39. 216.2 lb.
40. 24.2 T.; 30.25 T.; 36.3 T.

Exercise 84. Page 163

1. \$12.50.
2. \$3.90.
3. \$2.44.
4. \$13.50.
5. \$27.
6. \$6.65.
7. \$29.70.
8. \$115.20.
9. \$120.
10. \$4.38.
11. \$1.50.
12. \$363; \$242; \$242.
13. \$163.80.

Exercise 85. Page 165

1. 150.
2. 171.
3. 175 da.
4. 27 da.
5. 2 min. $11\frac{1}{4}$ sec.
6. 1023 lb.
7. 3 hr. $46\frac{4}{5}$ min.
8. 22.5.
9. 3480.
10. 8 ft. $5\frac{1}{5}$ in.
11. 2 hr. 12 min.
12. $2\frac{5}{8}$ mi.
13. \$15.
14. 63.
15. 5 bu.

Exercise 86. Page 167

1. 100.
2. 53.
3. 1280.
4. 960.
5. 1600.
6. \$4809.38.
7. \$3200.
8. \$152.
9. 220.
10. $3\frac{1}{2}$¢.

Exercise 87. Page 168

1. 100,412 T. 974 lb.
2. 8735 T. 1000 lb.
3. 5,119,737,264.
4. 19,711,840,608.
5. \$3000.
6. 500 lb.
7. \$241,600.
8. 70.
9. \$138.24; \$57.60.

Exercise 88. Page 169

1. \$0.90.
2. $\frac{2}{5}$¢.
3. \$0.13.
4. \$0.06.
5. \$1.62.
6. \$0.38.
7. \$2.64.
8. 24¢.
9. 10¢; 30¢.

Exercise 89. Page 170

1. 5.
2. 720.
3. 560.
4. 3454.70.
5. 16 in.
6. 24 in.
7. \$4.32.
8. 5 A.
9. $3055\frac{7}{18}$.
10. 20 rd.
11. 20 rd.
12. 660; $4\frac{1}{8}$.

Exercise 90. Page 172

1. \$27.60.
2. \$6.99.
3. \$7.09.
4. \$10.82.
5. \$10.84.
6. \$3.76.
7. \$11.54.
8. \$378.45.
9. \$456.60.
10. \$1328.50.
11. \$541.92.
12. \$355.85.
13. \$670.36.
14. \$523.74.
15. \$197.98.
16. \$641.61.
17. \$1086.10.
18. \$656.38.
19. \$86.09.

Exercise 91. Page 174

1. \$8.12.
2. \$182.75.
3. \$90.26.
4. \$232.60.
5. \$288.93.
6. \$252.72.

Exercise 92. Page 176

1. \$77.74.
2. \$50.28.
3. \$184.95.
4. \$157.80.
5. \$12.40.
6. \$16.07.
7. \$18.67.
8. \$13.75.
9. \$68.12.
10. \$14.60.
11. \$1183.50.
12. \$303.60.
13. \$52.15.
14. \$1115.60.
15. \$1450.59.
16. \$1500.09.

Exercise 94. Page 181

1. 432; 640; 1024; 3036; 8292; 4878.
2. 636; 444; 1002; 3027; 1104; 6777.
3. 830; 4225; 3010; 7200; 6305; 9270; 4005.
4. 53; 61; 97.
5. 2, 2, 2, 3, 5.
6. 2, 3, 5, 7.
7. 2, 3, 5, 5.
8. 2, 2, 5, 11.
9. 2, 5, 17.
10. 5, 5, 7.
11. 3, 3, 5, 5.
12. 3, 101.
13. 2, 3, 5, 5, 5.
14. 2, 3, 3, 37.
15. 7, 11, 13.
16. 3, 5, 83.
17. 11, 11, 11.
18. 3, 3, 3, 3, 3, 3, 5.
19. 2, 2, 2, 3, 11, 13.
20. 2, 2, 2, 2, 2, 2, 3, 3, 3.
21. 2, 2, 2, 2, 2, 3, 5, 11.
22. 2, 3, 7, 11, 13.
23. 37, 101.
24. 2, 2, 2, 3, 11, 37.

Exercise 95. Page 183

1. 2.
2. 4.
3. 3.
4. 6.
5. 7.
6. 15.
7. 8.
8. 17.
9. 9.
10. 19.
11. 5.
12. 21.
13. 11.
14. 23.
15. 6.
16. 2.
17. 3.
18. 4.
19. 3.
20. 5.
21. 4.
22. 7.
23. 8.
24. 9.
25. 5.
26. 12.
27. 6.
28. 16.
29. 6.
30. 13.
31. 21.
32. 11
33. 10.
34. 16.
35. 3.
36. 12.
37. 12.
38. 38.
39. 8.
40. 11.
41. 25.
42. 64.
43. 32.
44. 9.
45. 25.

Exercise 96. Page 184

1. 65.
2. 14.
3. 37.
4. 41.
5. 144.
6. 625.
7. 61.
8. 101.
9. 29.
10. 14.
11. 229.
12. 37.
13. 53.
14. 57.
15. 71.
16. 146.
17. 29.
18. 29.

Exercise 97. Page 186

1. 66.
2. 51.
3. 48.
4. 105.
5. 351.
6. 165.
7. 105.
8. 228.
9. 390.
10. 240.
11. 882.
12. 144.
13. 180.
14. 210.
15. 6552.
16. 25,389.
17. 150.
18. 192.
19. 396.
20. 756.
21. 120.
22. 720.
23. 66.
24. 4004.
25. 560.
26. 600.
27. 480.
28. 27,720.
29. 96.
30. 1260.
31. 6930.
32. 1260.
33. 576.
34. 1890.
35. 660.
36. 495.
37. 144.
38. 312.
39. 300.
40. 30 ft.
41. 24 qt.
42. 50¢.
43. 96 gal.
44. 509.
45. 127.

Exercise 98. Page 188

1. 36.
2. 8.
3. 9.
4. 6.
5. 10.
6. 8.
7. 3.
8. 2.
9. 8.
10. 16.
11. 9.
12. 252.
13. 28.
14. 144.
15. $1\frac{1}{2}$.
16. 84.
17. 40.
18. 140.
19. 384.
20. 70.
21. \$2.
22. 3.
23. 4.
24. 9.
25. 18.
26. 9.
27. \$16.
28. \$75 ; \$108.
29. 4.
30. 8.
31. 7.
32. 7.
33. 4.
34. 5.

Exercise 99. Page 192

1. $\frac{1}{4}$.
2. $\frac{3}{7}$.
3. $\frac{9}{11}$.
4. $\frac{4}{7}$.
5. $\frac{6}{11}$.
6. $\frac{1}{3}$.
7. $\frac{1}{3}$.
8. $\frac{1}{2}$.
9. $\frac{2}{3}$.
10. $\frac{8}{25}$.
11. $\frac{1}{3}$.
12. $\frac{1}{3}$.
13. $\frac{1}{4}$.
14. $\frac{3}{4}$.
15. $\frac{5}{9}$.
16. $\frac{3}{4}$.
17. $\frac{1}{4}$.
18. $\frac{1}{3}$.
19. $\frac{16}{39}$.
20. $\frac{6}{13}$.
21. $\frac{1}{2}$.
22. $\frac{1}{3}$.
23. $\frac{2}{5}$.
24. $\frac{11}{48}$.
25. $\frac{2}{3}$.
26. $\frac{9}{32}$.
27. $\frac{1}{7}$.
28. $\frac{2}{7}$.
29. $\frac{37}{40}$.
30. $\frac{11}{12}$.
31. $\frac{11}{12}$.
32. $\frac{12}{13}$.

Exercise 100. Page 193

1. $\frac{30}{60}$.
2. $\frac{32}{60}$.
3. $\frac{27}{60}$.
4. $\frac{22}{60}$.
5. $\frac{40}{80}$.
6. $\frac{25}{80}$.
7. $\frac{28}{80}$.
8. $\frac{18}{80}$.
9. $\frac{72}{144}$.
10. $\frac{12}{144}$.
11. $\frac{30}{144}$.
12. $\frac{14}{144}$.
13. $\frac{78}{156}$.
14. $\frac{8}{156}$.
15. $\frac{10}{156}$.
16. $\frac{14}{156}$.
17. $\frac{128}{256}$.
18. $\frac{48}{256}$.
19. $\frac{40}{256}$.
20. $\frac{28}{256}$.
21. $\frac{144}{288}$.
22. $\frac{24}{288}$.
23. $\frac{60}{288}$.
24. $\frac{42}{288}$.
25. $\frac{160}{320}$.
26. $\frac{60}{320}$.
27. $\frac{50}{320}$.
28. $\frac{45}{320}$.
29. $\frac{50}{100}$.
30. $\frac{30}{100}$.
31. $\frac{16}{100}$.
32. $\frac{18}{100}$.

Exercise 101. Page 194

1. 3.
2. $3\frac{1}{6}$.
3. $3\frac{1}{3}$.
4. 5.
5. 6.
6. $2\frac{1}{4}$.
7. $2\frac{1}{2}$.
8. $4\frac{1}{2}$.
9. 5.
10. 6.
11. $1\frac{1}{2}$.
12. 3.
13. 4.
14. 4.
15. 3.
16. 2.
17. $1\frac{1}{2}$.
18. $1\frac{1}{6}$.
19. $1\frac{3}{4}$.
20. 5.
21. $1\frac{1}{5}$.
22. $1\frac{1}{2}$.
23. $1\frac{4}{5}$.
24. $1\frac{9}{10}$.
25. 7.
26. 1.
27. $1\frac{1}{8}$.
28. $1\frac{1}{4}$.
29. $1\frac{3}{8}$.
30. 7.
31. $1\frac{1}{9}$.
32. $1\frac{1}{6}$.
33. $1\frac{1}{3}$.
34. 2.
35. 4.
36. 2.
37. $2\frac{1}{16}$.
38. 3.
39. 5.
40. 6.
41. $2\frac{1}{2}$.
42. $3\frac{3}{4}$.
43. $4\frac{1}{4}$.
44. $5\frac{1}{2}$.
45. $7\frac{1}{2}$.
46. 1.
47. $2\frac{1}{2}$; 3.

Exercise 102. Page 195

1. $\frac{31}{8}$.
2. $\frac{39}{8}$.
3. $\frac{61}{8}$.
4. $\frac{55}{8}$.
5. $\frac{77}{8}$.
6. $\frac{115}{16}$.
7. $\frac{85}{16}$.
8. $\frac{137}{16}$.
9. $\frac{151}{16}$.
10. $\frac{107}{16}$.
11. $\frac{53}{24}$.
12. $\frac{127}{24}$.
13. $\frac{203}{24}$.
14. $\frac{61}{24}$.
15. $\frac{161}{24}$.
16. $\frac{227}{32}$.
17. $\frac{293}{32}$.
18. $\frac{103}{32}$.
19. $\frac{201}{32}$.
20. $\frac{75}{32}$.
21. $\frac{293}{36}$.
22. $\frac{223}{36}$.
23. $\frac{155}{36}$.
24. $\frac{265}{36}$.
25. $\frac{89}{36}$.
26. $\frac{245}{48}$.
27. $\frac{391}{48}$.
28. $\frac{107}{48}$.
29. $\frac{305}{48}$.
30. $\frac{71}{48}$.
31. $\frac{71}{64}$.
32. $\frac{331}{64}$.
33. $\frac{465}{64}$.
34. $\frac{221}{64}$.
35. $\frac{421}{64}$.
36. $\frac{77}{72}$.
37. $\frac{439}{72}$.
38. $\frac{383}{72}$.
39. $\frac{2555}{72}$.
40. $\frac{5225}{72}$.
41. $\frac{7}{8}$; $\frac{24}{8}$; $\frac{31}{8}$.
42. $\frac{5}{16}$; $\frac{80}{16}$; $\frac{85}{16}$.
43. $\frac{7}{32}$; $\frac{96}{32}$; $\frac{103}{32}$.
44. 7 ; 16 ; 28.

Exercise 103. Page 196

1. $\frac{9}{24}$; $\frac{10}{24}$.
2. $\frac{20}{24}$; $\frac{21}{24}$.
3. $\frac{20}{32}$; $\frac{3}{32}$.
4. $\frac{35}{40}$; $\frac{32}{40}$.
5. $\frac{20}{48}$; $\frac{21}{48}$.
6. $\frac{28}{48}$; $\frac{27}{48}$.
7. $\frac{44}{48}$; $\frac{45}{48}$.
8. $\frac{45}{48}$; $\frac{38}{48}$.
9. $\frac{26}{32}$; $\frac{29}{32}$.
10. $\frac{52}{96}$; $\frac{15}{96}$.
11. $\frac{21}{96}$; $\frac{22}{96}$.
12. $\frac{45}{96}$; $\frac{58}{96}$.
13. $\frac{4}{8}$; $\frac{6}{8}$; $\frac{7}{8}$.
14. $\frac{8}{24}$; $\frac{6}{24}$; $\frac{15}{24}$.
15. $\frac{16}{24}$; $\frac{6}{24}$; $\frac{9}{24}$.
16. $\frac{16}{24}$; $\frac{18}{24}$; $\frac{15}{24}$.
17. $\frac{24}{120}$; $\frac{15}{120}$; $\frac{10}{120}$.
18. $\frac{48}{120}$; $\frac{45}{120}$; $\frac{50}{120}$.
19. $\frac{72}{120}$; $\frac{105}{120}$; $\frac{70}{120}$.
20. $\frac{15}{120}$; $\frac{12}{120}$; $\frac{10}{120}$.
21. $\frac{45}{120}$; $\frac{84}{120}$; $\frac{70}{120}$.
22. $\frac{45}{120}$; $\frac{36}{120}$; $\frac{50}{120}$.
23. $\frac{30}{48}$; $\frac{15}{48}$; $\frac{14}{48}$.
24. $\frac{42}{48}$; $\frac{21}{48}$; $\frac{14}{48}$.
25. $\frac{6}{12}$; $\frac{4}{12}$; $\frac{3}{12}$; $\frac{2}{12}$.
26. $\frac{6}{12}$; $\frac{8}{12}$; $\frac{3}{12}$; $\frac{10}{12}$.
27. $\frac{12}{24}$; $\frac{8}{24}$; $\frac{18}{24}$; $\frac{3}{24}$.
28. $\frac{12}{24}$; $\frac{16}{24}$; $\frac{18}{24}$; $\frac{9}{24}$.
29. $\frac{12}{24}$; $\frac{16}{24}$; $\frac{18}{24}$; $\frac{21}{24}$.
30. $\frac{8}{16}$; $\frac{12}{16}$; $\frac{10}{16}$; $\frac{3}{16}$.
31. $\frac{24}{32}$; $\frac{28}{32}$; $\frac{10}{32}$; $\frac{3}{32}$.
32. $\frac{12}{48}$; $\frac{6}{48}$; $\frac{3}{48}$; $\frac{2}{48}$.
33. $\frac{36}{48}$; $\frac{18}{48}$; $\frac{9}{48}$; $\frac{10}{48}$.
34. $\frac{40}{64}$; $\frac{4}{64}$; $\frac{6}{64}$; $\frac{1}{64}$.
35. $\frac{56}{64}$; $\frac{20}{64}$; $\frac{10}{64}$; $\frac{7}{64}$.
36. $\frac{168}{192}$; $\frac{8}{192}$; $\frac{6}{192}$; $\frac{15}{192}$.

Exercise 104. Page 197

1. $\frac{3}{4}$.
2. $1\frac{1}{4}$.
3. $\frac{5}{8}$.
4. $1\frac{1}{8}$.
5. $1\frac{3}{8}$.
6. $\frac{3}{8}$.
7. $\frac{5}{8}$.
8. $\frac{7}{8}$.
9. $1\frac{1}{8}$.
10. $1\frac{1}{8}$.
11. $1\frac{3}{8}$.
12. $1\frac{5}{8}$.
13. $\frac{5}{16}$.
14. $1\frac{3}{16}$.
15. $\frac{11}{16}$.
16. $1\frac{9}{16}$.
17. $\frac{19}{32}$.
18. $\frac{27}{32}$.
19. $\frac{27}{32}$.
20. $1\frac{9}{32}$.
21. $\frac{11}{24}$.
22. $1\frac{5}{24}$.
23. $\frac{2}{3}$.
24. $1\frac{1}{12}$.
25. $\frac{11}{48}$.
26. $\frac{25}{48}$.
27. $\frac{43}{48}$.
28. $1\frac{5}{48}$.
29. $\frac{25}{48}$.
30. $\frac{37}{48}$.
31. $1\frac{2}{7}$.
32. $\frac{31}{48}$.
33. $\frac{47}{48}$.
34. $1\frac{17}{48}$.
35. $\frac{5}{12}$.
36. $1\frac{1}{12}$.
37. $\frac{13}{64}$.
38. $\frac{83}{192}$.
39. $\frac{139}{192}$.
40. $1\frac{59}{64}$.

Exercise 105. Page 198

1. $6\frac{1}{8}$.
2. $10\frac{5}{8}$.
3. $10\frac{7}{8}$.
4. $8\frac{5}{6}$.
5. $11\frac{1}{6}$.
6. $12\frac{7}{12}$.
7. $70\frac{1}{8}$.
8. $64\frac{2}{15}$.
9. $62\frac{4}{15}$.
10. $81\frac{5}{16}$.
11. $89\frac{19}{24}$.
12. $109\frac{3}{16}$.
13. $9\frac{17}{24}$.
14. $10\frac{3}{5}$.
15. $12\frac{5}{8}$.
16. $15\frac{1}{16}$.
17. $15\frac{1}{3}$.
18. $13\frac{15}{16}$.
19. $5\frac{1}{48}$ in.
20. $478\frac{1}{4}$.
21. $11\frac{85}{96}$ qt.

Exercise 106. Page 199

1. $\frac{1}{8}$.
2. $\frac{3}{8}$.
3. $\frac{1}{8}$.
4. $\frac{3}{8}$.
5. $\frac{3}{8}$.
6. $\frac{3}{16}$.
7. $\frac{5}{16}$.
8. $\frac{3}{16}$.
9. $\frac{7}{16}$.
10. $\frac{5}{16}$.
11. $\frac{5}{16}$.
12. $\frac{13}{32}$.
13. $\frac{3}{32}$.
14. $\frac{5}{32}$.
15. $\frac{1}{32}$.
16. $\frac{31}{64}$.
17. $\frac{17}{64}$.
18. $\frac{9}{64}$.
19. $\frac{13}{64}$.
20. $\frac{17}{64}$.
21. $\frac{13}{64}$.
22. $\frac{9}{64}$.
23. $\frac{5}{64}$.
24. $\frac{19}{64}$.
25. $\frac{19}{64}$.
26. $\frac{5}{64}$.
27. $\frac{9}{64}$.
28. $\frac{13}{64}$.
29. $\frac{1}{6}$.
30. $\frac{5}{24}$.
31. $\frac{31}{72}$.
32. $\frac{25}{72}$.
33. $\frac{11}{72}$.
34. $\frac{19}{72}$.
35. $\frac{1}{9}$.
36. $\frac{1}{18}$.
37. $\frac{1}{18}$.
38. $\frac{1}{36}$.
39. $\frac{7}{144}$.
40. $\frac{17}{144}$.
41. $\frac{31}{144}$.
42. $\frac{83}{144}$.
43. $\frac{95}{144}$.
44. $\frac{33}{100}$.
45. $\frac{787}{1000}$.

Exercise 107. Page 200

1. $5\frac{5}{8}$.
2. $6\frac{1}{8}$.
3. $4\frac{1}{8}$.
4. $6\frac{3}{8}$.
5. $2\frac{1}{16}$.
6. $8\frac{7}{8}$.
7. $6\frac{5}{8}$.
8. $7\frac{7}{8}$.
9. $5\frac{7}{8}$.
10. $4\frac{7}{8}$.
11. $4\frac{1}{12}$.
12. $7\frac{11}{12}$.
13. $13\frac{11}{15}$.
14. $15\frac{13}{15}$.
15. $24\frac{53}{72}$.
16. $41\frac{1}{4}$ yd.
17. $185\frac{3}{10}$ A.
18. 1 hr. 45 min. A.M.

Exercise 108. Page 201

1. $3\frac{3}{5}$.
2. 6.
3. 10.
4. $13\frac{1}{2}$.
5. $8\frac{1}{4}$.
6. $16\frac{1}{4}$.
7. 28.
8. $6\frac{9}{16}$.
9. $13\frac{1}{8}$.
10. $24\frac{1}{2}$.
11. $67\frac{1}{2}$.
12. 88.
13. 15.
14. $27\frac{1}{2}$.
15. 42.
16. $58\frac{1}{2}$.
17. 77.
18. $112\frac{1}{2}$.
19. 216.
20. $263\frac{1}{2}$.
21. $52\frac{1}{2}$.
22. $81\frac{2}{3}$.
23. $137\frac{1}{2}$.
24. 45.
25. $66\frac{1}{2}$.
26. $115\frac{1}{2}$.
27. 143.
28. $292\frac{1}{2}$.
29. 63.
30. $114\frac{7}{12}$.
31. $\$12\frac{3}{4}$.
32. 110 mi.

Exercise 109. Page 202

1. 12.
2. 18.
3. 14.
4. $28\frac{4}{5}$.
5. 60.
6. $77\frac{1}{2}$.
7. 9.
8. 30.
9. $46\frac{2}{3}$.
10. 88.
11. $13\frac{1}{2}$.
12. 25.
13. 42.
14. $56\frac{1}{4}$.
15. 15.
16. $23\frac{1}{3}$.
17. $38\frac{1}{2}$.
18. 52.
19. $70\frac{5}{6}$.
20. $15\frac{5}{8}$.
21. $43\frac{3}{4}$.
22. 45¢; 54¢.
23. 84¢; 60¢.
24. 66¢; 33¢; 55¢.

Exercise 110. Page 203

1. $\frac{2}{3}$.
2. $\frac{3}{5}$.
3. $\frac{5}{8}$.
4. $\frac{11}{36}$.
5. $\frac{25}{48}$.
6. $\frac{5}{32}$.
7. $\frac{1}{10}$.
8. $\frac{21}{64}$.
9. $\frac{1}{2}$.
10. $\frac{3}{16}$.
11. $\frac{1}{6}$.
12. $\frac{1}{6}$.
13. $\frac{1}{40}$.
14. $\frac{1}{50}$.
15. $\frac{1}{9}$.
16. $\frac{1}{16}$.
17. $\frac{5}{12}$.
18. $\frac{9}{20}$.
19. $\frac{3}{8}$ yd.
20. $\frac{5}{12}$ A.
21. $\frac{7}{32}$ mi.
22. $\frac{5}{128}$ in.
23. $\frac{3}{5}$.
24. $\frac{2}{7}$.
25. $\frac{4}{5}$.
26. $\frac{1}{4}$.
27. $\frac{5}{16}$.
28. $\frac{2}{9}$.
29. $\frac{13}{24}$.
30. $\frac{3}{8}$.
31. $\frac{7}{27}$.
32. $\frac{1}{4}$.
33. $\frac{11}{15}$.
34. $\frac{1}{4}$.
35. $\frac{2}{5}$.
36. $\frac{1}{4}$.
37. $\frac{1}{3}$.
38. $\frac{9}{160}$.
39. $\frac{9}{40}$.
40. $\frac{105}{512}$.
41. $\frac{7}{32}$ in.
42. $\frac{7}{24}$ in.
43. $\frac{15}{128}$.
44. $\frac{9}{20}$.
45. $\$\frac{1}{2}$; $\$\frac{3}{8}$.
46. $\frac{3}{8}$ yd.
47. $\frac{59}{80}$ min.
48. $\$\frac{1}{2}$; $\$\frac{1}{4}$; $\$\frac{3}{8}$.
49. $1\frac{9}{40}$ yd.
50. $\frac{7}{12}$ yd.
51. $\frac{3}{8}$ yd.
52. $\frac{1}{16}$ yd.
53. $\frac{3}{8}$; $\frac{9}{16}$.
54. $\frac{5}{18}$ yd.; $\frac{5}{8}$ yd.
55. $\frac{9}{20}$ hr.; 27.
56. $\frac{2}{9}$.
57. $\frac{5}{8}$ in.
58. $\frac{8}{15}$.
59. $\frac{11}{20}$.
60. $\frac{5}{8}$ yd.

Exercise 111. Page 206

1. $35\frac{1}{3}$.
2. $78\frac{3}{4}$.
3. $56\frac{1}{4}$.
4. 143.
5. $86\frac{3}{4}$.
6. $76\frac{7}{8}$.
7. $341\frac{1}{4}$.
8. $133\frac{2}{3}$.
9. $76\frac{3}{4}$.
10. $244\frac{7}{12}$.
11. $32\frac{3}{8}$.
12. $51\frac{15}{16}$.
13. $101\frac{3}{4}$.
14. $317\frac{1}{2}$.
15. $471\frac{1}{2}$.
16. $90\frac{3}{16}$.
17. $296\frac{5}{8}$.
18. $775\frac{31}{32}$.
19. $554\frac{1}{4}$.
20. $584\frac{1}{16}$.
21. \$131.
22. $\$148\frac{3}{4}$.
23. \$213.
24. \$222.
25. 1410.
26. 3333.
27. $3661\frac{1}{2}$.
28. 3390.
29. 2680.
30. $4245\frac{1}{2}$.
31. $\$20\frac{1}{4}$.
32. $104\frac{3}{4}$.
33. 60¢.
34. 1386.
35. $9\frac{5}{8}$.
36. $43\frac{3}{4}$ mi.

Exercise 112. Page 207

1. 285.
2. 322.
3. 378.
4. 651.
5. $193\frac{3}{4}$.
6. $341\frac{1}{4}$.
7. 354.
8. $468\frac{3}{4}$.
9. 360.
10. $533\frac{3}{4}$.
11. 372.
12. $534\frac{5}{8}$.
13. 66.
14. 204.
15. 425.
16. 906.
17. $197\frac{1}{8}$.
18. $258\frac{3}{4}$.
19. $830\frac{1}{2}$.
20. $468\frac{7}{8}$.
21. 495.
22. 2660.
23. $1270\frac{1}{2}$.
24. $5432\frac{1}{2}$.
25. 3289.
26. 3573.
27. 3960.
28. 4825.
29. 158,477.
30. 192,054.

Exercise 113. Page 208

1. $8\frac{7}{16}$.
2. $15\frac{5}{6}$.
3. $8\frac{2}{3}$.
4. $43\frac{2}{5}$.
5. $27\frac{5}{8}$.
6. $78\frac{3}{4}$.
7. $88\frac{2}{3}$.
8. 250.
9. 250.
10. $152\frac{3}{4}$.
11. $2222\frac{2}{9}$.
12. 4020.
13. 2210.
14. $5614\frac{2}{7}$.
15. 9599.
16. \$400.
17. $121\frac{7}{8}$ mi.
18. $1\frac{35}{64}$ in.
19. $16\frac{5}{16}$ in.

Exercise 114. Page 209

1. $\frac{5}{48}$.
2. $\frac{3}{32}$.
3. $\frac{3}{64}$.
4. $\frac{1}{32}$.
5. $\frac{3}{32}$.
6. $\frac{5}{96}$.
7. $\frac{3}{80}$.
8. $\frac{3}{128}$.
9. $\frac{1}{72}$.
10. $\frac{1}{144}$.
11. $\frac{1}{150}$.
12. $\frac{1}{150}$.
13. $\frac{1}{128}$.
14. $\frac{1}{320}$.
15. $\frac{1}{144}$.
16. $\frac{1}{168}$.
17. $\frac{27}{17000}$.
18. $\frac{1}{900}$.
19. $\frac{3}{7000}$.
20. $\frac{3}{10000}$.
21. $\frac{11}{10000}$.

Exercise 115. Page 210

1. $\frac{5}{12}$.
2. $\frac{23}{35}$.
3. $\frac{17}{18}$.
4. $1\frac{3}{8}$.
5. $2\frac{1}{6}$.
6. $\frac{1}{8}$.
7. $2\frac{23}{24}$.
8. $4\frac{3}{28}$.
9. $3\frac{5}{8}$.
10. $6\frac{15}{16}$.
11. $4\frac{97}{112}$.
12. $8\frac{11}{16}$.
13. $28\frac{1}{2}$.
14. $16\frac{2}{3}$.
15. $6\frac{2}{3}$.
16. $22\frac{47}{48}$.
17. $17\frac{19}{24}$.
18. $13\frac{13}{16}$.
19. $\$\frac{3}{4}$.
20. $\$1\frac{1}{4}$.
21. $\$1\frac{1}{8}$.
22. $\$4\frac{1}{2}$.

Exercise 116. Page 211

1. 56.
2. 111.
3. 196.
4. 280.
5. 651.
6. 6.
7. 12.
8. 28.
9. 60.
10. $21\frac{7}{8}$.
11. 5.
12. 20.
13. 42.
14. 81.
15. 828.
16. 18.
17. 500.
18. 100.
19. 30.
20. 284.
21. 213.

Exercise 117. Page 212

1. $4\frac{1}{2}$.
2. $6\frac{2}{3}$.
3. $8\frac{3}{4}$.
4. 15.
5. $21\frac{1}{3}$.
6. 8.
7. $9\frac{1}{7}$.
8. $10\frac{4}{5}$.
9. 20.
10. 21.
11. $22\frac{1}{2}$.
12. 20.
13. 20.
14. 35.
15. 60.
16. 80.
17. 64.
18. $22\frac{6}{7}$.
19. $38\frac{2}{5}$.
20. 135.
21. $\frac{8}{9}$.
22. $1\frac{1}{8}$.
23. $\frac{3}{5}$.
24. $1\frac{2}{5}$.
25. $\frac{1}{2}$.
26. $\frac{6}{7}$.
27. $\frac{5}{9}$.
28. $\frac{7}{8}$.
29. $\frac{9}{14}$.
30. $\frac{15}{22}$.
31. $3\frac{1}{9}$.
32. $4\frac{8}{9}$.
33. $10\frac{1}{4}$.
34. $7\frac{1}{6}$.
35. $20\frac{1}{3}$.
36. $13\frac{4}{5}$.
37. 11.
38. $8\frac{3}{14}$.
39. $12\frac{5}{12}$.
40. $18\frac{9}{14}$.

Exercise 118. Page 213

1. $\frac{12}{29}$.
2. $1\frac{1}{19}$.
3. $1\frac{5}{13}$.
4. $1\frac{15}{41}$.
5. $1\frac{1}{7}$.
6. $\frac{8}{11}$.
7. $2\frac{2}{19}$.
8. $7\frac{1}{2}$.
9. $9\frac{3}{10}$.
10. $3\frac{21}{25}$.
11. $4\frac{8}{25}$.
12. $\frac{39}{50}$.
13. $\frac{18}{25}$.
14. $1\frac{1}{3}$.
15. $\frac{3}{10}$.
16. $\frac{9}{80}$.
17. $\frac{3}{10}$.
18. $\frac{1}{14}$.
19. $\frac{5}{62}$.
20. $\frac{9}{320}$.
21. $\frac{3}{160}$.
22. 2.
23. $3\frac{3}{4}$.
24. $1\frac{7}{8}$.
25. $1\frac{1}{3}$.
26. $\frac{1}{2}$.
27. 6.
28. $\$11\frac{1}{2}$.
29. 18.
30. \$33; $\$2\frac{3}{4}$.
31. \$69; $\$5\frac{3}{4}$.

Exercise 119. Page 214

1. $\frac{1}{3}$.
2. $\frac{1}{3}$.
3. $\frac{3}{7}$.
4. $\frac{1}{3}$.
5. $\frac{1}{5}$.
6. $\frac{1}{3}$.
7. 2.
8. 4.
9. $\frac{1}{5}$.
10. $\frac{1}{8}$.
11. $\frac{1}{16}$.
12. $\frac{1}{32}$.
13. $1\frac{1}{9}$.
14. $1\frac{1}{5}$.
15. $2\frac{4}{5}$.
16. $2\frac{2}{9}$.
17. $2\frac{4}{5}$.
18. $2\frac{7}{10}$.
19. $\frac{2}{3}$.
20. $\frac{34}{43}$.
21. 2.
22. $\frac{1}{2}$.
23. $\frac{1}{3}$.
24. $\frac{2}{3}$.
25. $\frac{1}{10}$.
26. $\frac{1}{5}$.
27. $\frac{1}{2}$.
28. $\frac{1}{4}$.
29. $\frac{3}{4}$.
30. $\frac{1}{8}$.
31. $\frac{1}{3}$.
32. $\frac{2}{3}$.
33. $\frac{1}{16}$.
34. $\frac{3}{8}$.
35. $\frac{5}{8}$.
36. $\frac{7}{8}$.
37. $\frac{1}{5}$.
38. $\frac{4}{5}$.
39. $\frac{1}{2}$.
40. $\$41\frac{1}{4}$; $\$57\frac{3}{4}$.
41. $\frac{39}{85}$; $\frac{46}{85}$.
42. $\frac{15}{16}$.
43. $\frac{3}{4}$.
44. $\frac{41}{1050}$.
45. $\frac{9}{64}$.
46. $\frac{104}{165}$.
47. $\frac{7}{10}$.
48. $\frac{2}{3}$.
49. $\frac{2}{5}$.
50. $\frac{3}{5}$; 1.

Exercise 120. Page 216

1. \$28.
2. \$72.
3. 108.
4. $21\frac{1}{2}$.
5. $304\frac{1}{4}$.
6. \$56.
7. \$96.
8. 45.
9. $42\frac{3}{4}$.
10. \$60.
11. \$100.
12. 112 ft.
13. 160 ft.
14. \$984.
15. \$1144.
16. 96.
17. 64.
18. 64.
19. 81.
20. 108.
21. 110.
22. 48.
23. 204.
24. 136.
25. 320.
26. 150.
27. 208.
28. 160.
29. 189.
30. 270.
31. 259.
32. 423.
33. 370.
34. 1400.
35. 1560.
36. 2750.
37. $\frac{15}{16}$.
38. $\frac{1}{10}$.
39. $\frac{1}{53}$.
40. $\frac{1}{56}$; $\frac{1}{112}$; $\frac{3}{112}$; $\frac{1}{32}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$.
41. $\frac{15}{58}$.
42. $\frac{1}{36}$.
43. $\frac{1}{3}$.
44. $\frac{1}{9}$.
45. $\frac{27}{200}$.
46. $\frac{1}{6}$; 6.

Exercise 121. Page 219

1. $\frac{7}{8}$.
2. $\frac{7}{16}$.
3. $\frac{9}{16}$.
4. $\frac{13}{16}$.
5. $1\frac{9}{16}$.
6. $\frac{5}{16}$.
7. $\frac{13}{16}$.
8. $1\frac{5}{16}$.
9. $1\frac{9}{16}$.
10. $1\frac{11}{16}$.
11. $\frac{23}{32}$.
12. $\frac{31}{32}$.
13. $1\frac{1}{32}$.
14. $1\frac{23}{32}$.
15. $\frac{17}{32}$.
16. $1\frac{1}{24}$.
17. $1\frac{11}{24}$.
18. $\frac{5}{6}$.
19. $1\frac{5}{6}$.
20. $\frac{21}{64}$.
21. $2\frac{103}{120}$.
22. $3\frac{1}{2}$.
23. $14\frac{1}{12}$.
24. $15\frac{5}{12}$.
25. $9\frac{11}{24}$.
26. $11\frac{17}{24}$.
27. $18\frac{7}{24}$.
28. $16\frac{13}{24}$.
29. $9\frac{1}{6}$.
30. $10\frac{11}{16}$.
31. $92\frac{13}{30}$.
32. $138\frac{13}{16}$.
33. $168\frac{1}{10}$.
34. $142\frac{2}{15}$.
35. $128\frac{2}{3}$.
36. $122\frac{49}{60}$.
37. $113\frac{11}{20}$.
38. $69\frac{49}{60}$.
39. $16\frac{3}{5}$.
40. $16\frac{5}{16}$.
41. $12\frac{29}{30}$.
42. $14\frac{27}{32}$.
43. $12\frac{13}{24}$.
44. 14.
45. $15\frac{21}{64}$.
46. $38\frac{5}{12}$.
47. $\frac{5}{8}$.
48. $\frac{1}{8}$.
49. $\frac{7}{16}$.
50. $\frac{9}{16}$.
51. $\frac{11}{32}$.
52. $\frac{23}{32}$.
53. $\frac{27}{64}$.
54. $27\frac{1}{8}$.
55. $57\frac{1}{4}$.
56. $63\frac{1}{10}$.
57. $44\frac{25}{32}$.
58. $61\frac{13}{16}$.
59. $64\frac{19}{24}$.
60. $70\frac{43}{48}$.
61. $2\frac{1}{16}$.
62. $5\frac{3}{16}$.
63. $2\frac{1}{16}$.
64. $4\frac{1}{16}$.
65. $3\frac{7}{32}$.
66. $3\frac{17}{32}$.
67. $14\frac{15}{16}$.
68. $1\frac{27}{32}$.
69. $34\frac{5}{12}$.
70. $37\frac{31}{60}$.
71. $42\frac{23}{60}$.
72. $55\frac{3}{4}$.
73. $21\frac{23}{48}$.
74. $66\frac{5}{32}$.
75. $1\frac{2}{3}$.
76. $1\frac{9}{10}$.
77. $\frac{23}{24}$.
78. $\frac{27}{40}$.
79. $\frac{5}{16}$.
80. $\frac{1}{8}$.
81. $\frac{1}{8}$.
82. $2\frac{1}{3}$.
83. $5\frac{5}{6}$.
84. $13\frac{4}{15}$.
85. $19\frac{3}{8}$.
86. 20.
87. $42\frac{1}{2}$.
88. 84.
89. $56\frac{1}{4}$.
90. $75\frac{5}{8}$.
91. $97\frac{1}{2}$.
92. 225.
93. $11\frac{1}{4}$.
94. 14.
95. 45.
96. 60.
97. 28.
98. 40.
99. 45.
100. $\frac{9}{16}$.
101. $\frac{3}{8}$.
102. $\frac{7}{18}$.
103. $\frac{7}{100}$.
104. $\frac{5}{96}$.
105. $\frac{3}{64}$.
106. $\frac{1}{24}$.
107. $\frac{1}{20}$.
108. $38\frac{1}{2}$.
109. $49\frac{1}{2}$.
110. $59\frac{1}{2}$.
111. $59\frac{17}{20}$.
112. $83\frac{3}{8}$.
113. 31,603.
114. $495\frac{1}{32}$.
115. $118\frac{3}{4}$.
116. $270\frac{11}{16}$.
117. $264\frac{17}{56}$.
118. $247\frac{1}{2}$.
119. 17,787.
120. 27,054.
121. $2345\frac{1}{8}$.
122. $8028\frac{3}{8}$.
123. $8035\frac{1}{2}$.
124. $14,708\frac{3}{5}$.
125. $17,917\frac{9}{10}$.
126. $1\frac{1}{5}$.
127. $1\frac{5}{9}$.
128. $1\frac{1}{7}$.
129. $\frac{25}{77}$.
130. $2\frac{9}{10}$.
131. $87\frac{3}{5}$.
132. $1\frac{53}{64}$.
133. $\frac{5}{6}$.
134. $1\frac{3}{32}$.
135. $\frac{7}{8}$.
136. $\frac{1}{7}$.
137. $3\frac{3}{35}$.
138. $1\frac{47}{70}$.
139. 4.
140. $\frac{9}{14}$.
141. $\frac{32}{35}$.
142. $\frac{3}{22}$.
143. $4\frac{24}{49}$.
144. 5.
145. $\frac{4}{5}$.
146. $7\frac{31}{77}$.
147. $\frac{1}{4}$.
148. $\frac{6}{25}$ cu. ft.
149. 20 sq. ft.
150. 56 sq. in.
151. $\frac{2}{5}$ cu. ft.
152. $\frac{1}{3}$ cu. in.
153. 432 sq. ft.
154. 78 sq. in.
155. $2\frac{7}{16}$.
156. $6\frac{10}{17}$.
157. $\frac{23}{1422}$.
158. $1\frac{3}{5}$.
159. $\frac{21}{121}$.
160. $\frac{1}{9}$.
161. $\frac{1}{9}$.
162. $\frac{1}{7}$.
163. $\frac{1}{2}$.
164. $\frac{856}{1675}$.
165. $\frac{1}{2}$.
166. $\frac{1}{2}$.
167. $\frac{1}{8}$.
168. $\frac{3}{8}$.
169. $\frac{1}{3}$.
170. $\frac{1}{6}$.
171. $\frac{8}{9}$.
172. $\frac{1}{9}$.
173. $\frac{1}{2}$.
174. $\frac{1}{4}$.
175. $\frac{1}{6}$.
176. $\frac{9}{71}$.
177. $\frac{20}{27}$.
178. $\frac{1}{7}$.
179. $\frac{1}{11}$.

Exercise 122. Page 222

1. $11{,}139\frac{3}{16}$.
2. $8812\frac{1}{2}$.
3. 24,990.
4. $213{,}511\frac{1}{9}$.
5. $9887\frac{5}{8}$.
6. 1824.
7. $262\frac{1}{2}$.
8. $5002\frac{113}{180}$.
9. 435.
10. $2914\frac{92}{147}$.
11. 16.
12. $1\frac{7}{8}$.
13. $\frac{9}{10}$.
14. 4.
15. 2.
16. $70\frac{5}{8}$.
17. $49\frac{1}{2}$.
18. $159\frac{1}{2}$ cu. in. larger.
19. $489\frac{1}{2}$ cu. in. larger.
20. 9120.
21. 1st; $370\frac{5}{32}$.
22. $1235\frac{1}{4}$.
23. 1st; $50\frac{55}{64}$ cu. in.
24. $869\frac{1}{2}$; $427\frac{31}{32}$; $1847\frac{11}{16}$; $1059\frac{45}{64}$.

Exercise 123. Page 224

1. $140\frac{5}{6}$.
2. $97\frac{8}{15}$.
3. $96\frac{11}{18}$.
4. $68\frac{13}{20}$.
5. $155\frac{5}{12}$.
6. $103\frac{7}{15}$.
7. $112\frac{13}{14}$.
8. $150\frac{3}{10}$.
9. $76\frac{19}{20}$.
10. $107\frac{1}{2}$.
11. $117\frac{3}{10}$.
12. $55\frac{9}{14}$.
13. $57\frac{3}{10}$.
14. $54\frac{17}{20}$.
15. $16\frac{11}{40}$.
16. $50\frac{5}{6}$.
17. $10\frac{19}{24}$.
18. $24\frac{13}{24}$.
19. $24\frac{7}{8}$.
20. $13\frac{5}{8}$.
21. $25\frac{1}{3}$.
22. 396.
23. $1741\frac{1}{2}$.
24. 2754.
25. $156\frac{1}{4}$.
26. $138\frac{3}{5}$.
27. 6684.
28. 2960.
29. 2074.
30. 4008.
31. $35\frac{14}{15}$.
32. 1128.
33. 4629.
34. $4984\frac{4}{5}$.
35. 3872.
36. 4560.
37. $22\frac{1}{10}$.
38. 1304.
39. 1.
40. $\frac{1}{7}$.
41. $\frac{3}{10}$ ft.
42. 30 ft.
43. 54 in.
44. $\frac{3}{4}$ ft.
45. $1\frac{3}{4}$ in.
46. 960 ft.
47. 54 in.
48. 69.
49. 200.
50. 259.
51. $250.
52. 57.
53. 50.
54. 72.
55. 160.
56. $3.60.
57. $\frac{1}{5}$.
58. $\frac{8}{21}$.
59. $1\frac{1}{2}$.
60. 1.
61. $3\frac{77}{81}$.
62. $\frac{3}{5}$.
63. $1\frac{1}{84}$.
64. 2.
65. $\frac{1}{2}$.
66. $1\frac{1}{7}$.
67. $1\frac{5}{16}$.
68. $\frac{10}{21}$.
69. $\frac{5}{14}$.
70. $\frac{1}{4}$.
71. $1\frac{1}{15}$.
72. $1\frac{1}{24}$.
73. $1\frac{11}{16}$.
74. $1\frac{1}{9}$.
75. $\frac{9}{10}$.
76. $1\frac{1}{5}$.
77. $\frac{7}{8}$.
78. $\frac{16}{25}$.
79. $\frac{3}{10}$.
80. $1\frac{1}{8}$.
81. $4\frac{2}{3}$.
82. $\frac{5}{16}$.
83. $\frac{12}{17}$.
84. $\frac{42}{155}$.
85. $4\frac{1}{18}$.
86. $\frac{5}{11}$.
87. $1\frac{71}{335}$.
88. $5\frac{17}{80}$.
89. $6\frac{1}{17}$.
90. $\frac{25}{77}$.
91. 2.
92. $6\frac{9}{10}$.
93. $\frac{93}{112}$.
94. $\frac{41}{111}$.
95. $1\frac{34}{333}$.
96. $\frac{39}{50}$.
97. $1\frac{1}{2}$.
98. $\frac{21}{55}$.
99. $2\frac{10}{77}$.
100. $4\frac{18}{23}$.
101. $5\frac{9}{11}$.
102. $9\frac{1}{36}$.
103. $18.38.
104. $19.
105. $34.
106. $220.
107. $165.67.
108. $126.

Exercise 124. Page 226

1. $4.
2. 80¢.
3. $1.13.
4. $1.79.
5. $1.09.
6. $19.06.
7. 1.12\frac{1}{2}$.
8. 6.87\frac{1}{2}$.
9. $7\frac{1}{5}$ mi.
10. $1.20.
11. 23.
12. $9\frac{1}{2}$¢.
13. 7¢.
14. 10 hr.
15. $2.69.
16. $2.40.
17. $92.50.
18. 23¢.
19. 28¢.
20. $40.

21. \$60.
22. \$1.12.
23. \$1.14.
24. 13 gal.; \$1.95.
25. \$4.80.
26. \$3.60.
27. \$4.50 ; $\frac{1}{3}$.
28. \$339 ; $\frac{113}{245}$.
29. \$2.43.
30. $\frac{125}{216}$.
31. $133\frac{49}{72}$ oz. ; $33\frac{121}{288}$ oz.
32. $449\frac{7}{32}$ lb.
33. $732\frac{27}{64}$ lb.
34. 231 lb. 9 oz.
35. 57.5 lb. ; $586\frac{47}{48}$ lb.
36. $38{,}994\frac{9}{64}$ lb.
37. \$7.94.
38. $8\frac{1}{2}$ mi.
39. \$0.65.
40. \$1.92.
41. \$4.40.
42. \$0.62.
43. \$0.07.
44. \$1.70.
45. \$1.57.
46. \$7.29.
47. 196.
48. $\frac{3}{5}$; 16.
49. 12.
50. $\frac{3}{10}$.
51. 12.
52. $\frac{1}{4}$.
53. 3 cups ; $1\frac{1}{2}$ cups.
54. \$0.61.
55. 312.
56. $7\frac{1}{7}$¢.
57. 1728.

Exercise 125. Page 232

1. $\frac{1}{10}$.
2. $\frac{1}{5}$.
3. $\frac{3}{10}$.
4. $\frac{2}{5}$.
5. $\frac{1}{2}$.
6. $\frac{1}{25}$.
7. $\frac{1}{20}$.
8. $\frac{3}{50}$.
9. $\frac{2}{25}$.
10. $\frac{9}{100}$.
11. 1.
12. 2.
13. 3.
14. $1\frac{1}{2}$.
15. $2\frac{3}{4}$.
16. $\frac{1}{3}$.
17. $\frac{3}{8}$.
18. $\frac{2}{3}$.
19. $\frac{5}{8}$.
20. $\frac{1}{6}$.
21. 7 ; 7 ; 70.
22. 10 ; 10 ; 100.
23. 33 ; 33 ; 330.
24. 36 ; 36 ; 360.
25. 72 ; 72 ; 720.

Exercise 126. Page 232

1. 0.25.
2. 0.3.
3. 0.5.
4. 0.75.
5. 0.8.
6. 0.125.
7. $0.33\frac{1}{3}$.
8. 0.375.
9. $0.66\frac{2}{3}$.
10. 0.875.
11. 1.25.
12. 1.5.
13. 1.75.
14. 2.5.
15. 3.75.
16. 0.375.
17. 0.165.
18. 0.348.
19. 0.625.
20. 0.783.
21. 0.5 ; 0.005.
22. 0.25 ; 0.0025.

Exercise 127. Page 233

1. 25%.
2. 75%.
3. 50%.
4. 80%.
5. 150%.
6. 250%.
7. 37.5%.
8. 62.5%.
9. $33\frac{1}{3}$%.
10. $66\frac{2}{3}$%.
11. 50% ; $12\frac{1}{2}$%.
12. 10% ; 1000%.

Exercise 128. Page 233

1. 50%.
2. 25%.
3. 75%.
4. 20%.
5. 60%.
6. $12\frac{1}{2}$%.
7. $37\frac{1}{2}$%.
8. $262\frac{1}{2}$%.
9. 375%.
10. $687\frac{1}{2}$%.
11. $33\frac{1}{3}$% ; $16\frac{2}{3}$%.
12. $6\frac{1}{4}$% ; $3\frac{1}{8}$%.

Exercise 129. Page 234

1. \$137.
2. \$93.
3. \$64.
4. \$211.
5. \$6.66.
6. \$7.45.
7. \$30.17.
8. \$23.10.
9. \$53.38.
10. \$59.25.
11. \$147.03.
12. \$244.50.
13. \$440.49.
14. 81 ; 99.
15. 132 ; 121.
16. 84 ; 112.
17. \$330 ; \$330.
18. \$1260 ; \$1260.
19. \$2700 ; \$2700.
20. \$5047 ; \$5047.
21. $\frac{1}{12}$; $8\frac{1}{3}$%.
22. $\frac{1}{8}$; $12\frac{1}{2}$%.
23. $\frac{1}{20}$; 5%.
24. $\frac{1}{3}$.
25. 20%.
26. 25% ; 50% ; 75%.
27. $\frac{1}{2}$; 15.
28. 0.52 ; 234 ; 216.
29. 0.225 ; 4140.
30. 630 men ; 840 women ; 210 boys.
31. \$3400.

Exercise 130. Page 236

1. 8.
2. 12.
3. 72.
4. 9.
5. 16.
6. $4.50.
7. $2.
8. $4.
9. $2.70.
10. $3.60.
11. 22.
12. 66.
13. 18.
14. 105.
15. 60.
16. 33.
17. 12.
18. 49.
19. $57\frac{3}{5}$.
20. $8\frac{2}{5}$.
21. $235\frac{1}{5}$ lb.
22. 60,000 lb.
23. 225.
24. 25.52.
25. 20.
26. $1480\frac{5}{8}$ lb.
27. 4.77 ft.
28. 39.37.
29. 6082.56.
30. 14; 10 in.; $163\frac{1}{3}$.

Exercise 131. Page 238

1. 60%.
2. 50%.
3. $33\frac{1}{3}\%$.
4. 20%.
5. 20%.
6. 6%.
7. $16\frac{2}{3}\%$.
8. $14\frac{2}{7}\%$.
9. $33\frac{1}{3}\%$.
10. 20%.
11. 5%.
12. 6%.
13. 5%.
14. 3%.
15. 5%.
16. $25; $16\frac{2}{3}\%$.
17. $650; 10%.
18. 10%.
19. 11%.
20. 5%; 10%.
21. $37\frac{1}{2}\%$.
22. $6\frac{1}{4}\%$.
23. 16%.
24. 8%.
25. $37\frac{1}{2}\%$.
26. 8%; 92%.
27. $53\frac{1}{3}\%$; $46\frac{2}{3}\%$.
28. $12\frac{1}{2}\%$.
29. $12\frac{1}{2}\%$.
30. 25%; 20%; 25%.

Exercise 132. Page 240

1. 6000.
2. 2000.
3. 3000.
4. 700.
5. 480.
6. 3840.
7. $1400.
8. $1500.
9. $2200.
10. $60.
11. $360.
12. $400.
13. $250.
14. $400.
15. $500.
16. $1200.
17. $7250.
18. $375.
19. 23,800.
20. 32,500; 37,700.
21. $2400.
22. 280.
23. $9.50; $8.55.
24. 20; $17\frac{1}{2}$.
25. February.
26. $3240.
27. 40.
28. $7500.
29. 840.

Exercise 134. Page 244

1. $33.
2. $50.40.
3. $86.25.
4. $28.
5. $41.04.
6. $74.75.
7. $84.96.
8. $106.25.
9. $270.
10. $431.25.
11. $783.
12. $189.88.
13. $207.58.
14. $280.
15. $661.25.
16. $867.30.
17. $560.
18. $720.
19. $810.
20. $194.40.
21. $435.40.
22. $53,521.25.
23. $33,519.50.
24. $94,062.50.
25. $51.
26. $69.70.
27. $56.16.
28. $67.50.
29. $57.20.
30. $81.60.
31. $165.
32. $623.
33. $810.
34. $675.
35. $735.
36. $530.
37. $780.
38. $720.
39. $654.
40. $169.05.
41. $21,800.
42. $18,480.
43. $75.
44. $82.
45. $68.
46. $75.
47. $82.
48. $175.
49. $825.
50. $840.
51. $575.
52. $450.
53. $969.60.
54. $842.16.
55. $296.
56. $320.
57. $460.
58. $720.
59. $260.
60. $14,725.

61. \$15,000.
62. \$16,500.
63. \$16,800.
64. \$41.25.
65. \$45.63.
66. \$661.22.
67. \$634.40.
68. \$1603.14.
69. \$41.31.
70. \$55.44.
71. \$88.70.
72. \$138.12.
73. \$130.
74. \$342.04.
75. \$352.37.
76. \$524.12.
77. \$1402.01.
78. \$623.70.
79. \$16.50.
80. \$13.80.
81. \$37.04.
82. \$14.50.
83. \$34.20.
84. \$87.87.
85. \$378.02.
86. \$315.90.
87. \$625.53.
88. \$461.50.
89. 40%, gain.
90. \$1006.25.
91. \$6000.
92. \$4647.50.
93. 20%; 75¢.
94. $38\frac{8}{9}$%.
95. \$1.
96. \$1.20; \$1.50; \$0.30; 25%.
97. 25%.
98. $45\frac{5}{11}$%.
99. CCX.
100. ACHX; HRY.
101. \$262.50.
102. \$360; 50%.
103. \$10; $33\frac{1}{3}$%.
104. \$337.50.
105. \$3.15.
106. \$15.40.
107. \$2.40, loss.
108. \$223.20.
109. \$150.
110. \$400.
111. 50%.
112. \$800, loss.
113. \$450.
114. 10%.
115. 25%.
116. 40%.
117. Neither.
118. \$112.50, loss.
119. \$50, loss.
120. \$3180.
121. The same.
122. $26\frac{9}{16}$%.

Exercise 135. Page 250

1. $3\frac{1}{3}$%.
2. 25%.
3. $13\frac{1}{3}$%.
4. 5%.
5. $11\frac{1}{9}$%.
6. 20%.
7. 60%.
8. $33\frac{1}{3}$%.
9. $16\frac{2}{3}$%.
10. $12\frac{1}{2}$%.
11. 58%.
12. 48%; 52%.
13. 75%.
14. $66\frac{2}{3}$%; 50%; $58\frac{1}{3}$%.
15. \$125.
16. \$300.
17. \$150.
18. 250 ft.
19. \$350.
20. 625.
21. 30.
22. 119.
23. \$337.
24. \$2812.50.
25. \$2110.
26. 145.
27. Yes.
28. \$90.
29. \$6250.
30. 120%; \$50.
31. 140; 259.
32. \$451.
33. \$1250.
34. 150.
35. 55 lb.
36. 61,000.
37. 7134.
38. \$76.50.
39. 3800.
40. $9\frac{5}{27}$%.
41. 390.
42. 52 ft.
43. 65.
44. \$1500.
45. 19.18 mi.
46. 54.
47. \$450; \$900.
48. 84 words.
49. 76 words.
50. 0.826%.
51. \$1512.
52. \$280.80.

Exercise 136. Page 255

1. \$25.
2. \$75.
3. \$67.50.
4. \$23.
5. \$40.50.
6. \$24.75.
7. \$25.
8. \$112.
9. \$270.
10. \$200.
11. \$157.50.
12. \$324.
13. \$437.
14. \$492.80.
15. \$658.75.
16. \$400.
17. \$630.
18. \$540.
19. \$43.75.
20. \$736.
21. \$780.20.
22. \$37.50.
23. \$3.
24. 10%.

25. $90.
26. $74.40.
27. $150.
28. $2115.
29. 8%.
30. $1350; $1255.50.
31. 9%.
32. 25%.
33. $520.
34. $16.40.
35. $39.60.
36. $385.
37. $1911.
38. $41.82.
39. $0.12.
40. $47.25.
41. $0.03.
42. 25%.
43. $385.40.
44. 6%.
45. $16.68.
46. $238.70.
47. $66.64.
48. $770; $825; $660; $858.
49. $1509.75.
50. $617.63.
51. $1750.
52. 8%.
53. The first; $0.42.
54. The second; $0.38.
55. The second; $0.14.

Exercise 137. Page 259

1. $127.93.
2. $372.97.
3. $160.06.
4. $75.02.
5. $16.12.
6. $154.79.
7. $13.58.
8. $984.50.
9. $280.87.
10. $396.55.
11. $403.36.
12. $222.86.
13. $167.96.
14. $481.01.
15. $226.41.
16. $563.77.
17. $463.24.

Exercise 138. Page 262

1. $169.20.
2. $262.20.
3. $407.40.
4. $285.60.
5. $378.
6. $162.
7. $307.80.
8. $325.92.
9. $393.66.
10. $170.10.
11. $1324.80.
12. $2358.75.
13. $194.40.
14. $51.84.
15. $51.84.
16. 20% and 10% by 15¢.
17. $40.50.
18. $528.
19. $119.88.
20. $144.
21. $480.
22. $594.81.

Exercise 139. Page 264

1. $126.
2. $907.20.
3. $164.45.
4. $53.82.
5. $846.45.
6. $24.37.
7. $1486.75.
8. $65.87.
9. $405.
10. $121.87.
11. $310.50.
12. $168.47.
13. $463.73.
14. $194.09.
15. $3317.28.
16. $3037.73.
17. $455.43.

Exercise 140. Page 266

1. 17.2%.
2. 20.1%.
3. 20.46%.
4. 21.74%.
5. 28%.
6. 37%.
7. 34$\frac{3}{8}$%.
8. 27.1%.
9. 42.625%.
10. 46%.
11. 49.6%.
12. 43%.
13. 46%.
14. 32$\frac{1}{2}$%.
15. 37%.
16. 40$\frac{15}{16}$%.
17. They are equal.
18. 24%.
19. They are equal.

Exercise 141. Page 267

1. $733.59.
2. $559.55.
3. $361.42.
4. $10.25.
5. 46.199%.
6. $650.
7. $357.20; 24%.
8. 58.96%.
9. The single discount; $4.17.
10. The trade discounts; $0.30.
11. The same.

Exercise 143. Page 270

1. $5.50.
2. $10.35.
3. $27.
4. $36.25.
5. $52.50.
6. $82.50.
7. $217.50.
8. $373.
9. $41.25.
10. $81.88.
11. $96.60.
12. $187.88.
13. $212.50.
14. $327.
15. $519.
16. $622.
17. $10.
18. $18.75.
19. $31.
20. $13.20.
21. $54.
22. $100.
23. $30.
24. $120.
25. $4794.
26. $6384.
27. $3493.
28. $392.
29. $576.
30. $807.50.
31. $1193.75.
32. $2737.
33. $5481.
34. $6176.
35. $1477.50.
36. $2172.50.
37. $3537.
38. $4620.
39. $264.48.
40. $563.50.
41. $678.21.
42. $5593.
43. $4189.50.
44. $3188.
45. $6360.
46. $1140.
47. $3104.
48. $4200.
49. $3000.
50. $247.
51. 15%.
52. $44.45.
53. 2½%.
54. 2½%.
55. 12½%.
56. $186.30.
57. $117.
58. $906.95.
59. $906.95.
60. 3%.
61. $935; 3%.
62. $473.60.
63. 5%.
64. $21.48.
65. $168.
66. 5⅓%.
67. $4.88.
68. $66.70.
69. $41.11.
70. $4750.
71. 8%.
72. 3%.
73. $25.53.
74. $93.75.
75. $18.75.
76. $824.50.
77. $42.50.
78. 255,000; $18.75.
79. $8787.50; 425,000; $31.25.
80. $786.60.
81. $0.21.
82. 1⅓¢.
83. 5%.
84. 227,500; $23,798.25.
85. 3%.
86. 3½%.
87. $4.75.
88. $308.14.
89. 5%.
90. $12,500.
91. $597.71.
92. $1680.
93. $3162; $15.81.
94. 350,000; $42,385.
95. The amounts are equal.
96. $11.44.
97. The amounts are equal.
98. $17.38.
99. $784.
100. Equal; $300.
101. $109.20; $153.40.
102. $253.20.

Exercise 145. Page 278

1. $12.
2. $12.
3. $12.
4. $36.
5. $36.
6. $36.
7. $30.
8. $45.
9. $195.
10. $27.50.
11. $42.
12. $55.20.
13. $78.75.
14. $21.20.
15. $63.75.
16. $152.40.
17. $105.
18. $231.25.
19. $15.
20. $12.50.
21. $39.
22. $22.50.
23. $56.25.
24. $87.75.
25. $79.20.
26. $121.50.
27. $88.
28. $78.
29. $107.25.
30. $97.88.
31. $58.45.
32. $63.83.
33. $78.91.
34. $47.31.
35. $367.50.
36. $491.56.
37. $15.
38. $30.
39. $24; $48; $12; $36; $60.
40. $24; $48; $12; $36; $60.
41. $12; $1; $2; $7.
42. $36; $3; $9; $30; $24.
43. $60; $72.50; $72.50; $77.50.
44. $29.
45. $29.
46. $54.
47. $60.
48. $50.
49. $52.50.
50. $52.50.
51. $85.
52. $164.
53. $101.25.
54. $114.
55. $95.
56. $156.
57. $195.
58. $247.50.
59. $206.25.
60. $396.
61. $342.
62. $285.
63. $637.50.
64. $26.16; $13.08; $8.72, $17.44; $100.28.
65. $18; $9; $15; $1.50; $49.50.
66. $23.46; $46.92; $164.22.
67. $3.04; $15.20; $139.84.
68. $21.60; $1.80; $0.90; $4.50; $9.60.
69. $7.02; $3.51; $2.34; $1.76; $0.59; $1.56.

Exercise 146. Page 280

1. $15.
2. $15.17.
3. $20.25.
4. $33.89.
5. $55.83.
6. $19.71.
7. $52.34.
8. $103.95.
9. $152.78.
10. $76.88.
11. $93.68.
12. $74.53.
13. $174.74.
14. $240.
15. $207.81.
16. $442.36.

Exercise 147. Page 281

1. 2 yr. 1 mo. 13 da.
2. 5 yr. 3 mo. 20 da.
3. 5 yr. 9 mo. 27 da.
4. 3 yr. 1 mo. 25 da.
5. 4 yr. 4 mo. 16 da.
6. $67.50.
7. $92.08.
8. $139.22.
9. $39.48.
10. $52.13.
11. $48.33.
12. $745.88.

Exercise 148. Page 283

1. Sept. 5, 1913; $303.
2. July 4, 1913; $512.50.
3. Feb. 8, 1914; $418.
4. June 7, 1913; $762.77.
5. July 14, 1914; $533.13.
6. June 15, 1914; $256.57.
7. $12.
8. $10.
9. $11.25.
10. $201.67.
11. $1218.
12. $394.28.
13. $371.
14. $386.25.
15. $825.
16. $78.75.
17. $293.75.
18. $133.75.
19. $282.08.
20. $369.69.
21. $311.25.
22. $261.25.
23. $372.75.
24. $546.75.
25. $6564.92.

Exercise 149. Page 286

1. $125.
2. $560.
3. $68.50.
4. $426.
5. $130.
6. $240.
7. $402.
8. $1042.
9. $1080.
10. 3 yr. 6 mo.
11. 2 yr. 3 mo.
12. 2 yr. 4 mo.
13. 2 yr. 6 mo.
14. 5%.
15. 4%.
16. 6%.
17. 16$\frac{2}{3}$ yr.
18. 20 yr.
19. $500.
20. $500.

Exercise 150. Page 288

1. $500.
2. 4%.
3. 2 yr.
4. 5%.
5. $400.
6. 2 yr.
7. $65.
8. $200.
9. 4%.
10. 4 yr.
11. $105.
12. $200.
13. 6%.
14. 3 yr.
15. $64.
16. $60.
17. $700.
18. 5%.
19. $84.
20. 3 yr.
21. $200.
22. 5%.
23. $120.
24. 4 yr.
25. $400.
26. 4%.
27. 3 yr.
28. $324.50.
29. 2 yr.
30. 5%.
31. 6%.
32. $316.25.

Exercise 151. Page 289

1. $4.51; $9.03.
2. $85.80; $87.97.
3. $171; $172.58.
4. $188.13; $190.46.
5. $114.58; $115.45.
6. $389.58; $392.26.
7. $490.07; $490.77.
8. $304.03; $305.35.
9. $180.25; $182; $187.25.
10. $243.11; $247.61.
11. $635.68; $698.18.
12. $1267.01; $1392.01.
13. $2384.14; $2759.51.
14. $48.13.
15. $593.75.
16. $88.73.
17. $38.96.
18. $21.95.
19. $25.08.
20. $105.08.
21. $268.54; $219.71; $292.95.
22. $63.56.
23. 5%.
24. 4%.
25. 4%.
26. 6%.
27. 4%.
28. 3%.
29. 3%.
30. 6%.
31. 5%.
32. 4%.
33. 4%.
34. $3\frac{1}{2}$%.
35. $5\frac{1}{2}$%.
36. $3\frac{1}{2}$%.
37. 2 yr. 6 mo.
38. 2 yr. 7 mo.
39. 1 yr. 9 mo.
40. 1 yr. 5 mo. 3 da.
41. 1 yr. 2 da.
42. 2 yr. 6 mo.
43. 1 yr. 8 mo.
44. 1 yr. 6 mo.
45. 2 yr.
46. 4 mo.
47. 25 yr.
48. 2 yr. 6 mo.
49. 2 yr. 6 mo.
50. 6 mo.; 2 yr. 1 mo. 28 da.; 2 yr. 7 mo. 28 da.
51. 11 yr.
52. 14 yr.
53. The first; $3.13.
54. 5%.
55. $30.
56. $55.
57. $405.
58. $1035.
59. $3327.50.
60. $5302.50.
61. $19\frac{9}{35}$%.